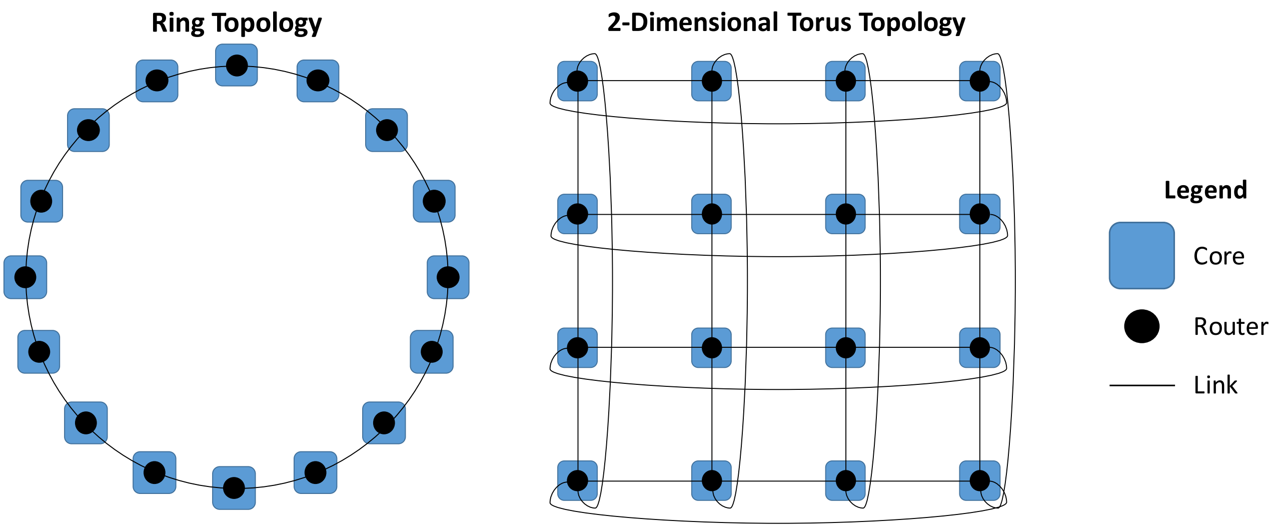
Part D: On-Chip Networks (24 points + 10 bonus)

You are choosing between two topologies for your on-chip network, shown below.



For a system of N nodes, the 2-D Torus topology consists of $k = \sqrt{N}$ rows and columns, where each row or column is a ring.

Question 1 (10 points)

Your first task is to compare the topologies along key metrics. Fill in the table below as a function of the number of nodes in the network, N. The units for each cell are hops or links. For average distance, assume uniform random traffic (where each node sends $1/N^{th}$ of the traffic to each destination, including itself).

To ease your derivations, you can define a variable $k = \sqrt{N}$, and assume k is an even integer.

For partial credit, give the asymptotic growth instead.

	Ring	**2-D Torus**
Number of links	N	$2N$
Diameter	$\frac{N}{2}$	$\sqrt{N}$
Average distance	$\frac{N}{4}$	$\frac{\sqrt{N}}{2}$
Bisection bandwidth	2	$2\sqrt{N}$

Question 2 (8 points)

As discussed in lecture, higher-dimensional topologies have smaller diameter, but the routers are more expensive. For what range of number of nodes, *N*, is the **average latency** in cycles of a 2-D torus topology lower than the **average latency** in cycles of a ring topology? (Provide an inequality on *N*.)

Assume the following:

- Traversing one link requires 1 cycle
- Traversing one router requires 1 cycles in the ring topology
- Traversing one router requires 4 cycles in the 2-D torus ring topology
- Entering and exiting the network at a router counts as traversing the router
- Traffic is uniform random

(For example, in a 4-node ring, the latency to send a flit from a node to a node two hops away is 5 cycles: 1 cycle to enter the network at the first router, 2 cycles for two link traversals, 1 cycle for one router traversal, and 1 cycle to exit the network at the last router.)

Consider the Ring topology.
Every hop takes 2 cycles (1 for link, 1 for router, including the exit router).
The average distance in hops for uniform random traffic is N/4.
Add 1 cycle to enter the network.
$L_{ring} = N/4 * 2 + 1 = N/2 + 1$

Consider the 2-D Torus topology.
Every hop takes 5 cycles (1 for link, 4 for router)
Average distance in hops is $\sqrt{N}/2$.
Add 4 cycles to enter the network.
$L_{torus} = \sqrt{N}/2 * 5 + 4 = 5/2\sqrt{N} + 4$

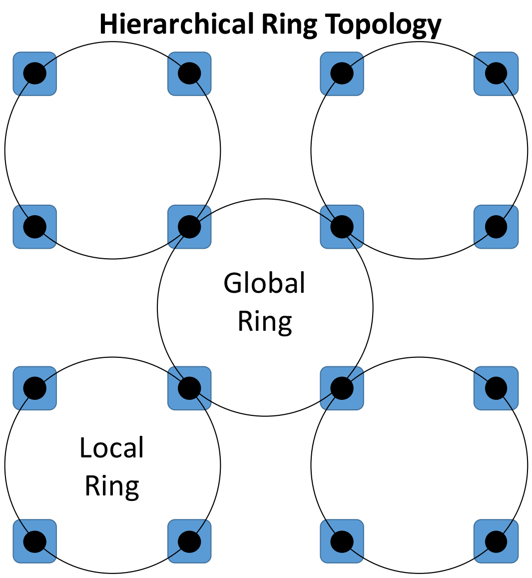
When is $L_{torus} < L_{ring}$?
$\Rightarrow 5/2\sqrt{N} + 4 < N/2 + 1$
$\Rightarrow 0 < N - 5\sqrt{N} - 6$
$\Rightarrow 0 < (\sqrt{N} - 6)(\sqrt{N} + 1)$
$\Rightarrow N > 36$

Since N is an even square, N >= 64.

Question 3 (6 points)

In a stroke of genius, you realize that a Hierarchical Ring topology (shown below) can bridge the low-latency routing of a Ring topology with the lower diameter of a 2-D Torus topology. For a system of N nodes, the Hierarchical Ring topology consists of one *global ring* surrounded by $k = \sqrt{N}$ *local rings*. Each local ring has one node that also bridges with the global ring.

Fill in the table below as a function of the number of nodes in the network, N. The units for each cell are hops or links. To ease your derivations, you can define a variable $k = \sqrt{N}$, and assume k is an even integer. *For partial credit, give the asymptotic growth instead.*



	Hierarchical Ring
Number of links	$N + \sqrt{N}$
Diameter	$\frac{3}{2}\sqrt{N}$
Bisection bandwidth	*2*

Each of the $\sqrt{N}$ local rings contains $\sqrt{N}$ links. The single global ring contains $\sqrt{N}$ links.

The diameter within a ring is $\frac{\sqrt{N}}{2}$. In the worst case, such a diameter is crossed for three rings: one end of a local ring to the bridge to enter the global ring, to the most distant bridge to exit the global ring, then to the end of that local ring.

Bonus Question (10 points)

Warning: This question is harder than others, so we recommend finishing as much of the quiz as you can before attempting it!

Derive the average distance (in hops or links) for a Hierarchical Ring topology as a function of the number of nodes in the network, *N*. Assume uniform random traffic (where each node sends *1/N^th* of the traffic to each destination, including itself). To ease your derivations, you can define a variable $k = \sqrt{N}$, and assume *k* is an even integer. *Only exact answers are accepted, no partial credit.*

With probability $\frac{1}{k}$ the source and destination are in the same ring.
The average distance in this case is $\frac{k}{4}$.

With probability $\frac{k-1}{k}$ the source and destination are in different rings.
The average distance to enter the global ring is $\frac{k}{4}$ (this includes when the source is a bridge node).
The average distance to exit the global ring is $\frac{k}{4}$ (this includes when the destination is a bridge).
The average distance to traverse the global ring is **not** $\frac{k}{4}$, because we have assumed the source and destination are in different rings; we scale the average distance for a ring by $\frac{k}{k-1}$.

The average distance is therefore:

$$\frac{1}{k}\cdot\frac{k}{4}+\frac{k-1}{k}\left(\frac{k}{4}+\frac{k}{4}+\frac{k}{4}\cdot\frac{k}{k-1}\right)=\frac{3\sqrt{N}-1}{4}$$