



Bluespec-7: Semantics of Bluespec

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Topics

- ◆ Guarded actions versus Conditional actions
- ◆ Naming expressions: the “let” statement
- ◆ Modules and methods with implicit conditions
- ◆ Rule composition

BS₀ : A simple language of Guarded Atomic Actions

a is an Action;
e is an Expression;
r is a register (state variable)

action a is guarded by predicate e

conditional action

compound atomic action

a ::= r := e | a when e | if e then a | a ; a
e ::= r | c | Op(e , e) | e ? e : e | e when e | ...

conditional expression

guarded expression

“;” is commutative and associative, i.e.

$$a1;a2 = a2;a1$$

$$a1;(a2;a3) = (a1;a2);a3$$

BS₀ Program

A program is a collection of registers and rules:

R, R_1, R_2, \dots are names for rules;

r, r_1, r_2, \dots are register names

Program ::=

Registers $r_1, \dots, ;$

Rule $R_1 a_1; \dots ;$ Rule $R_m a_m ;$

$a ::= r := e \mid a \text{ when } e \mid \text{if } e \text{ then } a \mid a ; a$

$e ::= r \mid c \mid \text{Op}(r_a, r_b) \mid e ? e : e \mid e \text{ when } e \mid \dots$

Guards vs If's

- ◆ A guard on one action of a group of actions affects every action within the group
(a1 when p1); (a2 when p2)
==> (a1; a2) when p1 & p2
- ◆ A condition of a Conditional action only affects the actions within the scope of the conditional action
(if p1 then a1); (if p2 then a2)
p1 has no effect on a2 ...

Canonicalizing BS_0

ignoring guards on expressions

Rules for canonicalization:

1. $(a1 \text{ when } p); a2$

$\implies (a1; a2) \text{ when } p$

2. $(a \text{ when } p1) \text{ when } p2$

$\implies a \text{ when } (p1 \ \& \ p2)$

3. $\text{if } p \text{ then } (a \text{ when } q)$

$\implies (\text{if } p \text{ then } a) \text{ when } (p\&q \ | \ !p)$

Conditionals & Cases

- ◆ if p then a1 else a2
= if p then a1; if !p then a2
- ◆ Similarly for cases

BS₀ : Canonical form

In the canonical form, expressions have no guards and an (or a compound) action has at most one guard and it occurs at the top level;

move all the
guards to the
top level

ag is an Action with guard

a is an Action without guard;

e is an Expression;

r is a register (state variable)

ag ::= a when e

a ::= r := e | if e then a | a ; a

e ::= r | c | Op(e , e) | e ? e : e | ...

Rules for Canonicalizing BS_0

- | | | |
|-----|--|--|
| 1. | $(a1 \text{ when } p); a2$ | $\implies (a1; a2) \text{ when } p$ |
| 2.1 | $(a \text{ when } p) \text{ when } q$ | $\implies a \text{ when } (p \ \& \ q)$ |
| 2.2 | $(e \text{ when } p) \text{ when } q$ | $\implies e \text{ when } (p \ \& \ q)$ |
| 3.1 | $\text{if } p \text{ then } (a \text{ when } q)$ | $\implies (\text{if } p \text{ then } a) \text{ when } (p\&q \mid !p)$ |
| 3.2 | $p \ ? \ (e1 \text{ when } q) : e2$ | $\implies (p \ ? \ e1 : e2) \text{ when } (p\&q \mid !p)$ |
| 3.3 | $p \ ? \ e1 : (e2 \text{ when } q)$ | $\implies (p \ ? \ e1 : e2) \text{ when } (p \mid !p\&q)$ |
| 4 | $r := (e \text{ when } q)$ | $\implies (r := e) \text{ when } q$ |
| 5.1 | $Op(e1 \text{ when } q, e2)$ | $\implies Op(e1, e2) \text{ when } q$ |
| 5.2 | $Op(e1, e2 \text{ when } q)$ | $\implies Op(e1, e2) \text{ when } q$ |
| 5.3 | $\text{if } (p \text{ when } q) \text{ then } a$ | $\implies (\text{if } p \text{ then } a) \text{ when } q$ |
| 5.4 | $(p \text{ when } q) \ ? \ e1 : e2$ | $\implies (p \ ? \ e1 : e2) \text{ when } q$ |

Theorem: Canonical form for an action exists and is unique up to the boolean simplification of the guard expression.

$BS_1 = BS_0 + \text{Let blocks}$ Introducing local names

t, t_1, t_2, \dots are identifiers (not registers)

Program ::=

Registers r_1, \dots, r_n ;

$t_1 = e_1; \dots; t_n = e_n;$

Rule $R_1 a_1; \dots; \text{Rule } R_m a_m ;$

$a ::= r := e \mid a \text{ when } e \mid \text{if } e \text{ then } a \mid a ; a$
 $\mid (t_1 = e_1; \dots; t_n = e_n; \text{in } a)$

$e ::= r \mid c \mid \text{Op}(r_a, r_b) \mid e ? e : e \mid e \text{ when } e \mid \dots$
 $\mid t \mid (t_1 = e_1; \dots; t_n = e_n; \text{in } e)$

BS₁ Lifting rules

- ◆ Unique local names (t₁, t₂, ...) can be introduced anywhere for sharing

$$e \implies (t = e \text{ in } t)$$

- ◆ Lifting rules for actions

- $r := (t = e \text{ in } e')$ $\implies (t = e \text{ in } r := e')$
- $a \text{ when } (t = e \text{ in } e')$ $\implies (t = e \text{ in } (a \text{ when } e'))$
- $(t = e \text{ in } (e' \text{ when } p))$ $\implies (t = e \text{ in } e') \text{ when } p$
- $\text{if } (t=e \text{ in } p) \text{ then } e1$ $\implies (t = e \text{ in } (\text{if } p \text{ then } e1))$
- $(t = e \text{ in } a1); a2$ $\implies (t = e \text{ in } (a1; a2))$
- $(t1 = e1 \text{ in } (t2 = e2 \text{ in } a))$ $\implies (t1 = e1; t2 = e2 \text{ in } a)$

Some renaming of local variables may be required

- ◆ Substitution & when clauses
 - $(t=e \text{ when } p \text{ in } e') \implies (t = e; t1 = p \text{ in } [(t \text{ when } t1)/t]e')$
- ◆ Lifting rules for expressions are similar

Lifting lets to the top level

Registers r_1, \dots, r_n ;

$t_1 = e_1 ; \dots ; t_n = e_n ;$

Rule $R_1 a_1 ; \dots ;$ Rule $R_m a_m ;$

Rule $R_i (t_{i1} = e_{i1} ; \dots ; t_{in} = e_{in} ; \text{in } a_i)$

\Rightarrow Registers r_1, \dots, r_n ;

$t_1 = e_1 ; \dots ; t_n = e_n ;$

$t_{i1} = e_{i1} ; \dots ; t_{in} = e_{in} ;$

Rule $R_1 a_1 ; \dots ;$ Rule $R_m a_m ;$

Rule $R_i a_i$

Some renaming of local variables may be required

BS₁: Canonical form

Program ::=

Registers r_1, \dots, r_n ;

$t_1 = e_1; \dots; t_n = e_n$;

Rule R_1 ac_1 ; ... ; Rule R_m ac_m ;

$ac ::= (t_1 = e_1; \dots; t_n = e_n; \text{in } aw)$

$ac' ::= (t_1 = e_1; \dots; t_n = e_n; \text{in } a)$

$aw ::= a \mid a \text{ when } e$

$a ::= r := e \mid \text{if } e \text{ then } ac' \mid a ; a$

$ec ::= (t_1 = e_1; \dots; t_n = e_n; \text{in } ew)$

$ec' ::= (t_1 = e_1; \dots; t_n = e_n; \text{in } e)$

$ew ::= e \mid e \text{ when } p$

$e ::= r \mid c \mid \text{Op}(r_a, r_b) \mid e ? ec' : ec' \dots \mid t$

$BS_2 = BS_1 + \text{Modules}$

A program is a collection of (instantiated) modules m, m_1, \dots ;

A module is a collection of rules and interface methods

f, f_1, f_2, \dots are names for “read methods”

g, g_1, g_2, \dots are names for “action methods”

$a ::= r := e \mid a \text{ when } e \mid \text{if } e \text{ then } a \mid a ; a \mid (t = e \text{ in } a)$
 $\mid m.g(e)$

$e ::= r \mid c \mid \text{Op}(r_a, r_b) \mid e ? e : e \mid \dots \mid e \text{ when } e$
 $\mid t \mid (t = e \text{ in } e) \mid m.f(e)$

BS₂ Program

A program is a collection of instantiated modules:

Program ::= $m_1 ; m_2 ; m_2 ; \dots$

Module ::=

Module name

[Register r]

[Rule R a];

Interface

Interface ::= [action method]; [read method]

action method ::= method $g(x) = a$

read method ::= method $f(x) = e$

Implicit conditions

- ◆ Every method has two parts: guard and body. These will be designated by subscripts G and B, respectively

- ◆ Making guards explicit in every method call:

$$m.h(e) == >$$
$$(p = m.h_G \text{ in } m.h_B(e) \text{ when } p)$$

BS₂ : Additional Lifting rules

- ◆ Only read methods can be named in a let block
 $m.f(e) \implies (t=m.f(e) \text{ in } t)$
- ◆ $m.g (t = e \text{ in } e')$ $\implies (t = e \text{ in } m.g(e'))$
- ◆ $m.g (e \text{ when } p)$ $\implies m.g(e) \text{ when } p$
 - similar rules for read methods

Some subtle issues

- ◆ Does it matter if we first make the guards explicit and then lift or can we lift at any stage?

BS₂ : Canonicalization procedure

1. Make guards of method calls explicit
2. Lift *lets* to the top
3. Get rid of the *whens* from the *lets*
4. Lift *whens* to the top

Getting ready for circuit generation

- ◆ We need to collect multiple conditional assignments to one register in one expression, i.e.,
 - ... if p1 then r := e1;
if p2 then r := e2;
(r := p3? e4: e5); ...

Notation for conditional assignment

◆ $r := e_1.p_1 + \dots + e_n.p_n$

where

- e_1, e_2, \dots are expressions
- p_1, p_2, \dots are booleans

◆ $e.p$ evaluates to e if p is true otherwise to False (zero's)

◆ if p_i 's are not pairwise mutually exclusive then the program is illegal

◆ $e_1.p_1 + \dots + e_n.p_n$ evaluates to some e_i or if all p_i 's are false then the value of r does not change

Collecting conditional assignments to a register

◆ Apply the following rules after the guards have been made explicit and the program has been canonicalized,

$$1. \text{ if } p \text{ then } a \quad \Longrightarrow a . p$$

$$2.1 \text{ (} r := e \text{) . } p \quad \Longrightarrow r := e . p$$

$$2.2 \text{ m.g}(e) . p \quad \Longrightarrow \text{m.g}(e . p)$$

$$3. (a1; a2) . p \quad \Longrightarrow a1 . p ; a2 . p$$

$$4.1 r := e1 ; r := e2 \quad \Longrightarrow r := e1 + e2$$

$$4.2 \text{ m.g}(e1); \text{ m.g}(e2) \quad \Longrightarrow \text{m.g}(e1 + e2)$$

Theorem: After applying the above rules to a Program in canonical form any action in it will be reduced the following form:

$r1 := e1; r2 := e2; \dots$

$\text{m.g}(e); \text{m1.g1}(e1); \dots$ where e 's may contain "." and "+"