Constructive Computer Architecture:

# Multirule systems and Concurrent Execution of Rules

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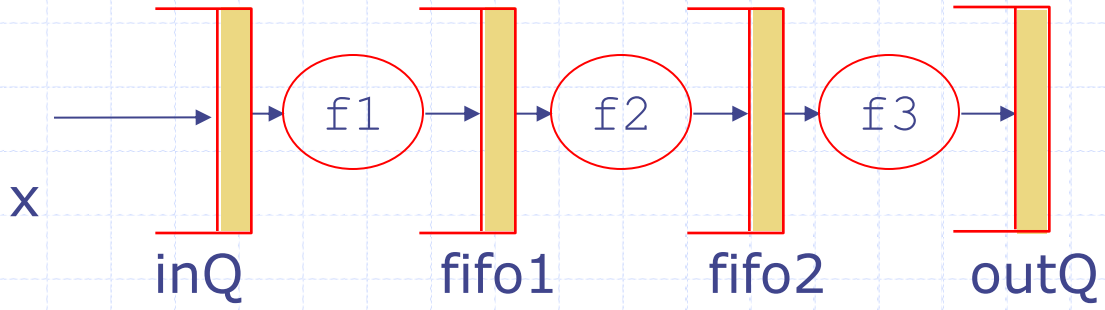
# Rewriting Elastic pipeline as a multirule system




```
rule stage1;
  if(inQ.notEmpty && fifo1.notFull)
    begin fifo1.enq(f1(inQ.first)); inQ.deq; end endrule
rule stage2;
  if(fifo1.notEmpty && fifo2.notFull)
    begin fifo2.enq(f2(fifo1.first)); fifo1.deq; end endrule
rule stage3;
  if(fifo2.notEmpty && outQ.notFull)
    begin outQ.enq(f3(fifo2.first)); fifo2.deq; end endrule
```

- How does such a system function?

# Bluespec Execution Model

*Repeatedly:*

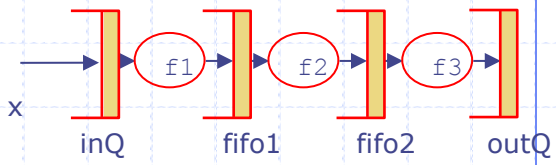
- Select a rule to execute
- Compute the state updates
- Make the state updates

Highly non-deterministic; User annotations can be used in rule selection

One-rule-at-a-time-semantics: Any legal behavior of a Bluespec program can be explained by observing the state updates obtained by applying only one rule at a time

However, for performance we need to execute multiple rules concurrently if possible

# Multi-rule versus single rule elastic pipeline




```
rule ArithPipe;
  if(inQ.notEmpty && fifo1.notFull)
    begin fifo1.enq(f1(inQ.first)); inQ.deq; end
  if(fifo1.notEmpty && fifo2.notFull)
    begin fifo2.enq(f2(fifo1.first)); fifo1.deq; end
  if(fifo2.notEmpty && outQ.notFull)
    begin outQ.enq(f3(fifo2.first)); fifo2.deq; end
endrule
```

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rule stage1;
  if(inQ.notEmpty && fifo1.notFull)
    begin fifo1.enq(f1(inQ.first)); inQ.deq; end endrule
rule stage2;
  if(fifo1.notEmpty && fifo2.notFull)
    begin fifo2.enq(f2(fifo1.first)); fifo1.deq; end endrule
rule stage3;
  if(fifo2.notEmpty && outQ.notFull)
    begin outQ.enq(f3(fifo2.first)); fifo2.deq; end endrule
```

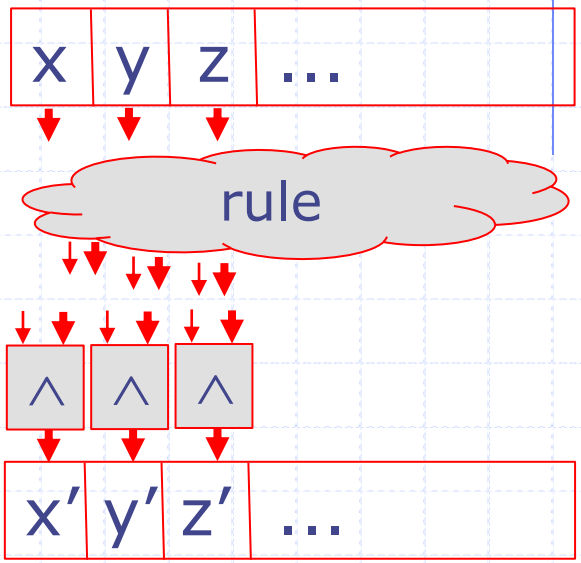
*How are these two systems the same (or different)?*

# Elastic pipeline

- Do the system see all the same state changes?
  - The single rule system – fills up the pipeline and then processes a message at every pipeline stage for every rule firing – no more than one slot in any fifo would be filled unless the OutQ blocks
  - The multirule system has many more possible states. It can mimic the behavior of one-rule system but one can also execute rules in different orders, e.g., stage1; stage1; stage2; stage1, stage3, stage2,stage3, … (assuming stage fifos have more than one slot)
- When can some or all the rules in a multirule system execute concurrently?

# Evaluating or applying a rule

- The state of the system s is defined as the value of all its registers
- An *expression* is evaluated by computing its value on the current state
- An *action* defines the next value of some of the state elements based on the current value of the state
- A *rule* is evaluated by evaluating the corresponding action and simultaneously updating all the affected state elements




Given action *a* and state S, let *a*(S) represent the state after the application of action *a*

# One-rule-at-a-time semantics

- Given a program with a set of rules {rule $r_i$ $a_i$} and an initial state $S_0$ , S is a legal state if and only if there exists a sequence of rules $r_{j1}$,…., $r_{jn}$ such that S= $a_{jn}$(…($a_{j1}$($S_0$))…)

# Concurrent scheduling of rules

- rule $r_1$ $a_1$ and rule $r_2$ $a_2$ can be scheduled concurrently, preserving one-rule-at-a-time semantics, if and only if
  - Either $\forall S.\ (a_1; a_2)(S) = a_2(a_1(S))$
    or $\forall S.\ (a_1; a_2)(S) = a_1(a_2(S))$

- rule $r_1$ $a_1$ to rule $r_n$ $a_n$ can be scheduled concurrently, preserving one-rule-at-a-time semantics, if and only if there exists a permutation $(p_1,\ldots,p_n)$ of $(1,\ldots,n)$ such that
  - for all S. $(a_1;\ldots;a_n)(S) = a_{pn}(\ldots(a_{p1}(S))$

# Extending CM to rules

- CM between two rules is computed exactly the same way as CM for the methods of a module
  - Given rule r1 a1 and rule r2 a2 such that

    mcalls(a1)={g11,g12...g1n}

    mcalls(a2)={g21,g22...g2m}
  - Compute
    - CM[r1,r2] = conflict(g11,g21) ∩ conflict(g11,g22) ∩...
      ∩ conflict(g12,g21) ∩ conflict(g12,g22) ∩...
      ...
      ∩ conflict(g1n,g21) ∩ conflict(g12,g22) ∩...
    - Conflict(x,y) = if x and y are methods of the same module then CM[x,y] else CF

# Using CMs for concurrent scheduling of rules

Theorem: Given rule $r_1$ $a_1$ … rule $r_n$ $a_n$, if there exists a permutation $p_1$, $p_2$ … $p_n$ such that

$$\forall\ i < j.\ CM(a_{pi}, a_{pj}) \text{ is CF or } <$$

then $\forall\ S.\ (a_1|\ldots|a_n)(S) = a_{pn}(\ldots(a_{p1}(S)))$.

Thus rules $r_1$, $r_2$ … $r_n$ can be scheduled concurrently with the effect $\forall\ i, j.\ r_{pi}$ happens before $r_{pj}$

# Example 1: Compiler Analysis

```
rule ra;
   if (z>10)
      x <= x+1;
endrule

rule rb;
   if (z>20)
   y <= y+2;
endrule
```

mcalls(ra) = {z.r, x.w, x.r}
mcalls(rb) = {z.r, y.w, y.r}

CM(ra, rb) =
conflict(z.r, z.r) ∩ conflict(z.r, y.w)
∩ conflict(z.r, y.r) ∩ conflict(x.w, z.r)
∩ conflict(x.w, y.w) ∩ conflict(x.w, y.r)
∩ conflict(x.r, z.r) ∩ conflict(x.r, y.w)
∩ Conflict(x.r, y.r)
= CF ∩ CF ∩ CF ∩ CF … = CF

Rules ra and rb can be scheduled together without violating the one-rule-at-a-time-semantics. We say rules ra and rb are CF

# Example 2: Compiler Analysis

```
rule ra;
   if (z>10)
   x <= y+1;
endrule

rule rb;
   if (z>20)
   y <= x+2;
endrule
```

mcalls(ra) = {z.r, x.w, y.r}
mcalls(rb) = {z.r, y.w, x.r}

CM(ra, rb) =
  conflict(z.r, z.r) ∩ conflict(z.r, y.w)
∩ conflict(z.r, x.r) ∩ conflict(x.w, z.r)
∩ conflict(x.w, y.w) ∩ conflict(x.w, x.r)
∩ conflict(y.r, z.r) ∩ conflict(y.r, y.w)
∩ Conflict(y.r, x.r)
= CF ∩ CF
∩ CF ∩ CF
∩ CF ∩ >
∩ CF ∩ <
∩ CF = C

Rules ra and rb cannot be scheduled together without violating the one-rule-at-a-time-semantics. Rules ra and rb are C

# Example 3: Compiler Analysis

```
rule ra;
 if (z>10)
   x <= y+1;
endrule

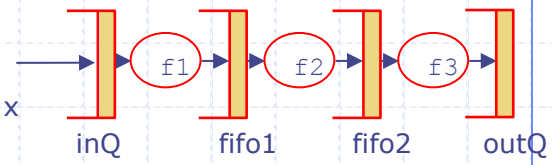
rule rb;
 if (z>20)
   y <= y+2;
endrule
```

mcalls(ra) = {z.r, x.w, y.r}
mcalls(rb) = {z.r, y.w, y.r}

CM(ra, rb) =
  conflict(z.r, z.r) ∩ conflict(z.r, y.w)
∩ conflict(z.r, y.r) ∩ conflict(x.w, z.r)
∩ conflict(x.w, y.w) ∩ conflict(x.w, y.r)
∩ conflict(y.r, z.r) ∩ conflict(y.r, y.w)
∩ Conflict(y.r, y.r)
= CF ∩ CF
∩ CF ∩ CF
∩ CF ∩ CF
∩ CF ∩ <
∩ CF = <

Rules ra and rb can be scheduled together without violating the one-rule-at-a-time-semantics. Rule ra < rb

# Multi-rule versus single rule elastic pipeline

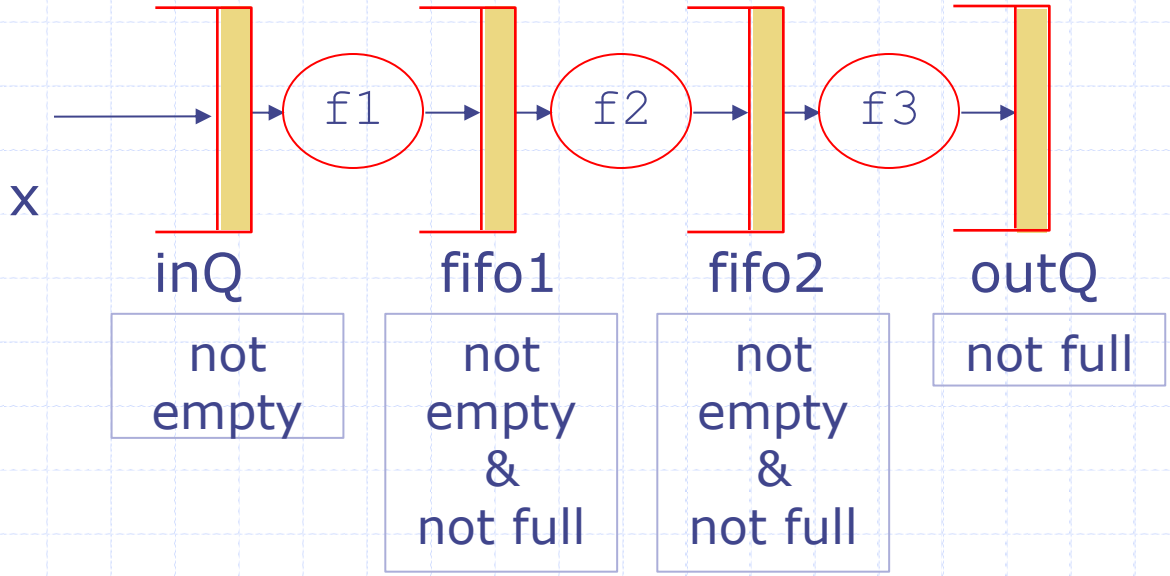



```
rule ArithPipe;
      if(inQ.notEmpty && fifo1.notFull)
        (fifo1.enq(f1(inQ.first) ; inQ.deq)
    ; if(fifo1.notEmpty && fifo2.notFull)
        (fifo2.enq(f2(fifo1.first) ; fifo1.deq)
    ; if(fifo2.notEmpty && outQ.notFull)
        (outQ.enq(f3(fifo2.first) ;fifo2.deq)
```

```
rule stage1;
  if(inQ.notEmpty && fifo1.notFull)
     (fifo1.enq(f1(inQ.first) ; inQ.deq) endrule;
rule stage2;
  if(fifo1.notEmpty && fifo2.notFull)
     (fifo2.enq(f2(fifo1.first) ; fifo1.deq) endrule;
rule stage3;
  if(fifo2.notEmpty && outQ.notFull)
     (outQ.enq(f3(fifo2.first) ;fifo2.deq) endrule;
```

If we do concurrent scheduling in the multirule system then the multi-rule system behaves like the single rule system

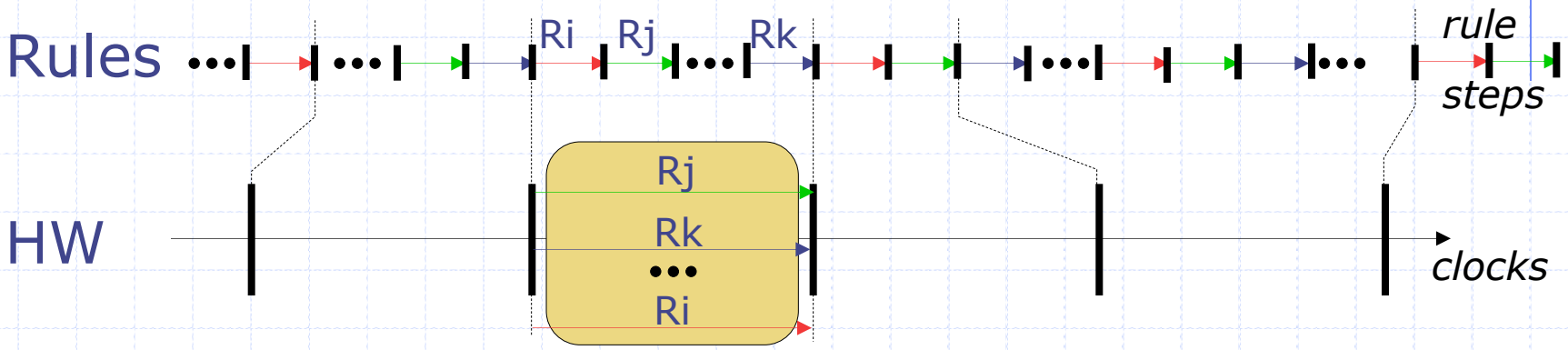
# Concurrency when the FIFOs do not permit concurrent enq and deq




At best alternate stages in the pipeline will be able to fire concurrently

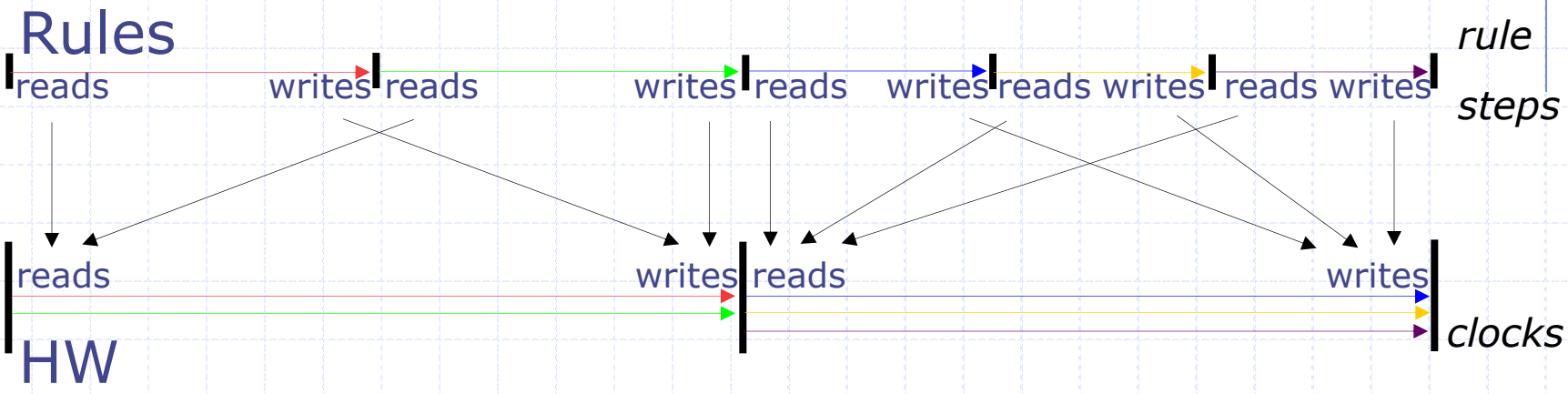
*some insight into*

# Concurrent rule firing



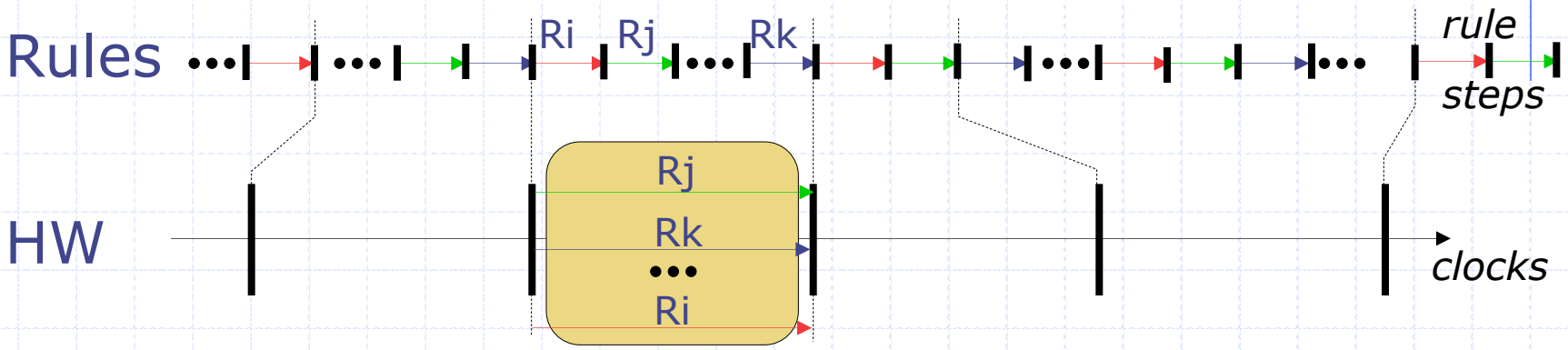

- There are more intermediate states in the rule semantics (a state after each rule step)
- In the HW, states change only at clock edges

# Parallel execution reorders reads and writes




- In the rule semantics, each rule sees (reads) the effects (writes) of previous rules
- In the HW, rules only see the effects from previous clocks, and only affect subsequent clocks

# Correctness




- The compiler will schedule rules concurrently only if the net state change is equivalent to sequential rule execution (which is what our theorem ensures)