Combinational Circuits in Bluespec

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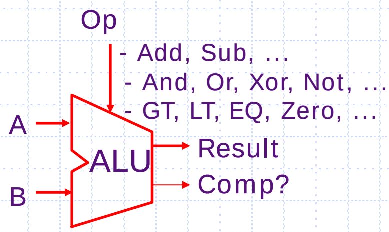
Combinational circuits are acyclic interconnections of gates

- And, Or, Not
- Nand, Nor, Xor
- …

Arithmetic-Logic Unit (ALU)



ALU performs all the arithmetic and logical functions

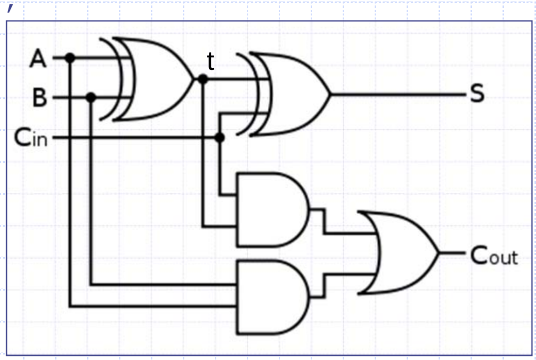
Each individual function can be described as a combinational circuit

Full Adder: A one-bit adder

```
function fa(a, b, c_in);
   t = (a ^ b);
   s = t ^ c_in;
   c_out = (a & b) | (c_in & t);
   return {c_out,s};
endfunction
```

Structural code – only specifies interconnection between boxes

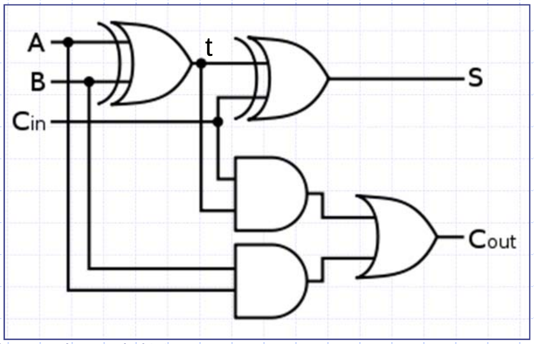
Not quite correct – needs type annotations



Full Adder: A one-bit adder

corrected

```
function Bit#(2) fa(Bit#(1) a, Bit#(1) b,
                                    Bit#(1) c_in);
   Bit#(1) t = a ^ b;
   Bit#(1) s = t ^ c_in;
   Bit#(1) c_out = (a & b) | (c_in & t);
   return {c_out,s};
endfunction
```



"Bit#(1) a" type declaration says that a is one bit wide

{c_out,s} represents bit concatenation

How big is {c_out,s}?

Types

- A type is a grouping of values:
  - Integer: 1, 2, 3, …
  - Bool: True, False
  - Bit: 0,1
  - A pair of Integers: Tuple2#(Integer, Integer)
  - A function fname from Integers to Integers:

    `function Integer fname (Integer arg)`
- Every expression and variable in a BSV program has a type; sometimes it is specified explicitly and sometimes it is deduced by the compiler
- Thus we say an expression has a type or belongs to a type

The type of each expression is unique

Parameterized types:

- A type declaration itself can be parameterized by other types
- Parameters are indicated by using the syntax '#'
  - For example `Bit#(n)` represents n bits and can be instantiated by specifying a value of n
    `Bit#(1), Bit#(32), Bit#(8), …`

Type synonyms

```
typedef bit [7:0] Byte;
typedef Bit#(8) Byte;
```
The same

```
typedef Bit#(32) Word;

typedef Tuple2#(a,a) Pair#(type a);
```

```
typedef Int#(n) MyInt#(type n);
typedef Int#(n) MyInt#(numeric type n);
```
The same

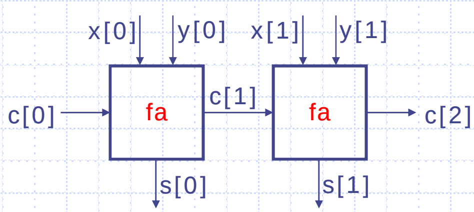
Type declaration versus deduction

- The programmer writes down types of some expressions in a program and the compiler deduces the types of the rest of expressions
- If the type deduction cannot be performed or the type declarations are inconsistent then the compiler complains

```
function Bit#(2) fa(Bit#(1) a, Bit#(1) b,
                               Bit#(1) c_in);
   Bit#(1) s = (a ^ b)^ c_in;
   Bit#(2) c_out = (a & b) | (c_in & (a ^ b));
   return {c_out,s};
endfunction
```

type error?

2-bit Ripple-Carry Adder



fa can be used as a black-box long as we understand its type signature

```
function Bit#(3) add(Bit#(2) x, Bit#(2) y,
                                  Bit#(1) c0);
   Bit#(2) s = 0;    Bit#(3) c=0; c[0] = c0;
   let cs0 = fa(x[0], y[0], c[0]);
             c[1] = cs0[1];  s[0] = cs0[0];
   let cs1 = fa(x[1], y[1], c[1]);
             c[2] = cs1[1];  s[1] = cs1[0];
  return {c[2],s};
endfunction
```

The “let” syntax avoids having to write down types explicitly

"let" syntax

- The "let" syntax: avoids having to write down types explicitly
  - `let cs0 = fa(x[0], y[0], c[0]);`
  - `Bits#(2) cs0 = fa(x[0], y[0], c[0]);`

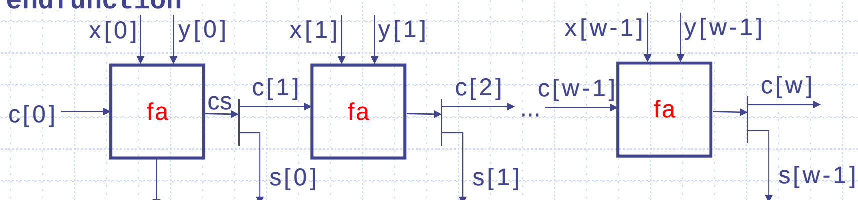
The same

An w-bit Ripple-Carry Adder

```
function Bit#(w+1) addN(Bit#(w) x, Bit#(w) y,
                                    Bit#(1) c0);
   Bit#(w) s; Bit#(w+1) c=0; c[0] = c0;
   for(Integer i=0; i<w; i=i+1)
   begin
      let cs = fa(x[i],y[i],c[i]);
      c[i+1] = cs[1]; s[i] = cs[0];
   end
return {c[w],s};
endfunction
```

Not quite correct

Unfold the loop to get the wiring diagram



Instantiating the parametric Adder

```
function Bit#(w+1) addN(Bit#(w) x, Bit#(w) y,
                                   Bit#(1) c0);
```

How do we define a add32, add3 … using addN ?

```
// concrete instances of addN!
function Bit#(33) add32(Bit#(32) x, Bit#(32) y,
                        Bit#(1) c0) =
                                      addN(x,y,c0);
```

The numeric type w on the RHS implicitly gets instantiated to 32 because of the LHS declaration

```
function Bit#(4) add3(Bit#(3) x, Bit#(3) y,
                      Bit#(1) c0) = addN(x,y,c0);
```


valueOf(w) versus w

- Each expression has a type and a value and these come from two entirely disjoint worlds
- w in Bit#(w) resides in the types world
- Sometimes we need to use values from the types world into actual computation, e.g., i<w
  - But i<w is not type correct
- The function valueOf allows us to lift a numeric type to a value
  - Making i<valueOf(w) type correct

TAdd#(w,1) versus w+1

- Sometimes we need to perform operations in the types world that are very similar to the operations in the value world
  - Examples: `Add`, `Mul`, `Log`
- We define a few special operators in the types world for such operations
  - Examples: `TAdd#(m,n)`, `TMul#(m,n)`, …

Integer versus Int#(32)

- In mathematics integers are unbounded but in computer systems integers always have a fixed size
- BSV allows us to express both types of integers, though unbounded integers are used only as a programming convenience

```
for(Integer i=0; i<valw; i=i+1)
   begin
      let cs = fa(x[i],y[i],c[i]);
      c[i+1] = cs[1]; s[i] = cs[0];
   end
```

A w-bit Ripple-Carry Adder

corrected

```
function Bit#(TAdd#(w,1)) addN(Bit#(w) x, Bit#(w) y,
                                     Bit#(1) c0);
   Bit#(w) s; Bit#(TAdd#(w,1)) c; c[0] = c0;
   let valw = valueOf(w);
   for(Integer i=0; i<valw; i=i+1)
   begin
      let cs = fa(x[i],y[i],c[i]);
      c[i+1] = cs[1]; s[i] = cs[0];
   end
return {c[valw],s};
endfunction
```


Static Elaboration phase

- When BSV programs are compiled, first type checking is done and then the compiler gets rid of many constructs which have no direct hardware meaning, like Integers, loops

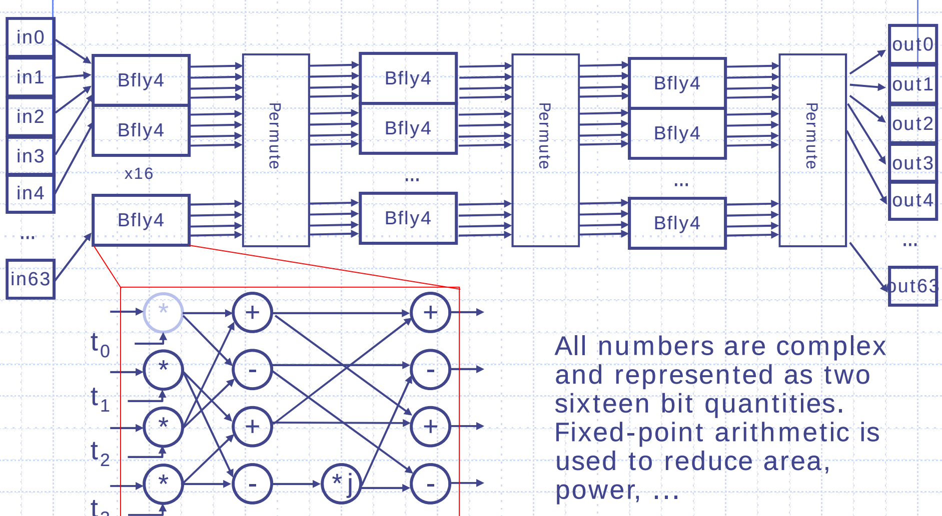
```
for(Integer i=0; i<valw; i=i+1) begin
   let cs = fa(x[i],y[i],c[i]);
   c[i+1] = cs[1]; s[i] = cs[0];
end
```

```
cs0 = fa(x[0], y[0], c[0]); c[1]=cs0[1]; s[0]=cs0[0];
cs1 = fa(x[1], y[1], c[1]); c[2]=cs1[1]; s[1]=cs1[0];
…
csw = fa(x[valw-1], y[valw-1], c[valw-1]);
      c[valw] = csw[1]; s[valw-1] = csw[0];
```

Complex combinational circuits

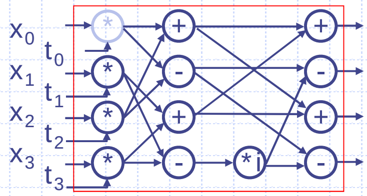
IFFT

Combinational IFFT



All numbers are complex and represented as two sixteen bit quantities. Fixed-point arithmetic is used to reduce area, power, ...

4-way Butterfly Node

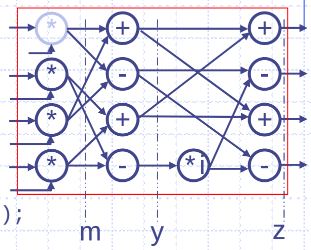



```
function Vector#(4,Complex) bfly4
            (Vector#(4,Complex) t,  Vector#(4,Complex) x);
```

- t's (twiddle coefficients) are mathematically derivable constants for each bfly4 and depend upon the position of bfly4 the in the network

BSV code: 4-way Butterfly

```
function Vector#(4,Complex#(s)) bfly4
            (Vector#(4,Complex#(s)) t,  Vector#(4,Complex#(s)) x);

  Vector#(4,Complex#(s)) m, y, z;

  m[0] = x[0] * t[0]; m[1] = x[1] * t[1];
  m[2] = x[2] * t[2]; m[3] = x[3] * t[3];

  y[0] = m[0] + m[2]; y[1] = m[0] – m[2];
  y[2] = m[1] + m[3]; y[3] = i*(m[1] – m[3]);

  z[0] = y[0] + y[2]; z[1] = y[1] + y[3];
  z[2] = y[0] – y[2]; z[3] = y[1] – y[3];

  return(z);
endfunction
```



Polymorphic code: works on any type of numbers for which *, + and - have been defined

Note: Vector does not mean storage; just a group of wires with names

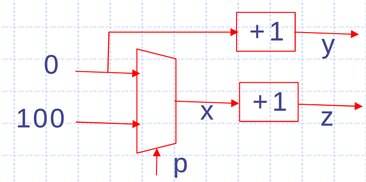
Language notes: Sequential assignments

- Sometimes it is convenient to reassign a variable (x is zero every where except in bits 4 and 8):

```
Bit#(32) x = 0;
x[4] = 1; x[8] = 1;
```

- This will usually result in introduction of muxes in a circuit as the following example illustrates:

```
Bit#(32) x = 0;
let y = x+1;
if(p) x = 100;
let z = x+1;
```



Complex Arithmetic

- Addition
  - $z_R = x_R + y_R$
  - $z_I = x_I + y_I$
- Multiplication
  - $z_R = x_R * y_R - x_I * y_I$
  - $z_I = x_R * y_I + x_I * y_R$

Representing complex numbers as a **struct**

```
typedef struct{
  Int#(t) r;
  Int#(t) i;
} Complex#(numeric type t) deriving (Eq,Bits);
```

Notice the Complex type is parameterized by the size of Int chosen to represent its real and imaginary parts

If x is a struct then its fields can be selected by writing x.r and x.i

BSV code for Addition

```
typedef struct{
  Int#(t) r;
  Int#(t) i;
} Complex#(numeric type t) deriving (Eq,Bits);

function Complex#(t) cAdd
          (Complex#(t) x, Complex#(t) y);
  Int#(t) real = x.r + y.r;
  Int#(t) imag = x.i + y.i;
  return(Complex{r:real, i:imag});
endfunction
```

What is the type of this + ?

Overloading (Type classes)

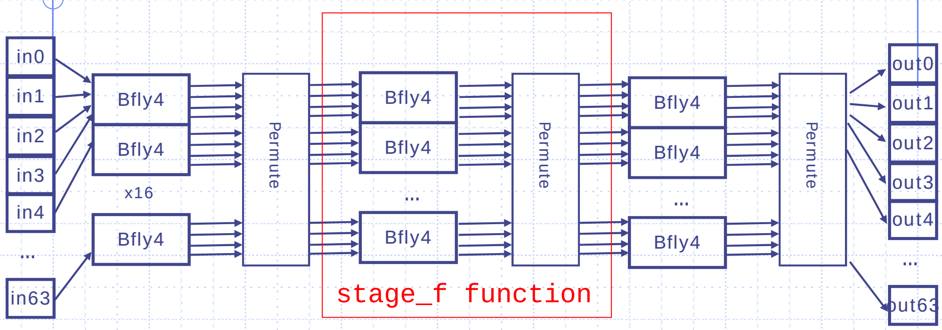
- The same symbol can be used to represent different but related operators using Type classes
- A type class groups a bunch of types with similarly named operations. For example, the type class Arith requires that each type belonging to this type class has operators +,-, *, / etc. defined
- We can declare Complex type to be an instance of Arith type class

Overloading +, *

```
instance Arith#(Complex#(t));
function Complex#(t) \+
               (Complex#(t) x, Complex#(t) y);
   Int#(t) real = x.r + y.r;
   Int#(t) imag = x.i + y.i;
   return(Complex{r:real, i:imag});
endfunction

function Complex#(t) \*
               (Complex#(t) x, Complex#(t) y);
   Int#(t) real = x.r*y.r – x.i*y.i;
   Int#(t) imag = x.r*y.i + x.i*y.r;
   return(Complex{r:real, i:imag});
endfunction
…
endinstance
```

The context allows the compiler to pick the appropriate definition of an operator

Combinational IFFT




```
function Vector#(64, Complex#(n)) stage_f
      (Bit#(2) stage, Vector#(64, Complex#(n)) stage_in);
```

```
function Vector#(64, Complex#(n)) ifft
      (Vector#(64, Complex#(n)) in_data);
```

repeat stage_f three times

BSV Code: Combinational IFFT

```
function Vector#(64, Complex#(n)) ifft
                    (Vector#(64, Complex#(n)) in_data);
//Declare vectors
   Vector#(4,Vector#(64, Complex#(n))) stage_data;

   stage_data[0] = in_data;
   for (Bit#(2) stage = 0; stage < 3; stage = stage + 1)
    stage_data[stage+1] = stage_f(stage,stage_data[stage]);
return(stage_data[3]);
endfunction
```

The for-loop is unfolded and stage_f is inlined during static elaboration

Note: no notion of loops or procedures during execution

BSV Code for `stage_f`

```
function Vector#(64, Complex#(n)) stage_f
        (Bit#(2) stage, Vector#(64, Complex#(n)) stage_in);
Vector#(64, Complex#(n)) stage_temp, stage_out;
   for (Integer i = 0; i < 16; i = i + 1)
    begin
      Integer idx = i * 4;
      Vector#(4, Complex#(n)) x;
      x[0] = stage_in[idx];   x[1] = stage_in[idx+1];
      x[2] = stage_in[idx+2]; x[3] = stage_in[idx+3];
      let twid = getTwiddle(stage, fromInteger(i));
      let y = bfly4(twid, x);
      stage_temp[idx]   = y[0]; stage_temp[idx+1] = y[1];
      stage_temp[idx+2] = y[2]; stage_temp[idx+3] = y[3];
    end
  //Permutation
   for (Integer i = 0; i < 64; i = i + 1)
      stage_out[i] = stage_temp[permute[i]];
  return(stage_out);
endfunction
```