Constructive Computer Architecture

Combinational circuits

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Content

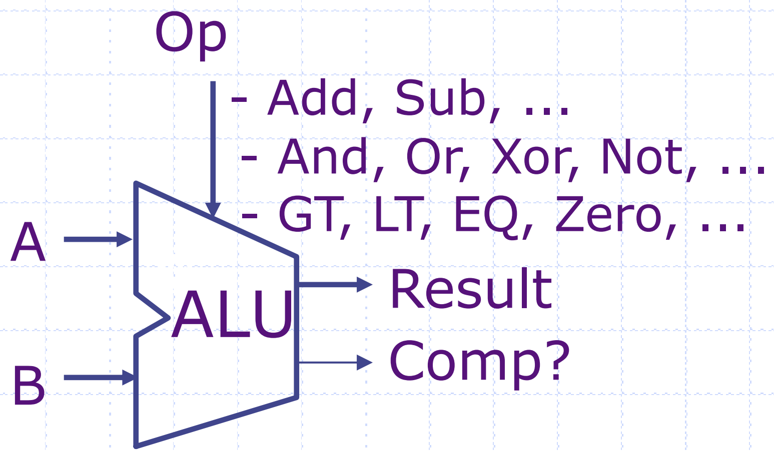
- Design of a combinational ALU starting with primitive gates And, Or and Not
- Combinational circuits as acyclic wiring diagrams of primitive gates
- Introduction to BSV
  - Intro to types – enum, typedefs, numeric types, int#(32) vs integer, bool vs bit#(1), vectors
  - Simple operations: concatenation, conditionals, loops
  - Functions
  - Static elaboration and a structural interpretation of the textual code

Combinational circuits

Combinational circuits are acyclic interconnections of gates

- And, Or, Not
- Nand, Nor, Xor
- …

Arithmetic-Logic Unit (ALU)

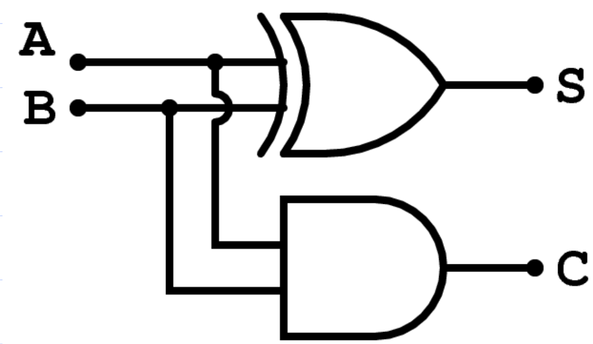


ALU performs all the arithmetic and logical functions

Each individual function can be described as a combinational circuit

Half Adder

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



Boolean equations

s = (~a·b) + (a·~b)

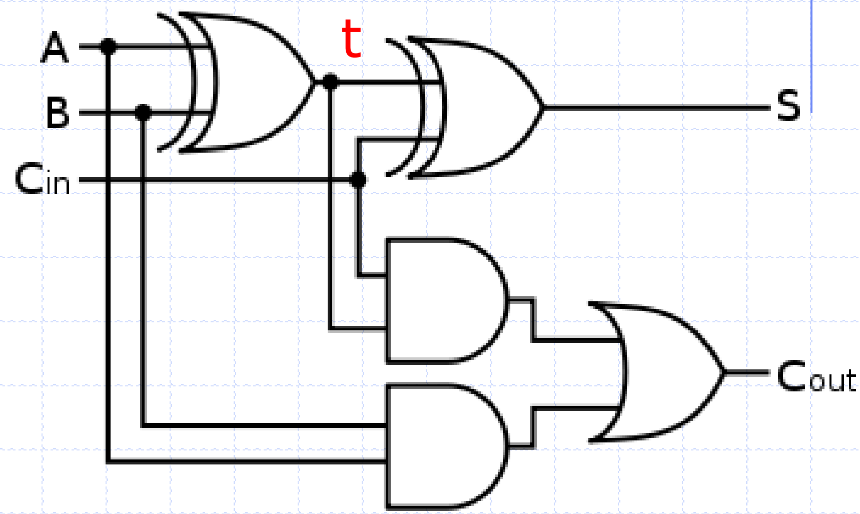
c = a·b

"Optimized"

s = a ⊕ b

Full Adder

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Boolean equations

$s = (\sim a \cdot \sim b \cdot c_{in}) + (\sim a \cdot b \cdot \sim c_{in}) + (a \cdot \sim b \cdot \sim c_{in}) + (a \cdot b \cdot c_{in})$

$c_{out} = (\sim a \cdot b \cdot c_{in}) + (a \cdot \sim b \cdot c_{in}) + (a \cdot b \cdot \sim c_{in}) + (a \cdot b \cdot c_{in})$

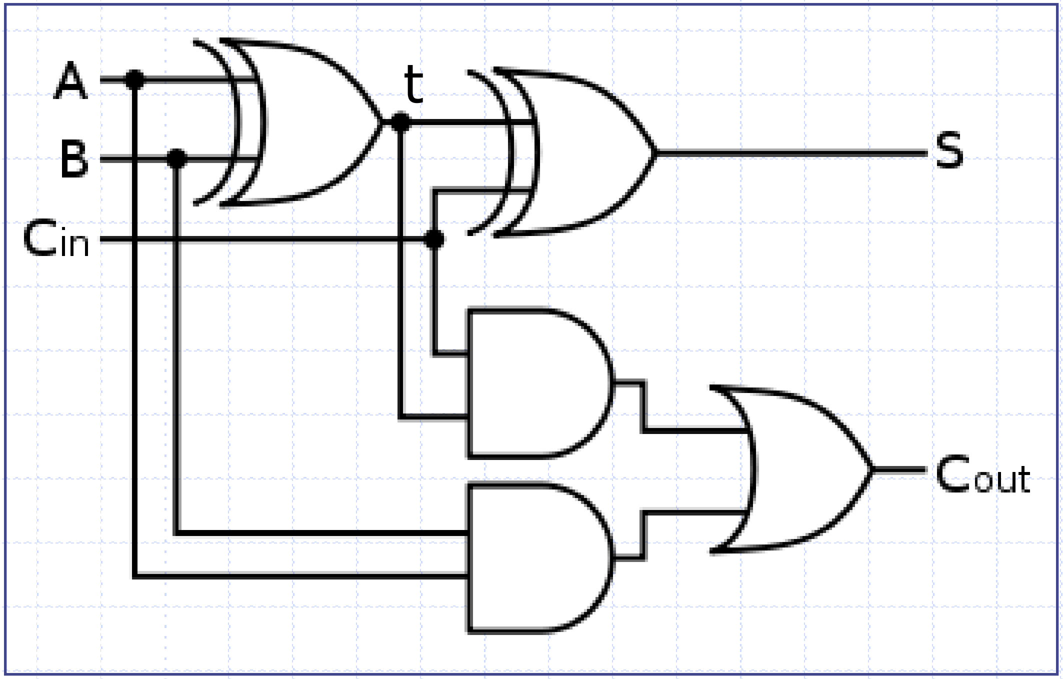
"Optimized"

$t = a \oplus b$ $\quad s = t \oplus c_{in}$ $\quad c_{out} = a \cdot b + c_{in} \cdot t$

Full Adder: A one-bit adder

```
function fa(a, b, c_in);
    t = (a ^ b);
    s = t ^ c_in;
    c_out = (a & b) | (c_in & t);
    return {c_out,s};
endfunction
```

Structural code – only specifies interconnection between boxes

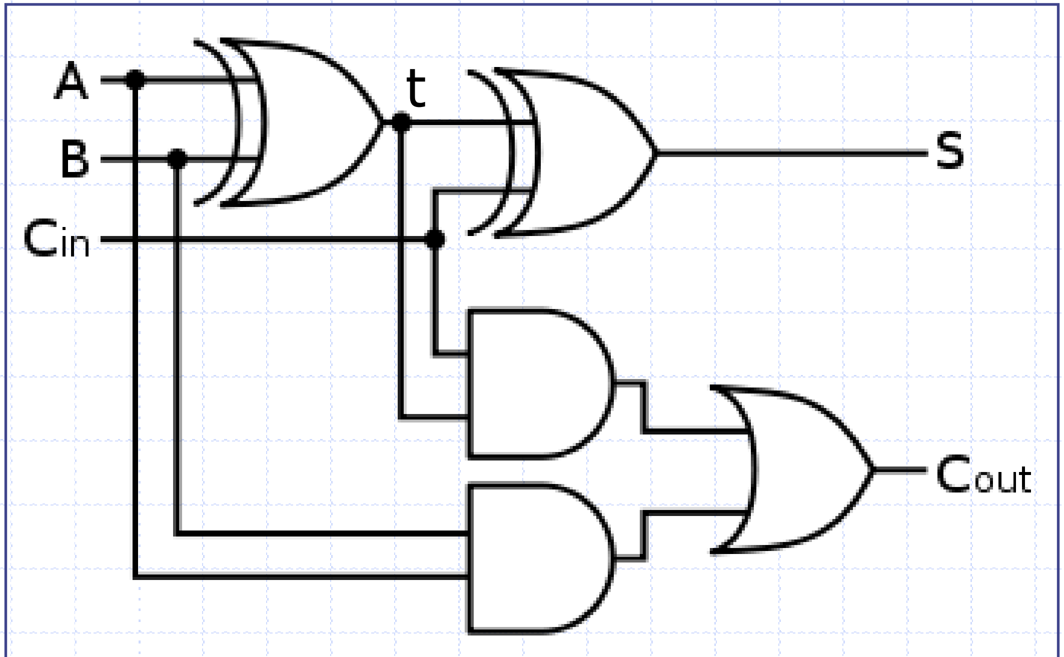


Not quite correct – needs type annotations

Full Adder: A one-bit adder

corrected

```
function Bit#(2) fa(Bit#(1) a, Bit#(1) b,
                                        Bit#(1) c_in);
   Bit#(1) t = a ^ b;
   Bit#(1) s = t ^ c_in;
   Bit#(1) c_out = (a & b) | (c_in & t);
   return {c_out,s};
endfunction
```



"`Bit#(1) a`" type declaration says that `a` is one bit wide

`{c_out,s}` represents bit concatenation

How big is `{c_out,s}`?

2 bits

Types

- A type is a grouping of values:
  - Integer: `1, 2, 3, …`
  - Bool: `True, False`
  - Bit: `0,1`
  - A pair of Integers: `Tuple2#(Integer, Integer)`
  - A function `fname` from Integers to Integers:

```
function Integer fname (Integer arg)
```

- Every expression in a BSV program has a type; sometimes it is specified explicitly and sometimes it is deduced by the compiler
- Thus we say an expression has a type or belongs to a type

The type of each expression is unique

Parameterized types:

- A type declaration itself can be parameterized by other types
- Parameters are indicated by using the syntax '#'
  - For example `Bit#(n)` represents n bits and can be instantiated by specifying a value of n

    `Bit#(1), Bit#(32), Bit#(8), …`

Type synonyms

```
typedef bit [7:0] Byte;

typedef Bit#(8) Byte;
```

The same

```
typedef Bit#(32) Word;

typedef Tuple2#(a,a) Pair#(type a);

typedef Int#(n) MyInt#(type n);

typedef Int#(n) MyInt#(numeric type n);
```

The same

Type declaration versus deduction

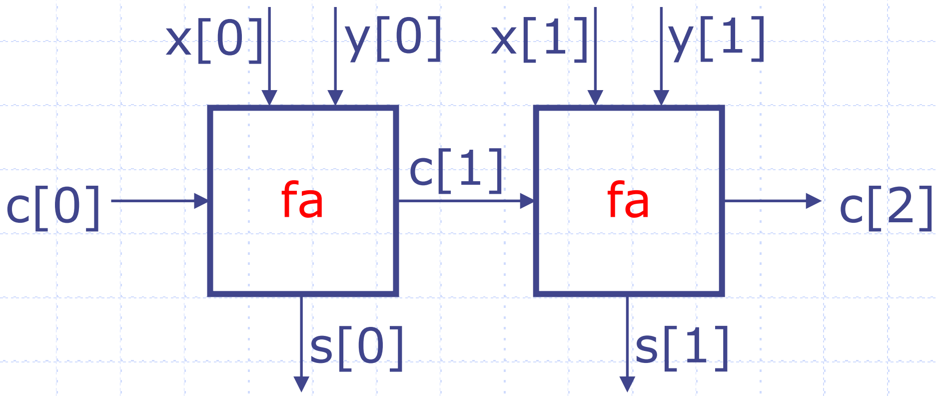
- The programmer writes down types of some expressions in a program and the compiler deduces the types of the rest of expressions
- If the type deduction cannot be performed or the type declarations are inconsistent then the compiler complains

```
function Bit#(2) fa(Bit#(1) a, Bit#(1) b,
                              Bit#(1) c_in);
   Bit#(1) t = a ^ b;
   Bit#(1) s = t ^ c_in;
   Bit#(2) c_out = (a & b) | (c_in & t);
   return {c_out,s};
endfunction
```

type error

Type checking prevents lots of silly mistakes

2-bit Ripple-Carry Adder



fa can be used as a black-box as long as we understand its type signature

```
function Bit#(3) add(Bit#(2) x, Bit#(2) y,
                                            Bit#(1) c0);
   Bit#(2) s = 0;    Bit#(3) c=0; c[0] = c0;
   let cs0 = fa(x[0], y[0], c[0]);
             c[1] = cs0[1];  s[0] = cs0[0];
   let cs1 = fa(x[1], y[1], c[1]);
             c[2] = cs1[1];  s[1] = cs1[0];
  return {c[2],s};
endfunction
```

The "let" syntax avoids having to write down types explicitly

"let" syntax

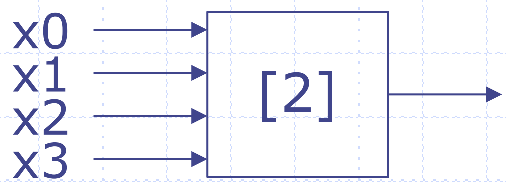
- The "let" syntax: avoids having to write down types explicitly
  - `let cs0 = fa(x[0], y[0], c[0]);`
  - `Bits#(2) cs0 = fa(x[0], y[0], c[0]);`

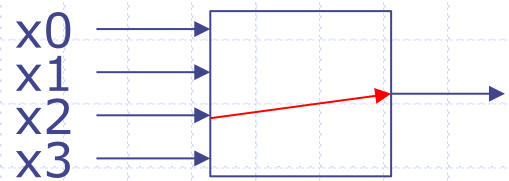
The same

Selecting a wire: x[i]

assume x is 4 bits wide

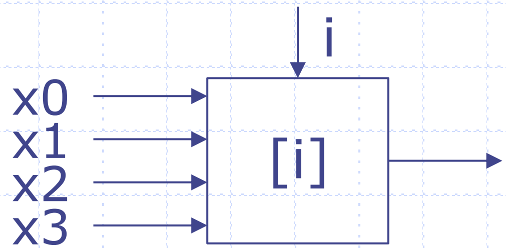
- Constant Selector: e.g., x[2]

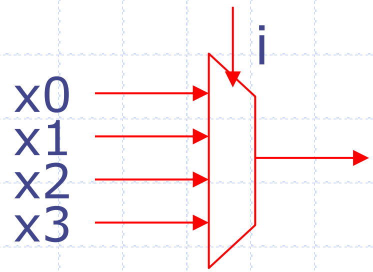




no hardware; x[2] is just the name of a wire

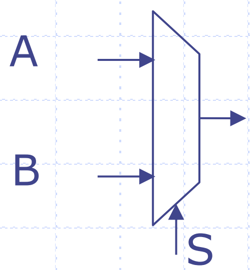
- Dynamic selector: x[i]

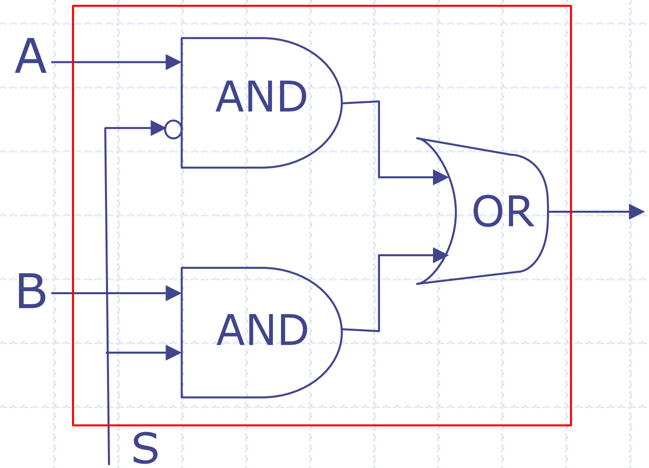




4-way mux

A 2-way multiplexer





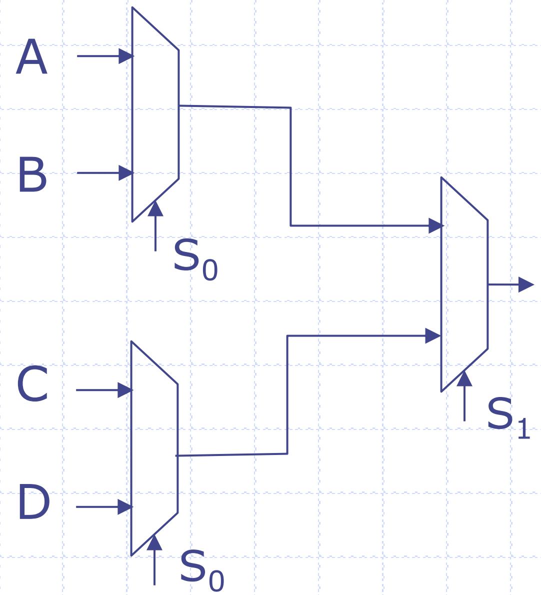
`(s==0)? A : B`

Gate-level implementation

Conditional expressions are also synthesized using muxes

A 4-way multiplexer

```
case {s1,s0} matches
    0:  A;
    1:  B;
    2:  C;
    3:  D;
endcase
```

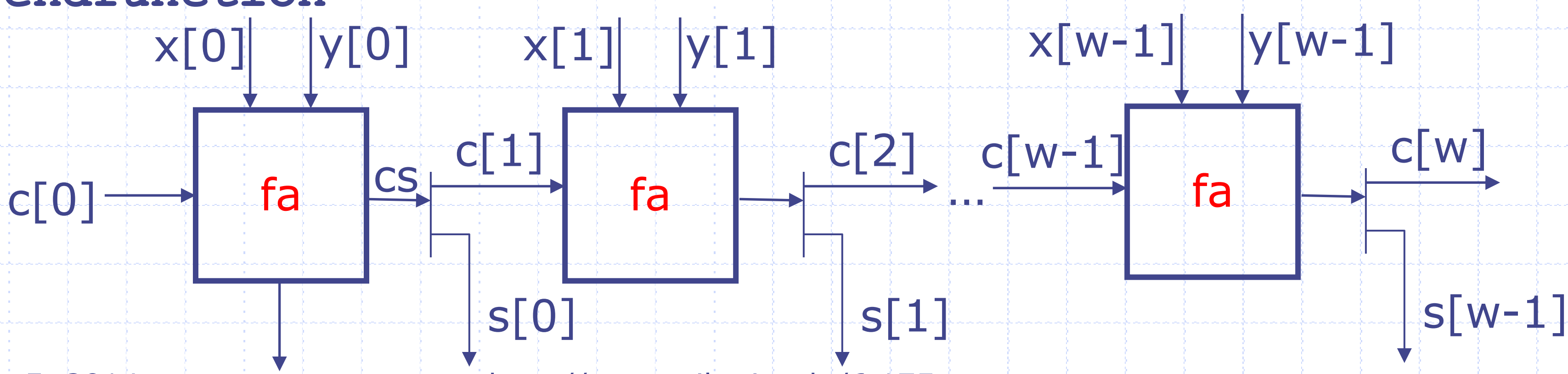


An w-bit Ripple-Carry Adder

```
function Bit#(w+1) addN(Bit#(w) x, Bit#(w) y,
                                    Bit#(1) c0);
   Bit#(w) s; Bit#(w+1) c=0; c[0] = c0;
   for(Integer i=0; i<w; i=i+1)
   begin
      let cs = fa(x[i],y[i],c[i]);
      c[i+1] = cs[1]; s[i] = cs[0];
   end
return {c[w],s};
endfunction
```

Not quite correct

Unfold the loop to get the wiring diagram



Instantiating the parametric Adder

```
function Bit#(w+1) addN(Bit#(w) x, Bit#(w) y,
                                    Bit#(1) c0);
```

Define `add32, add3` … using `addN`

```
// concrete instances of addN!
function Bit#(33) add32(Bit#(32) x, Bit#(32) y,
                        Bit#(1) c0) = addN(x,y,c0);


function Bit#(4) add3(Bit#(3) x, Bit#(3) y,
                        Bit#(1) c0) = addN(x,y,c0);
```

`valueOf(w)` versus `w`

- Each expression has a type and a value and these come from two entirely disjoint worlds
- `w` in `Bit#(w)` resides in the types world
- Sometimes we need to use values from the types world into actual computation. The function `valueOf` allows us to do that
  - Thus

    `i<w` is not type correct

    `i<valueOf(w)` is type correct

`TAdd#(w,1)` versus `w+1`

- Sometimes we need to perform operations in the types world that are very similar to the operations in the value world
  - Examples: `Add`, `Mul`, `Log`
- We define a few special operators in the types world for such operations
  - Examples: `TAdd#(m,n)`, `TMul#(m,n)`, …

A w-bit Ripple-Carry Adder
corrected

```
function Bit#(TAdd#(w,1)) addN(Bit#(w) x, Bit#(w) y,
                                         Bit#(1) c0);
   Bit#(w) s; Bit#(TAdd#(w,1)) c; c[0] = c0;
   let valw = valueOf(w);
   for(Integer i=0; i<valw; i=i+1)
   begin
      let cs = fa(x[i],y[i],c[i]);
      c[i+1] = cs[1]; s[i] = cs[0];
   end
return {c[valw],s};
endfunction
```

types world equivalent of w+1

Lifting a type into the value world

Structural interpretation of a loop – unfold it to generate an acyclic graph

Static Elaboration phase

- When BSV programs are compiled, first type checking is done and then the compiler gets rid of many constructs which have no direct hardware meaning, like Integers, loops

```
for(Integer i=0; i<valw; i=i+1) begin
   let cs = fa(x[i],y[i],c[i]);
   c[i+1] = cs[1]; s[i] = cs[0];
end
```

```
cs0 = fa(x[0], y[0], c[0]); c[1]=cs0[1]; s[0]=cs0[0];
cs1 = fa(x[1], y[1], c[1]); c[2]=cs1[1]; s[1]=cs1[0];
…
csw = fa(x[valw-1], y[valw-1], c[valw-1]);
      c[valw] = csw[1]; s[valw-1] = csw[0];
```

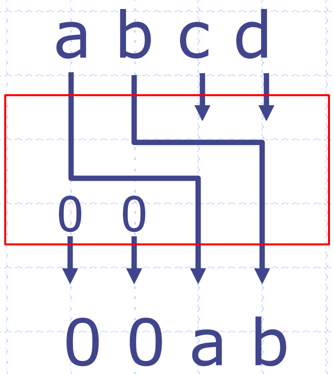
Integer versus Int#(32)

- In mathematics integers are unbounded but in computer systems integers always have a fixed size
- BSV allows us to express both types of integers, though unbounded integers are used only as a programming convenience

```
for(Integer i=0; i<valw; i=i+1)
   begin
      let cs = fa(x[i],y[i],c[i]);
      c[i+1] = cs[1]; s[i] = cs[0];
   end
```

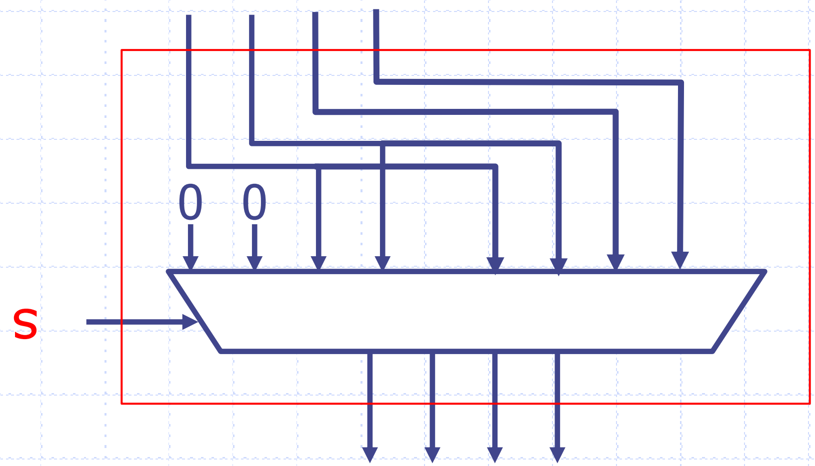
Shift operators

Logical right shift by 2



- Fixed size shift operation is cheap in hardware – just wire the circuit appropriately
- Rotate, sign-extended shifts – all are equally easy

Conditional operation: shift versus no-shift

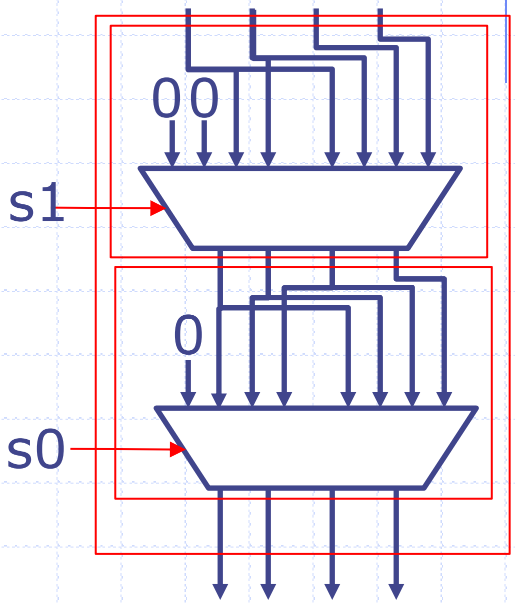


- We need a mux to select the appropriate wires: if s is one the mux will select the wires on the left otherwise it would select wires on the right

```
(s==0)?{a,b,c,d}:{0,0,a,b};
```

Logical right shift by *n*

- Shift *n* can be broken down in log *n* steps of fixed-length shifts of size 1, 2, 4, …
  - Shift 3 can be performed by doing a shift 2 and shift 1
- We need a mux to omit a particular size shift
- Shift circuit can be expressed as log *n* nested conditional expressions



A digression on types

- Suppose we have a variable c whose values can represent three different colors
  - We can declare the type of c to be Bit#(2) and say that 00 represents Red, 01 Blue and 10 Green
- A better way is to create a new type called `Color` as follows:

```
typedef enum {Red, Blue, Green}
Color deriving(Bits, Eq);
```

Types prevent us from mixing bits that represent color from raw bits

The compiler will automatically assign some bit representation to the three colors and also provide a function to test if the two colors are equal. If you do not use "deriving" then you will have to specify the representation and equality

Enumerated types

```
typedef enum {Red, Blue, Green}
Color deriving(Bits, Eq);

typedef enum {Eq, Neq, Le, Lt, Ge, Gt, AT, NT}
BrFunc deriving(Bits, Eq);

typedef enum {Add, Sub, And, Or, Xor, Nor, Slt, Sltu,
LShift, RShift, Sra}
AluFunc deriving(Bits, Eq);
```

Each enumerated type defines a new type

Combinational ALU

```
function Data alu(Data a, Data b, AluFunc func);
  Data res = case(func)
     Add   : (a + b);
     Sub   : (a - b);
     And   : (a & b);
     Or    : (a | b);
     Xor   : (a ^ b);
     Nor   : ~(a | b);
     Slt   : zeroExtend( pack( signedLT(a, b) ) );
     Sltu  : zeroExtend( pack( a < b ) );
     LShift: (a << b[4:0]);
     RShift: (a >> b[4:0]);
     Sra   : signedShiftRight(a, b[4:0]);
  endcase;
  return res;
endfunction
```

Given an implementation of the primitive operations like `addN`, `Shift`, `etc.` the ALU can be implemented simply by introducing a mux controlled by `op` to select the appropriate circuit

Comparison operators

```
function Bool aluBr(Data a, Data b, BrFunc brFunc);
   Bool brTaken = case(brFunc)
     Eq  : (a == b);
     Neq : (a != b);
     Le  : signedLE(a, 0);
     Lt  : signedLT(a, 0);
     Ge  : signedGE(a, 0);
     Gt  : signedGT(a, 0);
     AT  : True;
     NT  : False;
   endcase;
   return brTaken;
 endfunction
```

ALU including Comparison operators