6.5930/1

Hardware Architectures for Deep Learning

Computational Transforms

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FC – Vector Operation Counts

$$O_m = I_{chw} \times F_{m,chw}$$

$$O_m = I_{chw1,chw0} \times F_{m,chw1,chw0}$$

Order: m, chw1, chw0

Factors: M, C*H*W/L, L

```
// Level 2 loops
for m in [0, M):
  for chw1 in [0, C*H*W/L):
// Level 1 loops
    parallel_for chw0 in [0, L):
      o[m] += i[L*chw1+chw0] * f[C*H*W*m + L*chw1+chw0];
```

L == Lanes

Vector operation on L lanes

FC – Vector Operation Counts

$$O_m = I_{chw1,chw0} \times F_{m,chw1,chw0}$$

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```

L == Lanes

How many MACs?

How many reads of “inputs”

How many reads of “weights”

How many writes of “outputs”

Measuring reads/writes in units of 32-bit integers

Compute Intensity (MACs/Read)

```
// Level 2 loops
for m in [0, M):
  for chw2 in [0, C*H*W/L:
// Level 1 loops
    parallel_for chw1 in [0, L):
      o[m] += i[L*chw2+chw1] * f[C*H*W*m + L*chw2+chw1];
```

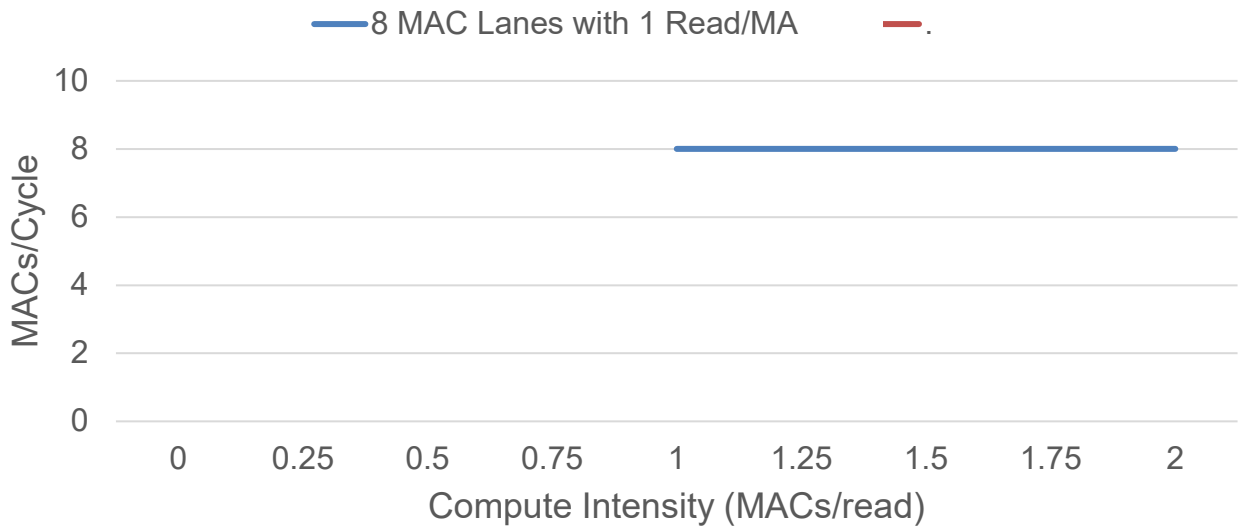
L == Lanes

MACs/Read?

$$\frac{C*H*W*M}{C*H*W*M + C*H*W*M} \sim \frac{1}{2}$$

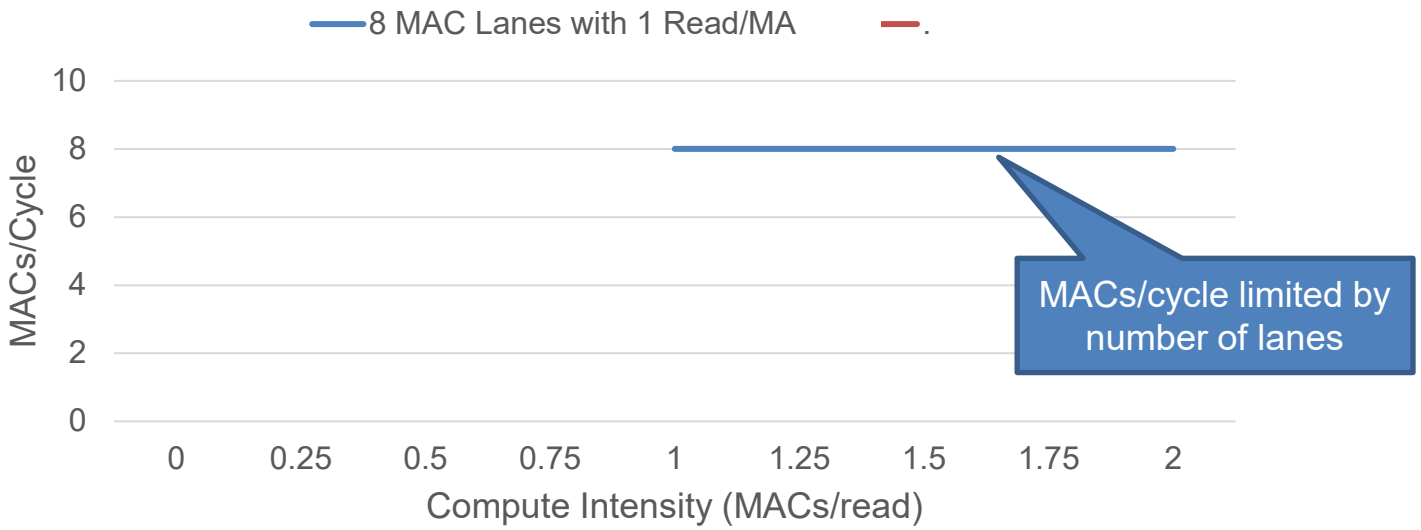
If system can support 1 Read/MAC
will system run at full throttle?

Roofline Model



Williams, Samuel, Andrew Waterman, and David Patterson. "Roofline: an insightful visual performance model for multicore architectures." Communications of the ACM 52.4 (2009): 65-76.

Roofline Model



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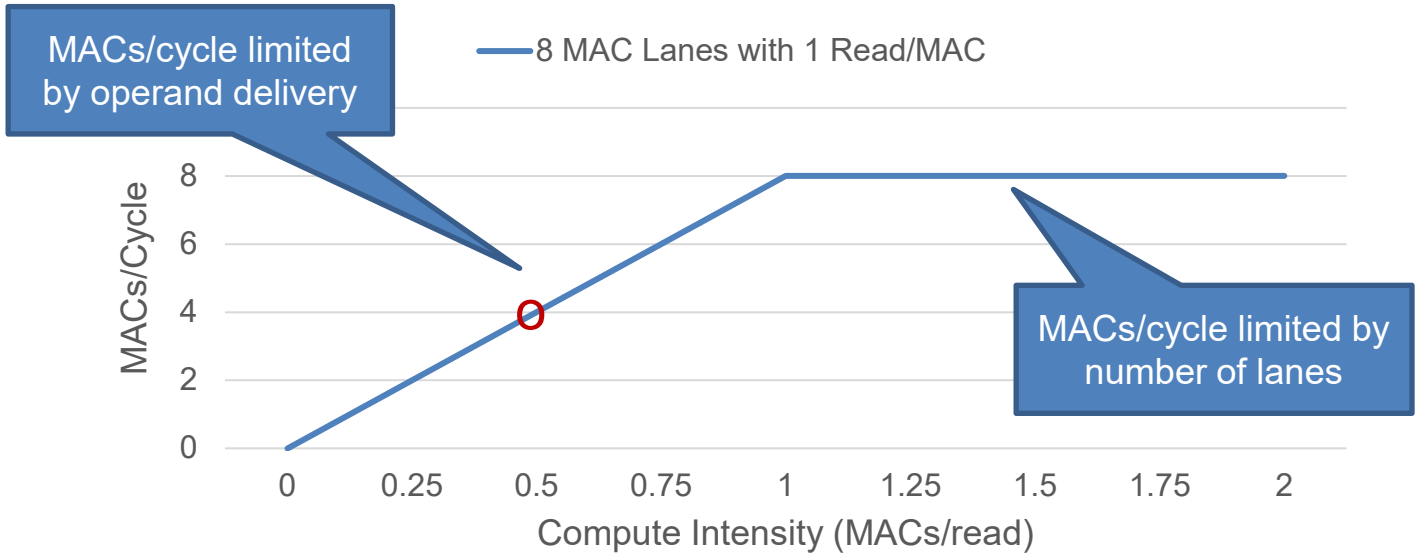

Where will the previous slide's code be?

How can we change the compute intensity?

FC – Reordered + loop invariant hoisted

$$O_m = I_{chw} \times F_{m,chw}$$

$$O_{m2,m1} = I_{chw} \times F_{m2,m1,chw}$$

Order: m2, chw, m1

Factors: M/L, C*H*W, L

```
// Level 2 loops
for m2 in [0, M/L):
   for chw in [0, C*H*W):
// Level 1 loops
     parallel_for m1 in [0, L):
        o[m2*L+m1] += i[chw] * f[CHW*(m2*L+m1) + chw]
```

L == Lanes

Strided?

Strided?

FC – Reordered + loop invariant hoisted

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```

L == Lanes

```
// Level 2 loops
for m2 in [0, M/L):
  for chw in [0, C*H*W):
    i_chw = i[chw]
// Level 1 loops
      parallel_for m1 in [0, L):
        o[m2*L+m1] += i_chw * f[CHW*(m2*L+m1) + chw]
```

L == Lanes

Loop invariant hoisted

Use loop invariant in register

FC – Operation Counts

$$O_{m2,m1} = I_{chw} \times F_{m2,m1,chw}$$

L == Lanes

```
// Level 2 loops
for m2 in [0, M/L):
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Measuring reads/writes in units of 32-bit integers

Compute Intensity

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MACs/Read?

Compute Intensity

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```
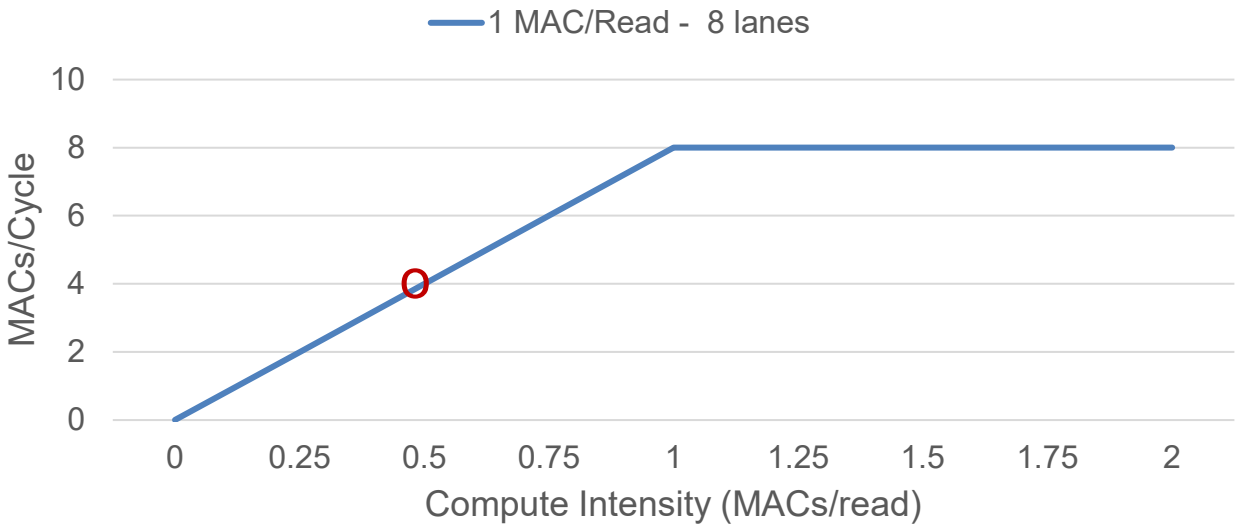

MACs/Read?

$$\frac{C*H*W*M}{C*H*W*M + C*H*W*M/L} \sim \frac{L}{1+L}$$

If system can support 1 Read/MAC
will system run at full throttle?

Roofline Model



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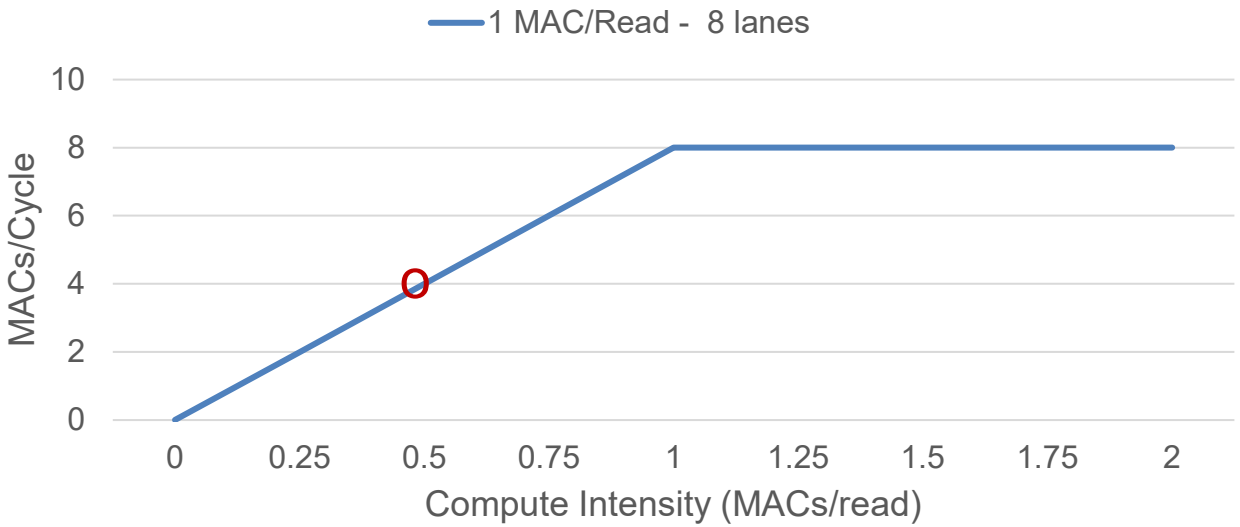

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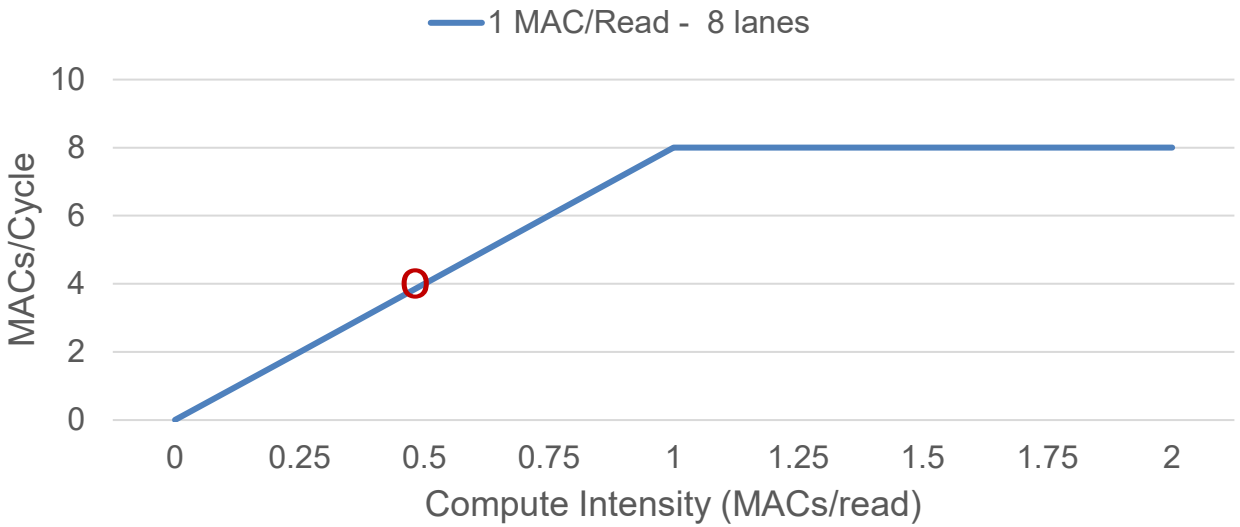

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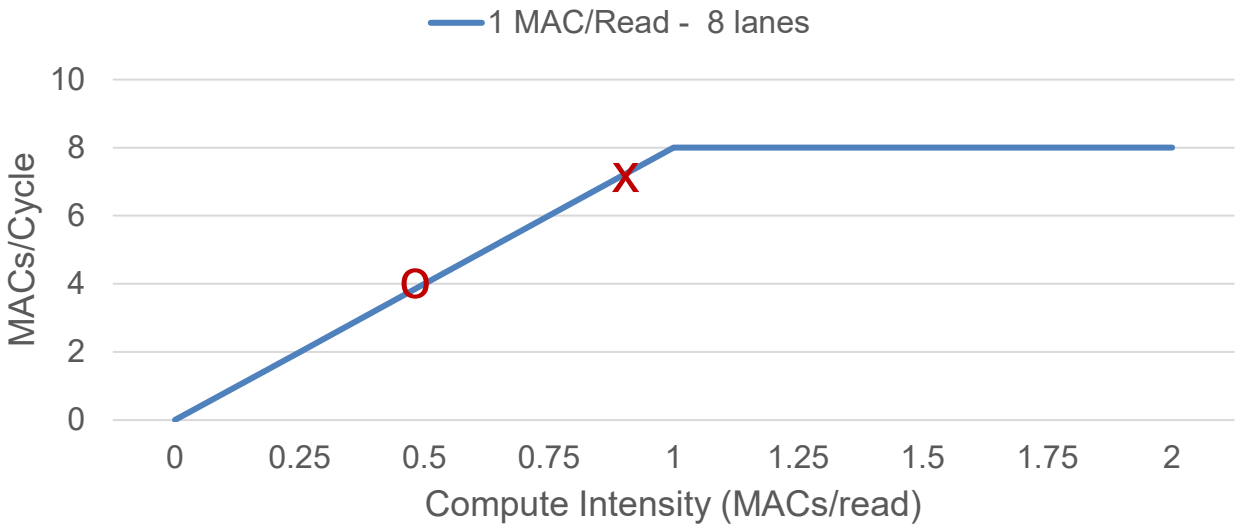


Where will the previous slide's code be?

Why might points be below the line?

Is being on the flat part always best?

Computation Transformations

- Goal: Bitwise same result, but reduce number of operations
- Focuses mostly on compute

Gauss's Multiplication Algorithm

$$(a+bi)(c+di) = (ac-bd) + (bc+ad)i.$$

4 multiplications + 3 additions

$$k_1 = c \cdot (a + b)$$

$$k_2 = a \cdot (d - c)$$

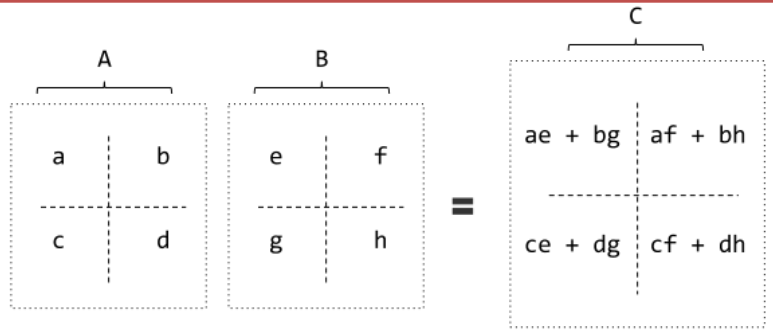
$$k_3 = b \cdot (c + d)$$

$$\text{Real part} = k_1 - k_3$$

$$\text{Imaginary part} = k_1 + k_2.$$

3 multiplications + 5 additions

Strassen



8 multiplications + 4 additions

P1 = a(f – h)
P2 = (a + b)h
P3 = (c + d)e
P4 = d(g – e)
P5 = (a + d)(e + h)
P6 = (b - d)(g + h)
P7 = (a – c)(e + f)

$$AB = \begin{bmatrix} P5 + P4 - P2 + P6 & P1 + P2 \\ P3 + P4 & P1 + P5 - P3 - P7 \end{bmatrix}$$

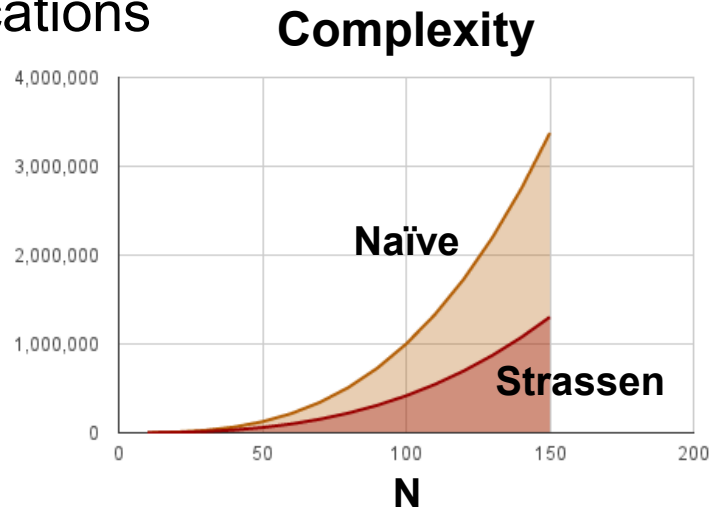
7 multiplications + 18 additions

7 multiplications + 13 additions (for constant B matrix – weights)

[Cong et al., ICANN, 2014]

Strassen

- Reduce the complexity of matrix multiplication from $\Theta(N^3)$ to $\Theta(N^{2.807})$ by reducing multiplications



Comes at the price of reduced numerical stability and requires significantly more memory

Image Source: http://www.stoimen.com/blog/2012/11/26/computer-algorithms-strassens-matrix-multiplication/

Python to C++ Chart

Version	*Implementation*	*Running time (s)*	*GFLOPS*	*Absolute speedup*	*Relative speedup*	*Fraction of peak*
1	Python	25,552.48	0.005	1	—	0.00%
2	Java	2,372.68	0.058	11	10.8	0.01%
3	C	542.67	0.253	47	4.4	0.03%
4	Parallel loops	69.80	1.969	366	7.8	0.24%
5	Parallel divide-and-conquer	3.80	36.180	6,727	18.4	4.33%
6	+ vectorization	1.10	124.914	23,224	3.5	14.96%
7	+ AVX intrinsics	0.41	337.812	62,806	2.7	40.45%
8	Strassen	0.38	361.177	67,150	1.1	43.24%

[**Leiserson**, There's plenty of room at the top, *Science*, 2020]

Tensor Computations

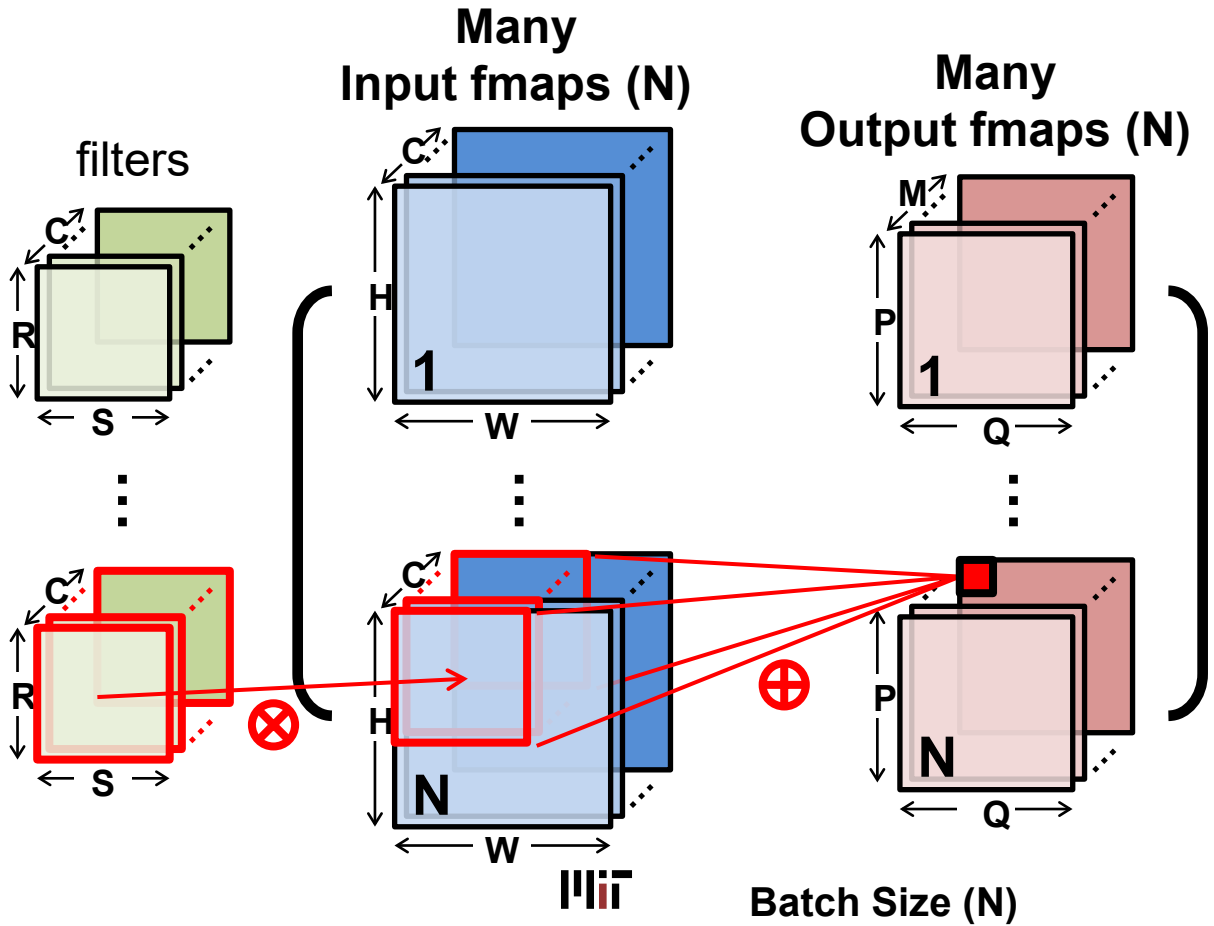
Matrix Multiply

$$O_{n,m} = I_{n,chw} \times F_{m,chw}$$

CONV Layer

$$O_{n,m,p,q} = I_{n,c,Up+r,Uq+s} \cdot F_{m,c,r,s}$$

Convolution (CONV) Layer



CONV Layer Implementation

Naïve 7-layer for-loop implementation:

```
for n in [0..N):
    for m in [0..M):
        for q in [0..Q):
            for p in [0..P):
```

} for each output fmap value

convolve a window and apply activation

```
O[n][m][p][q] = B[m];
for c in [0..C):
    for r in [0..R):
        for s in [0..S):
            O[n][m][p][q] += I[n][c][Up+r][Uq+s]
                           × F[m][c][r][s];

O[n][m][p][q] = Activation(O[n][m][p][q]);
```

Winograd 1D – F(2,3)

- Targeting convolutions instead of matrix multiply
- Notation: F(size of output, filter size)

inputs filter outputs

$$\text{F(2,3)} = \begin{bmatrix} i_0 & i_1 & i_2 \\ i_1 & i_2 & i_3 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} o_0 \\ o_1 \end{bmatrix}$$

6 multiplications + 4 additions

[Lavin et al., CVPR 2016]

Winograd 1D – F(2,3)

- Targeting convolutions instead of matrix multiply
- Notation: F(size of output, filter size)

inputs filter outputs

$$F(2,3) = \begin{bmatrix} i_0 & i_1 & i_2 \\ i_1 & i_2 & i_3 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} k_1 + k_2 + k_3 \\ k_2 - k_3 - k_4 \end{bmatrix}$$

$$k_1 = (i_0 - i_2) f_0$$

$$k_2 = (i_1 + i_2) \frac{f_0 + f_1 + f_2}{2}$$

$$k_3 = (i_2 - i_1) \frac{f_0 - f_1 + f_2}{2}$$

$$k_4 = (i_1 - i_3) f_2$$

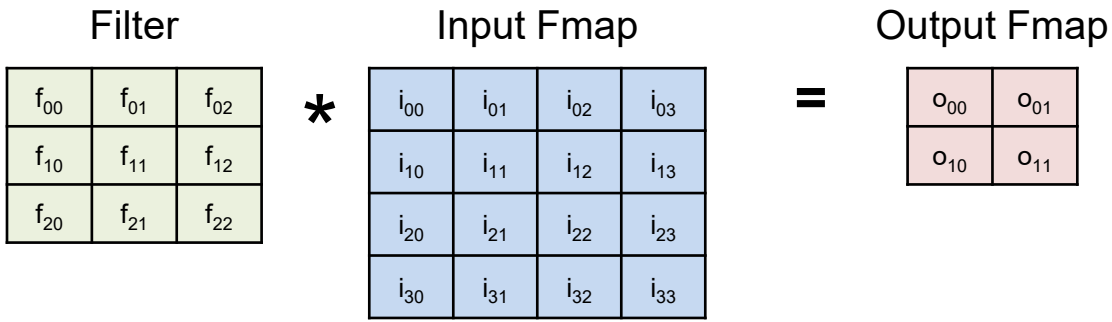
4 multiplications + 12 additions + 2 shifts

4 multiplications + 8 additions (for constant weights)

[Lavin et al., CVPR 2016]

Winograd 2D - F(2x2, 3x3)

- 1D Winograd is nested to make 2D Winograd

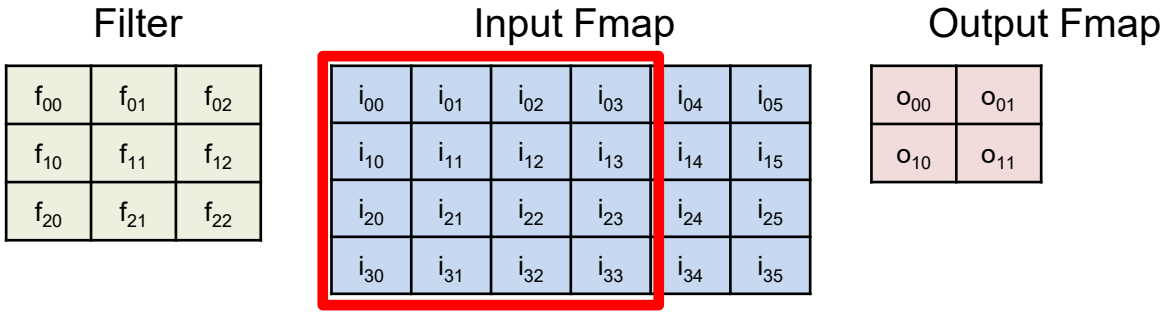


Original: 36 multiplications

Winograd: 16 multiplications → 2.25 times reduction

Winograd Halos

- Winograd works on a small region (tile) of output at a time, and therefore uses inputs repeatedly



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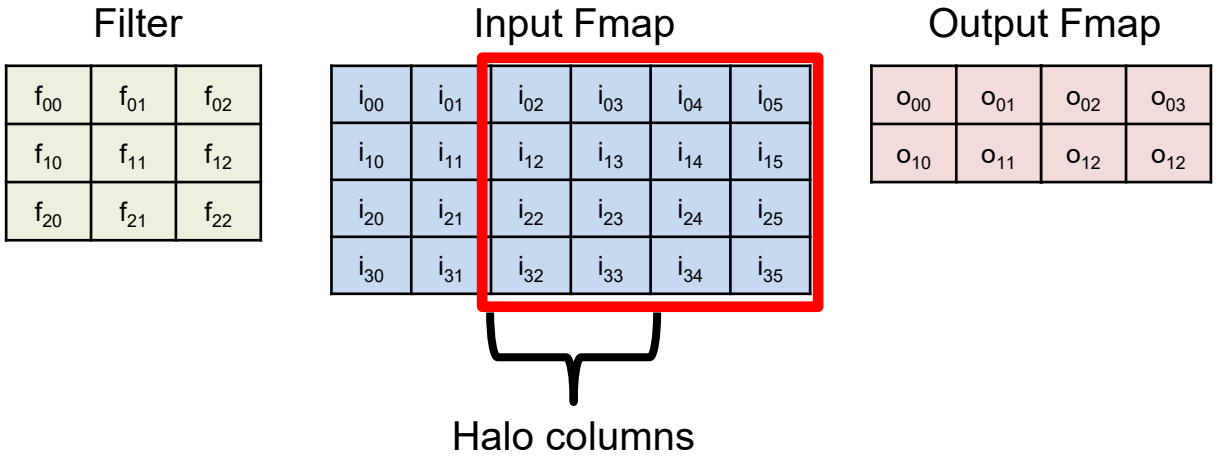
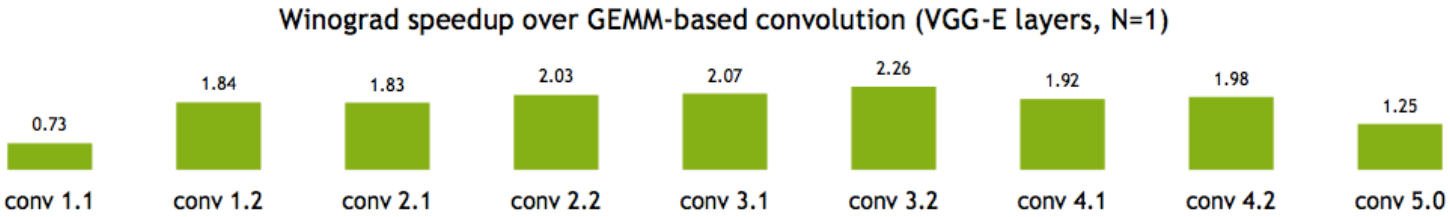


Winograd Performance Varies

Optimal convolution algorithm depends on convolution layer dimensions



Meta-parameters (data layouts, texture memory) afford higher performance

Using texture memory for convolutions: 13% inference speedup

(GoogLeNet, batch size 1)

Source: Nvidia

Winograd Summary

- Winograd is an optimized computation for convolutions
- It can significantly reduce multiplies
  - For example, for 3x3 filter by 2.25X
- But, each filter size (and output size) is a different computation.

Winograd as a Transform

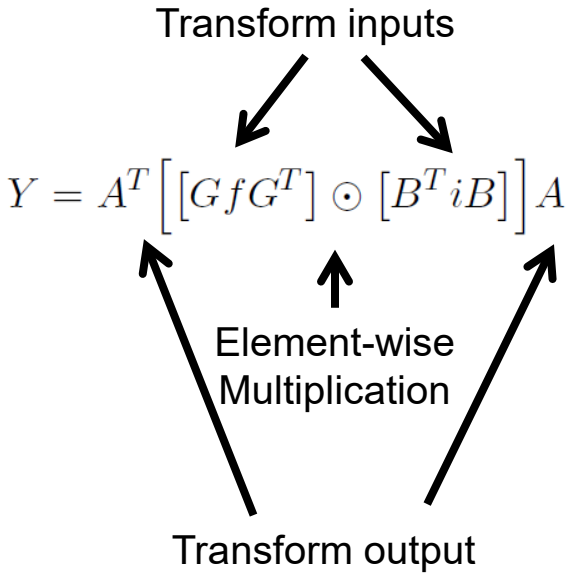
$$B^T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

filter $f = \begin{bmatrix} f_0 & f_1 & f_2 \end{bmatrix}^T$

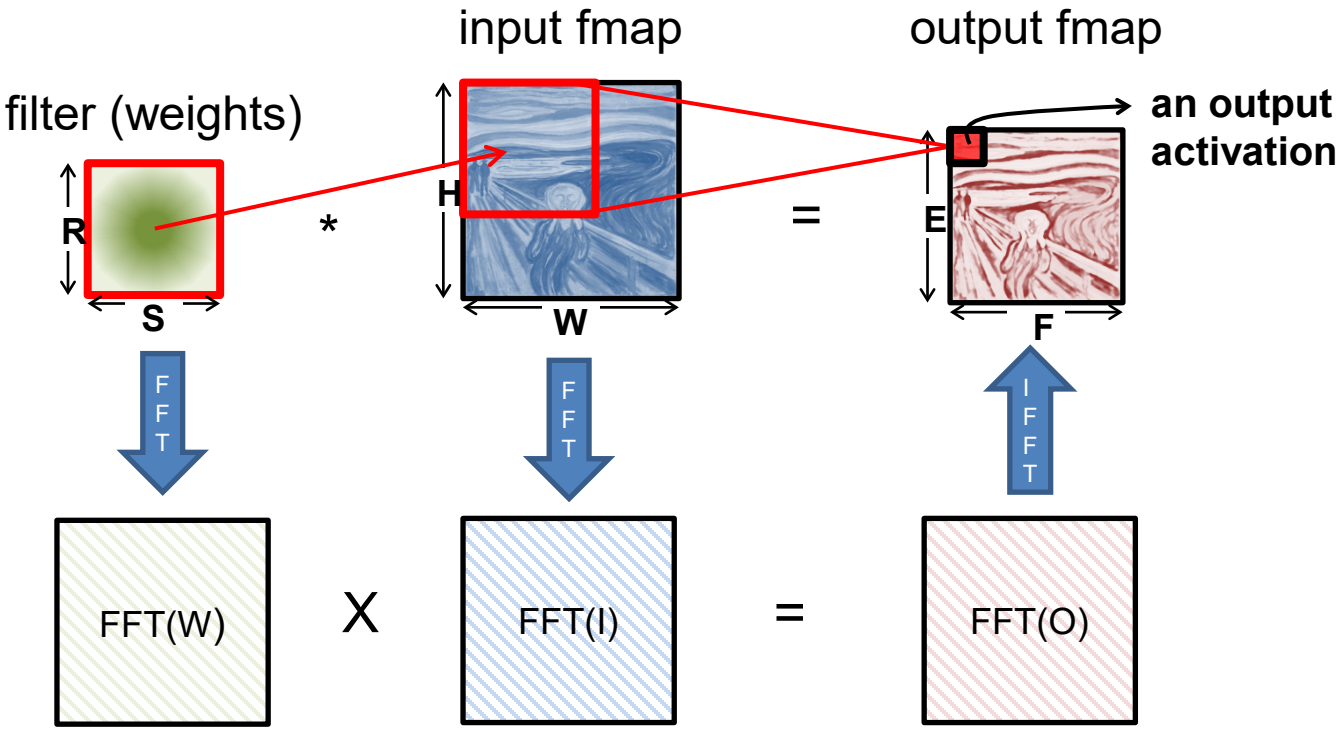
input $i = \begin{bmatrix} i_0 & i_1 & i_2 & i_3 \end{bmatrix}^T$



Note: GfG^T can be precomputed

[**Lavin**, *ArXiv* 2015]

Fast Fourier Transform (FFT) Flow



FFT Overview

- Convert filter and input to frequency domain to make convolution a simple multiply then convert back to space domain.

- Convert direct convolution $O(N_o^2 N_f^2)$ computation to $O(N_o^2 \log_2 N_o)$

[**Mathieu**, *ArXiv* 2013], [**Vasilache**, *ArXiv* 2014]

FFT Overview

- Convert filter and input to frequency domain to make convolution a simple multiply then convert back to space domain.

- Convert direct convolution $O(N_o^2 N_f^2)$ computation to $O(N_o^2 \log_2 N_o)$

- Note that computational benefit of FFT decreases with decreasing size of filter

[**Mathieu**, *ArXiv* 2013], [**Vasilache**, *ArXiv* 2014]

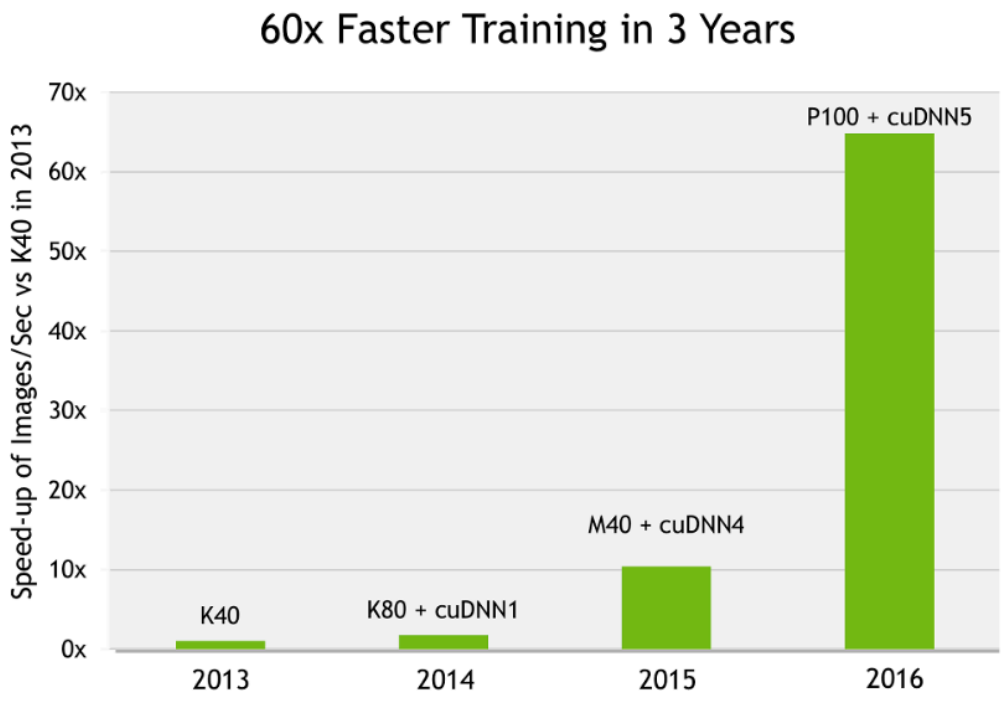
FFT Costs

- Input and Filter matrices are ‘0-completed’,
  - i.e., expanded to size P+R-1 x Q+S-1
- Frequency domain matrices are same dimensions as input, but complex.
- FFT often reduces computation, but requires much more memory space and bandwidth

Optimization opportunities

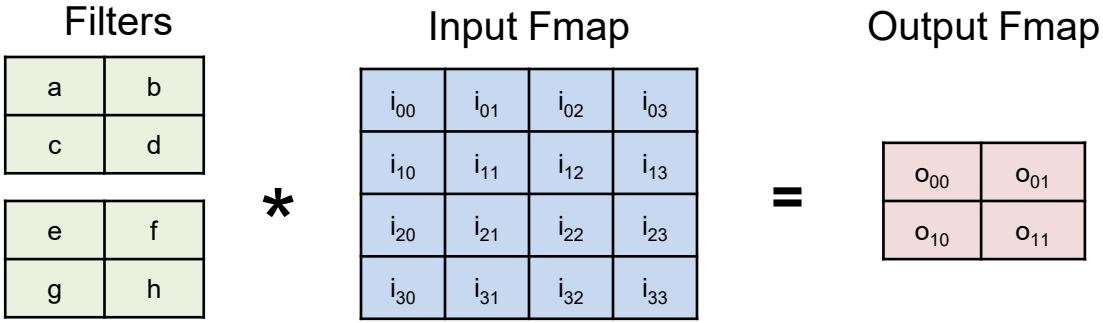
- FFT of real matrix is symmetric allowing one to save ½ the computes
- Filters can be pre-computed and stored, but convolutional filter in frequency domain is much larger than in space domain
- Can reuse frequency domain version of input for creating different output channels to avoid FFT re-computations
- Can accumulate across channels before performing inverse transform to reduce number of IFFT

cuDNN: Speed up with Transformations



Source: Nvidia

UCNN – Convolution (Simplified)

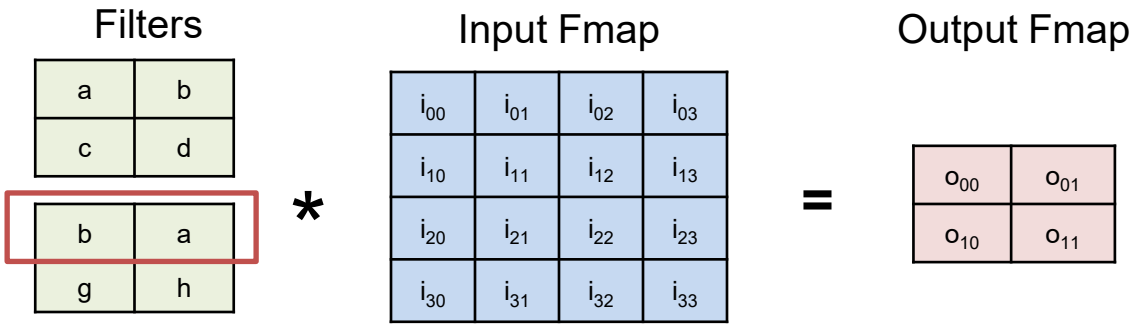


$$o_{00} = a\, i_{00} + b\, i_{01} + \mathrm{c}\, i_{10} + d\, i_{11} + e\, i_{00} + f\, i_{01} + g\, i_{10} + \mathrm{h}\, i_{11}$$

7 additions
8 multiplications

[**Hegde**, *ISCA* 2018]

UCNN – Convolution (Simplified)



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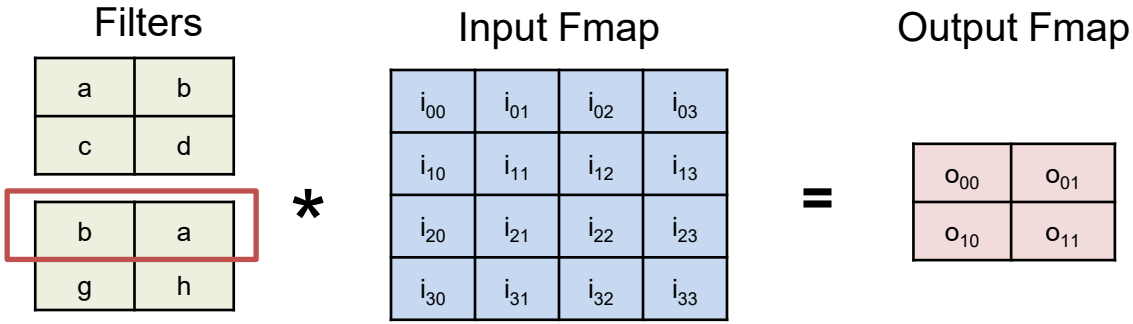

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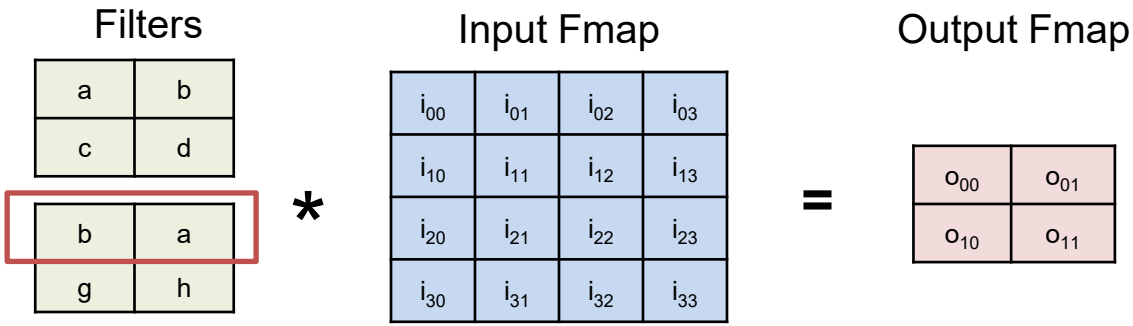

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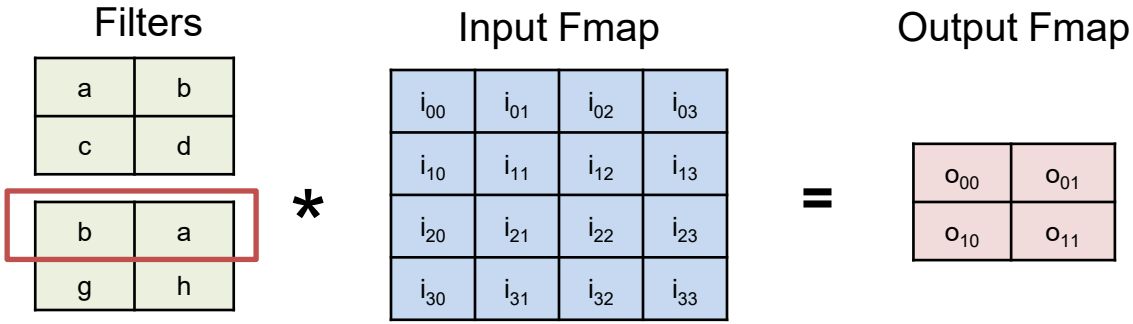
$$o_{00} = (a+b) i_{00} + (a+b)\, i_{01} + \mathrm{c}\, i_{10} + d\, i_{11} + g\, i_{10} + \mathrm{h}\, i_{11}$$

7 additions
8 multiplications

[**Hegde**, *ISCA* 2018]

UCNN – Convolution (Simplified)



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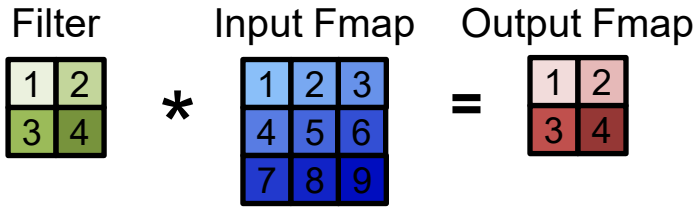
7 additions
8 multiplications

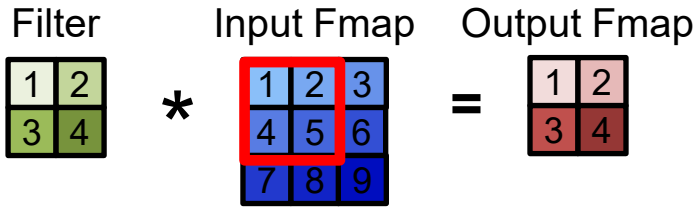
6 additions
6 multiplications

[**Hegde**, *ISCA* 2018]

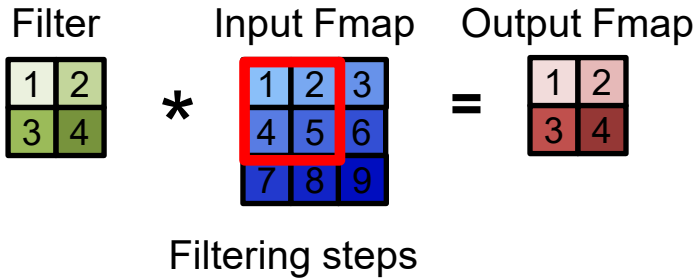
Convolution (CONV) Layer



Convolution (CONV) Layer



Convolution (CONV) Layer



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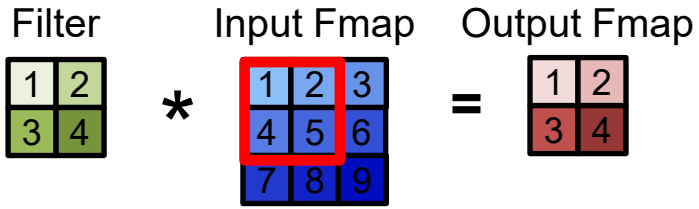

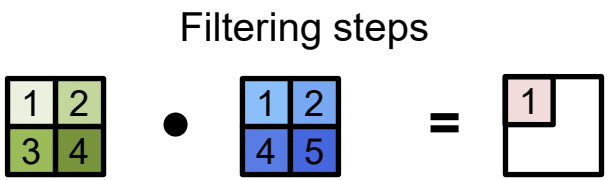

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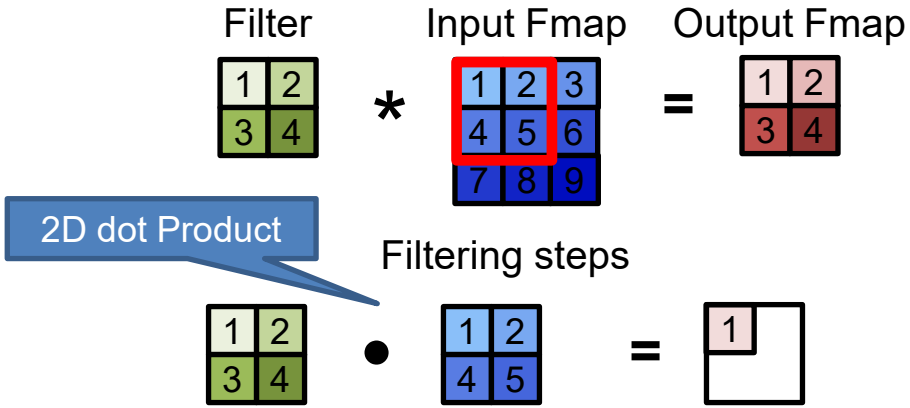

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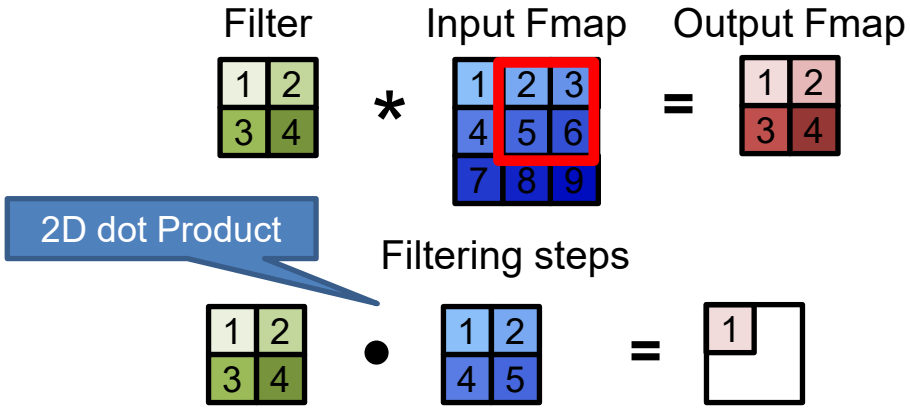

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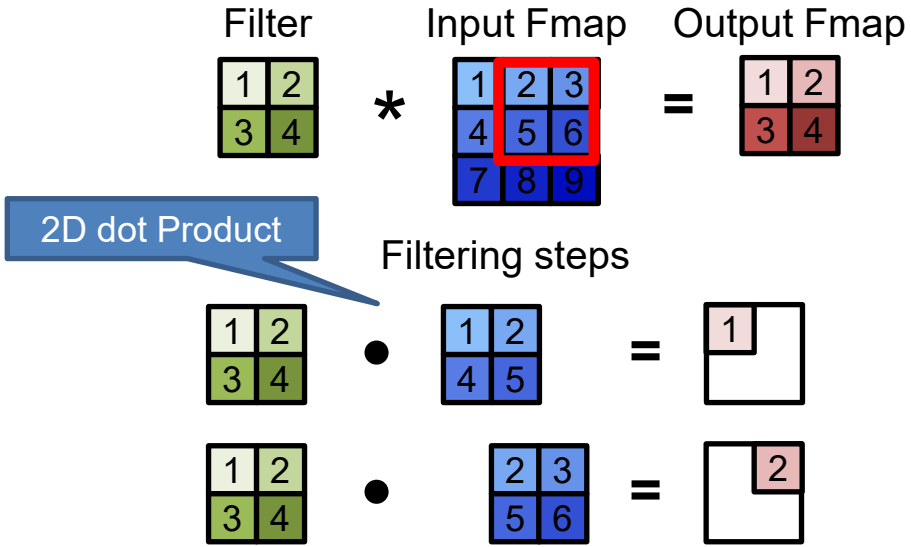

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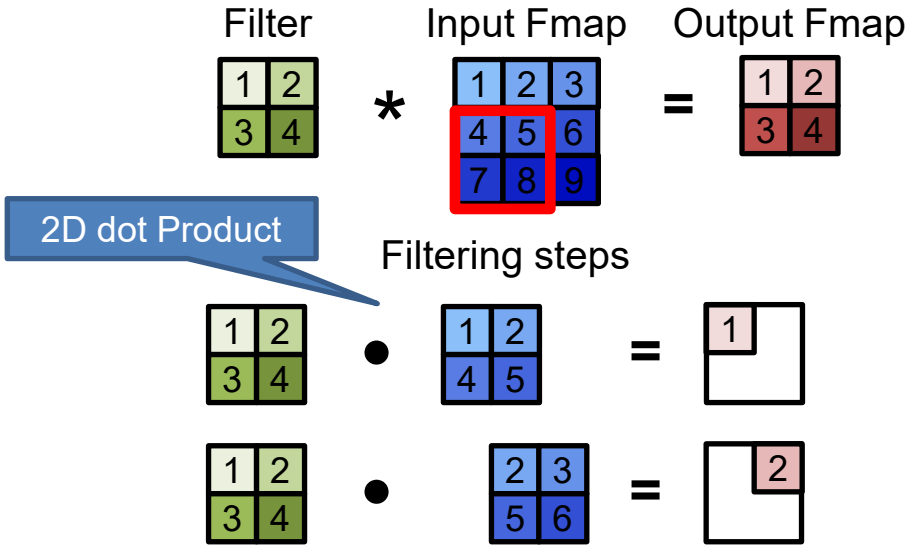

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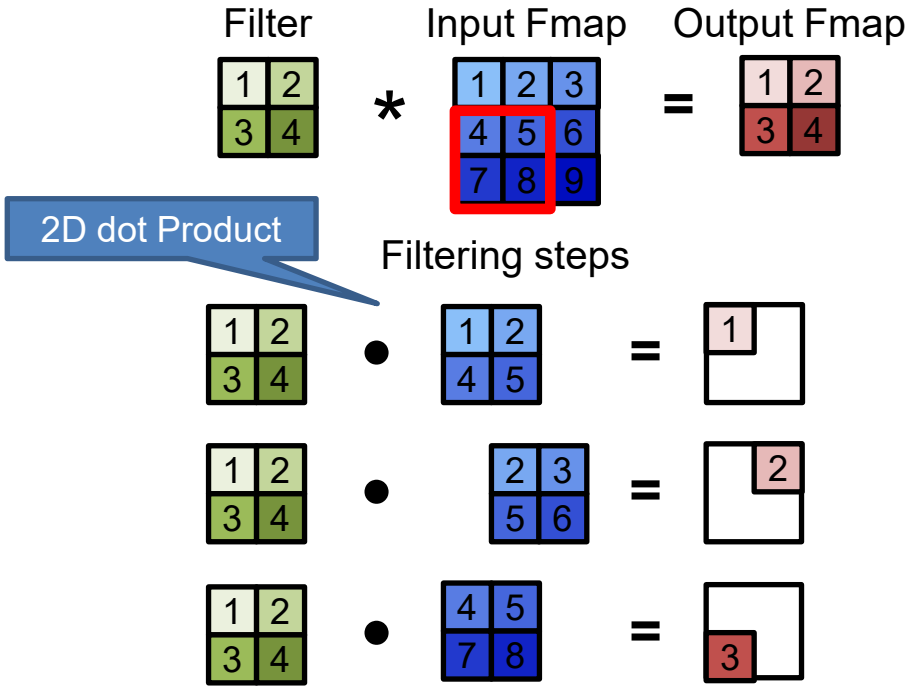

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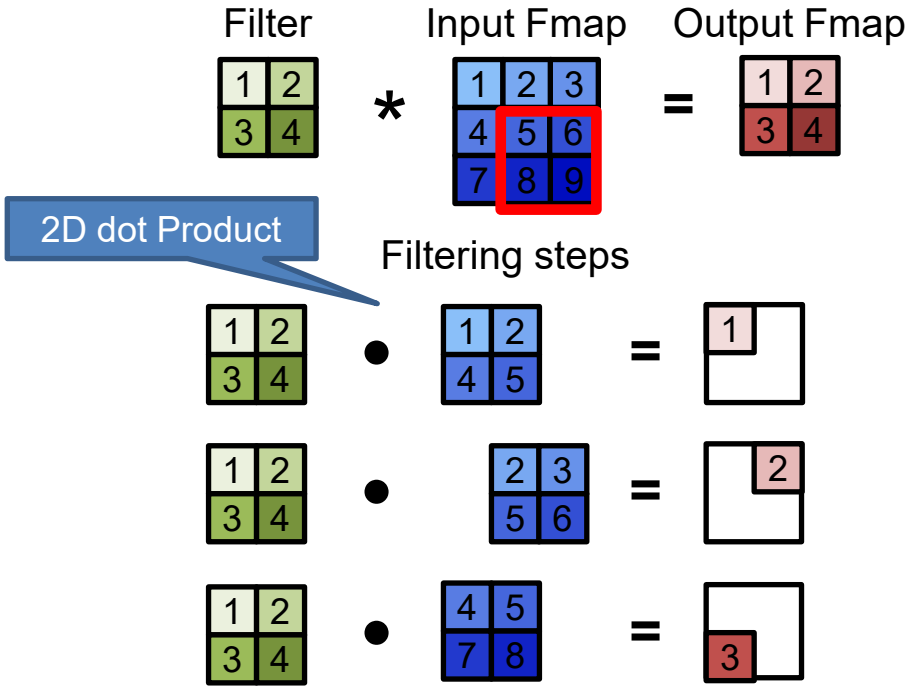

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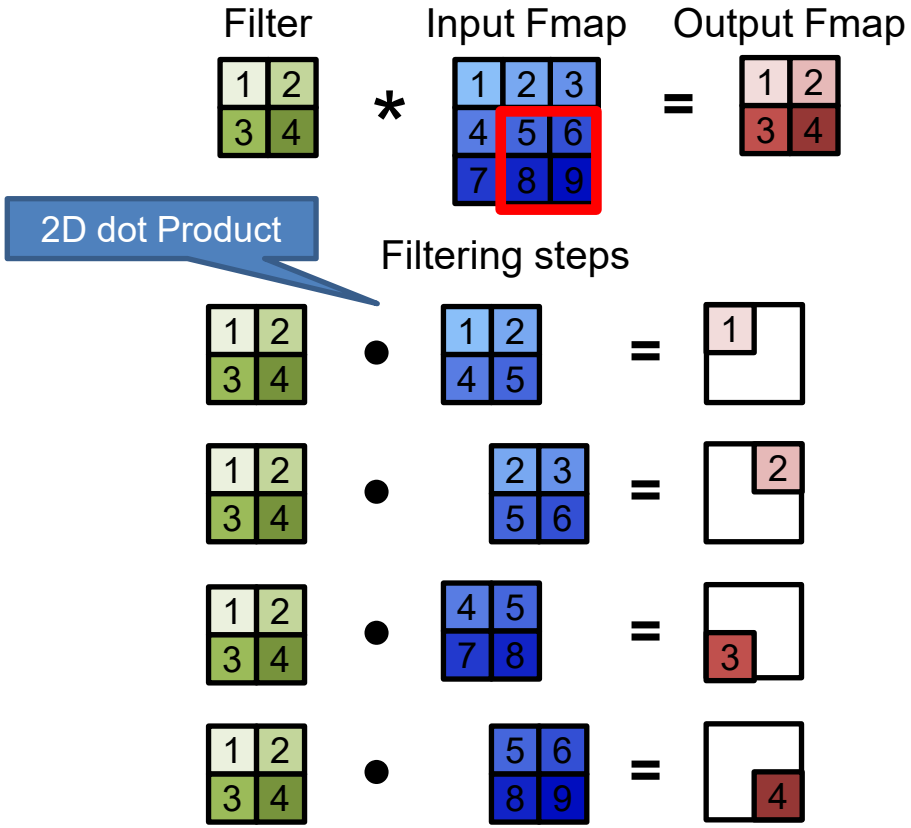

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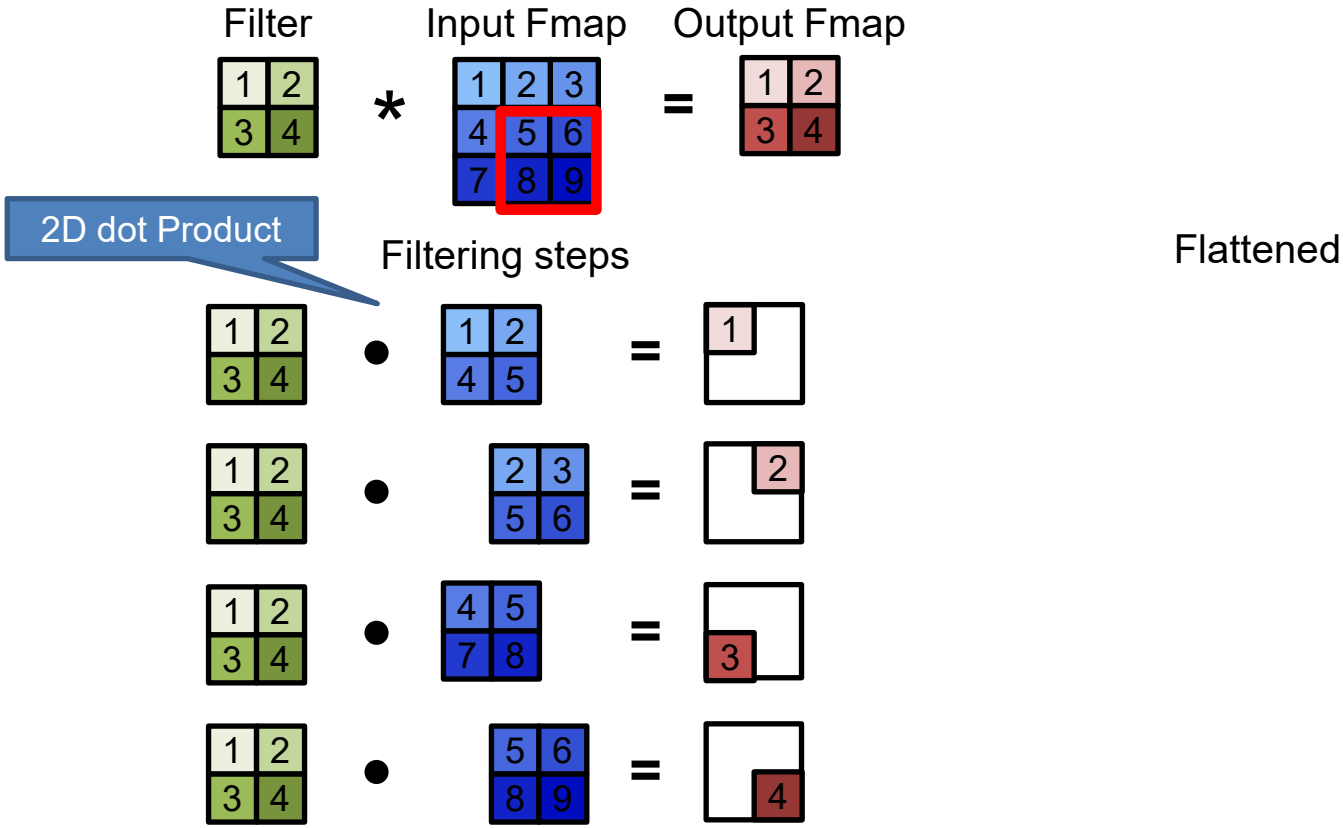

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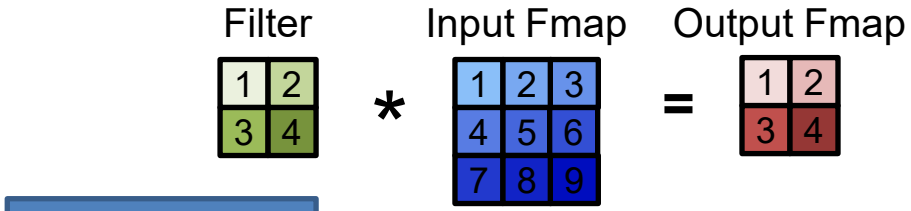
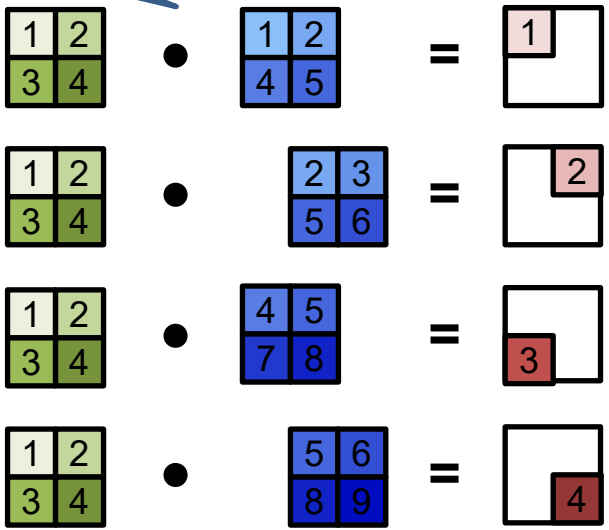


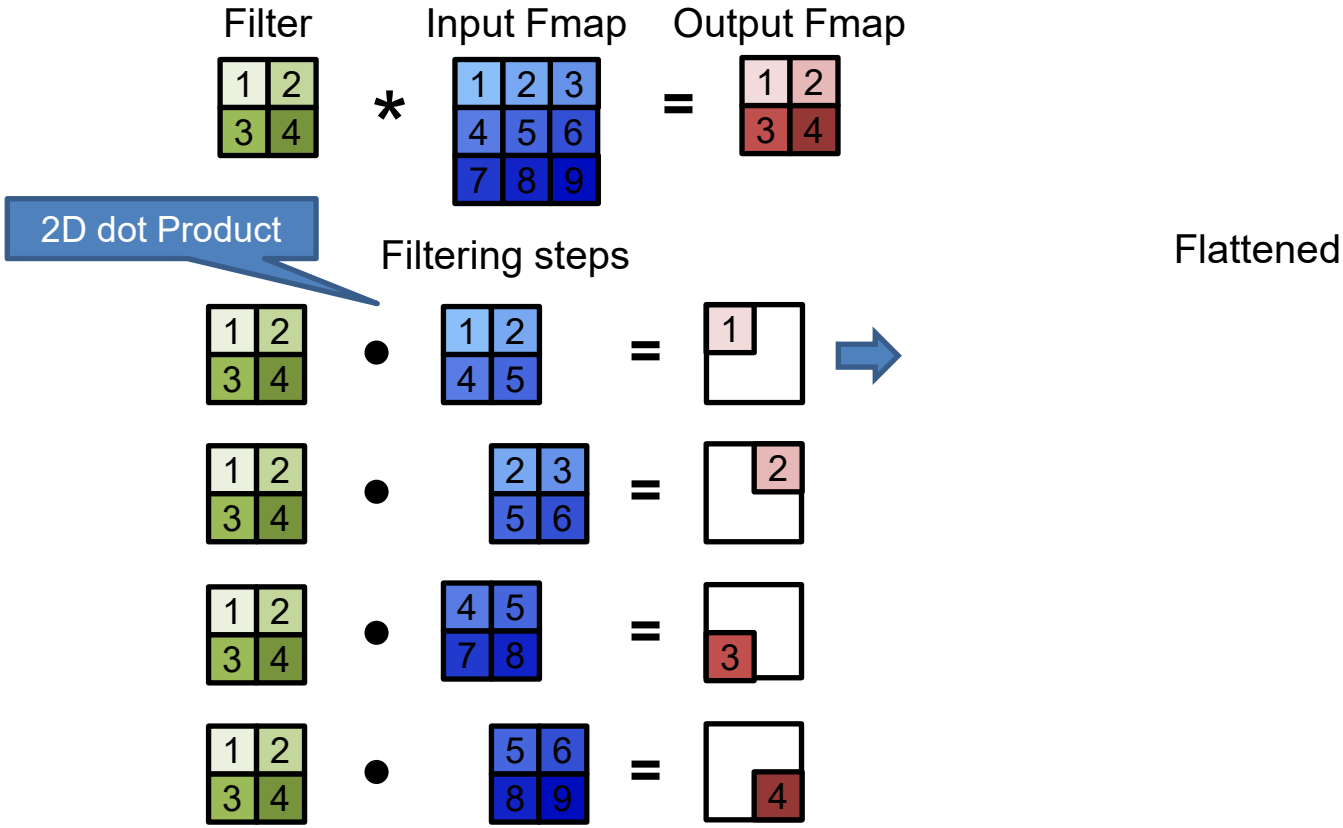
2D dot Product

Filtering steps



Flattened

Convolution (CONV) Layer



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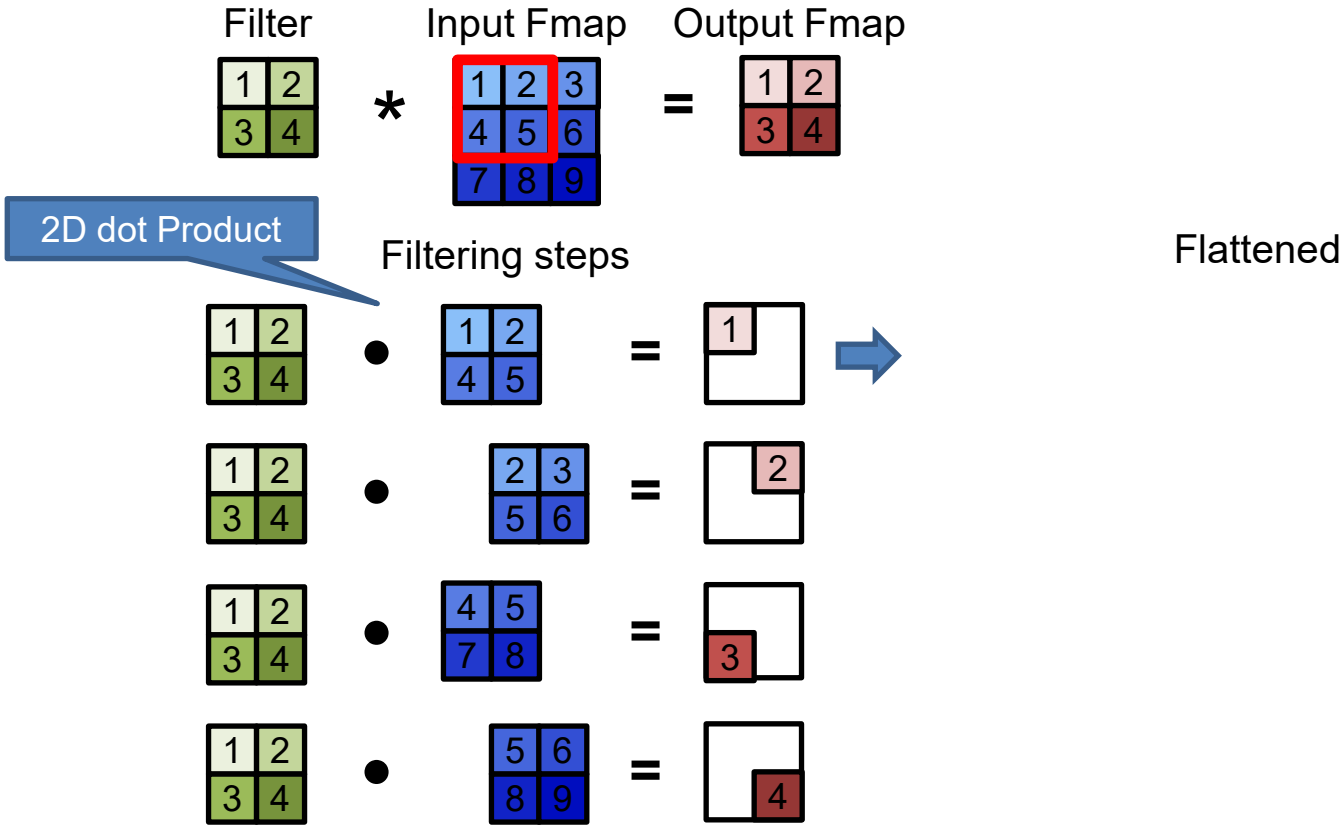

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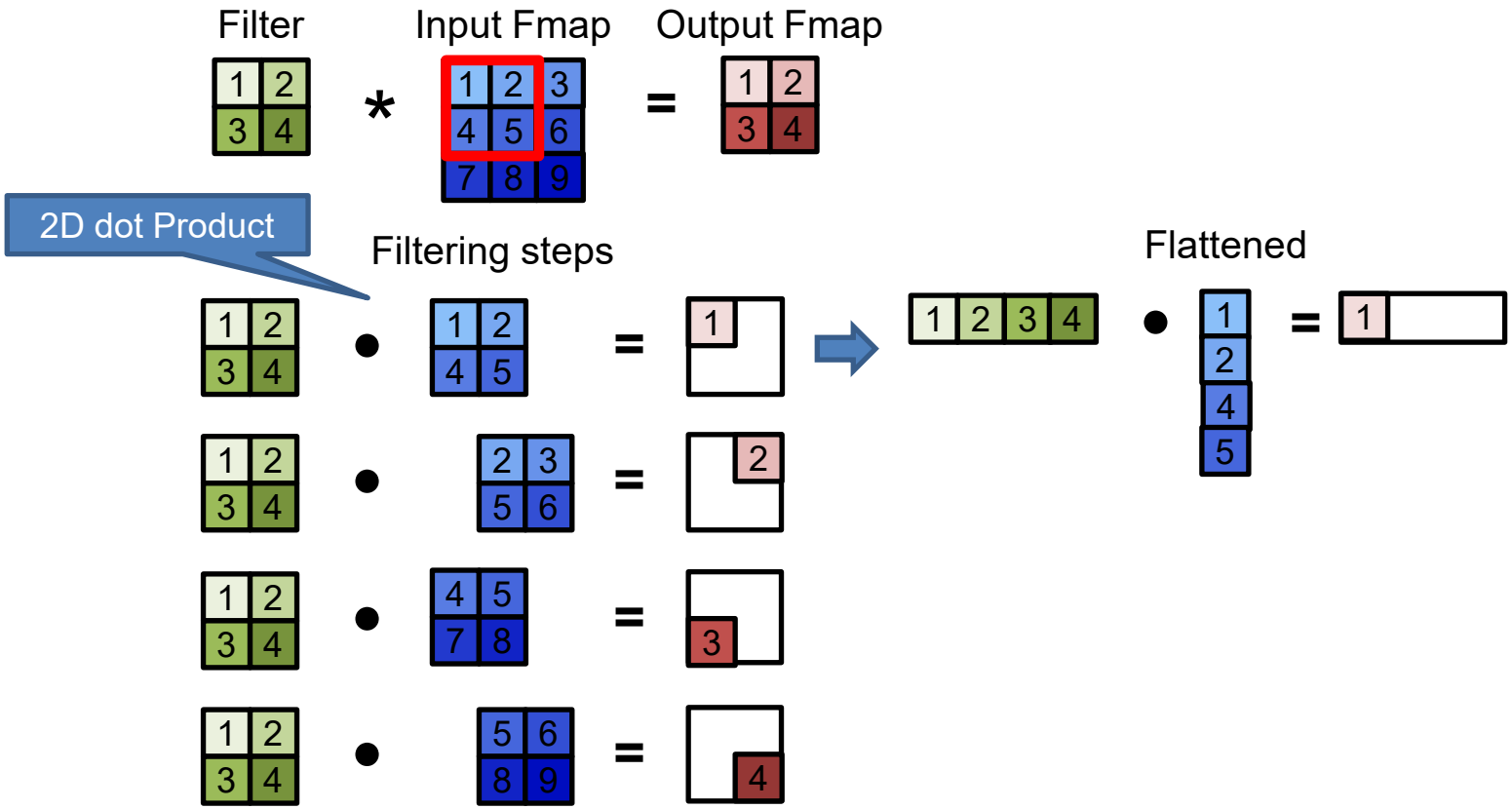

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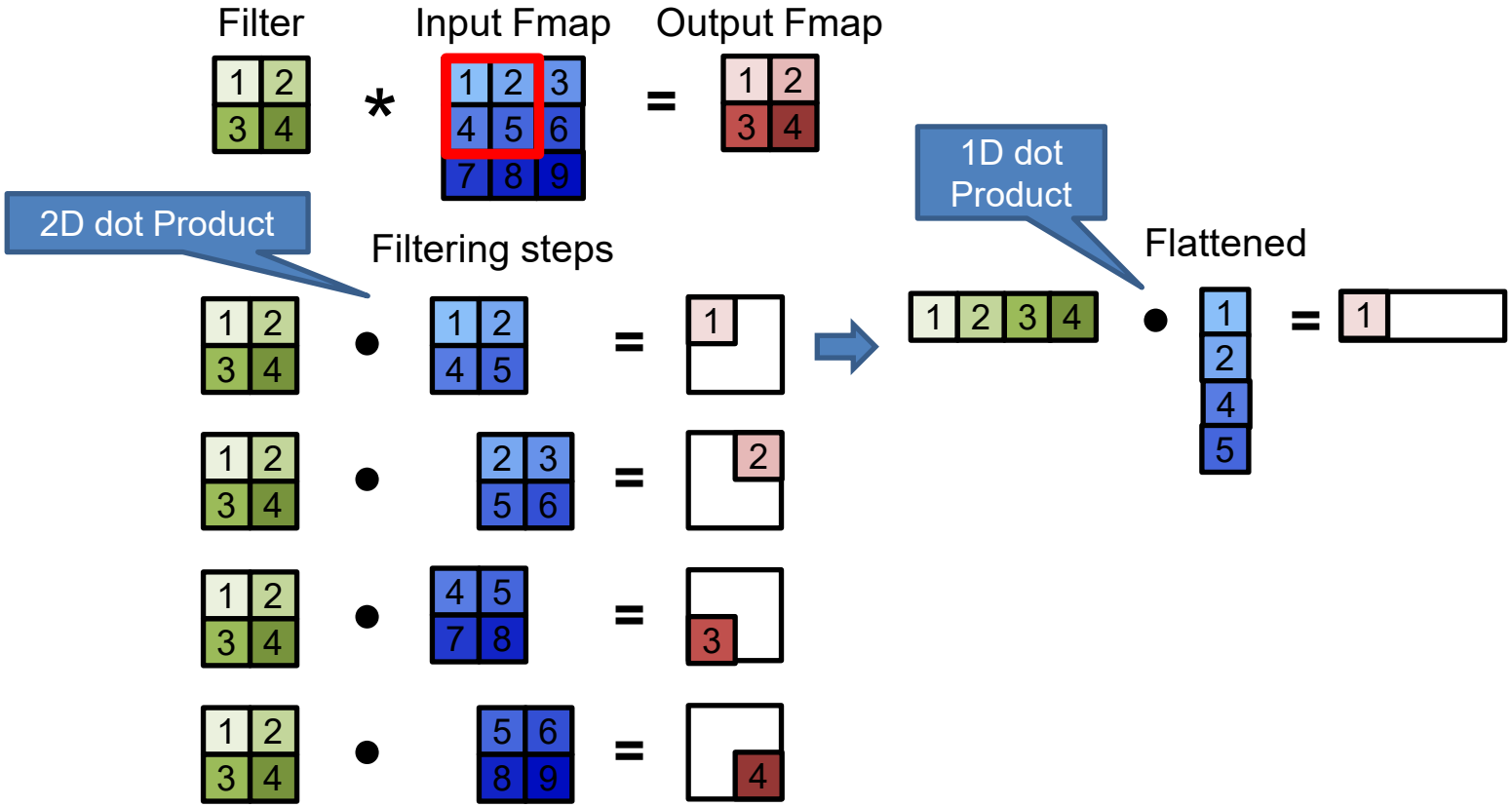

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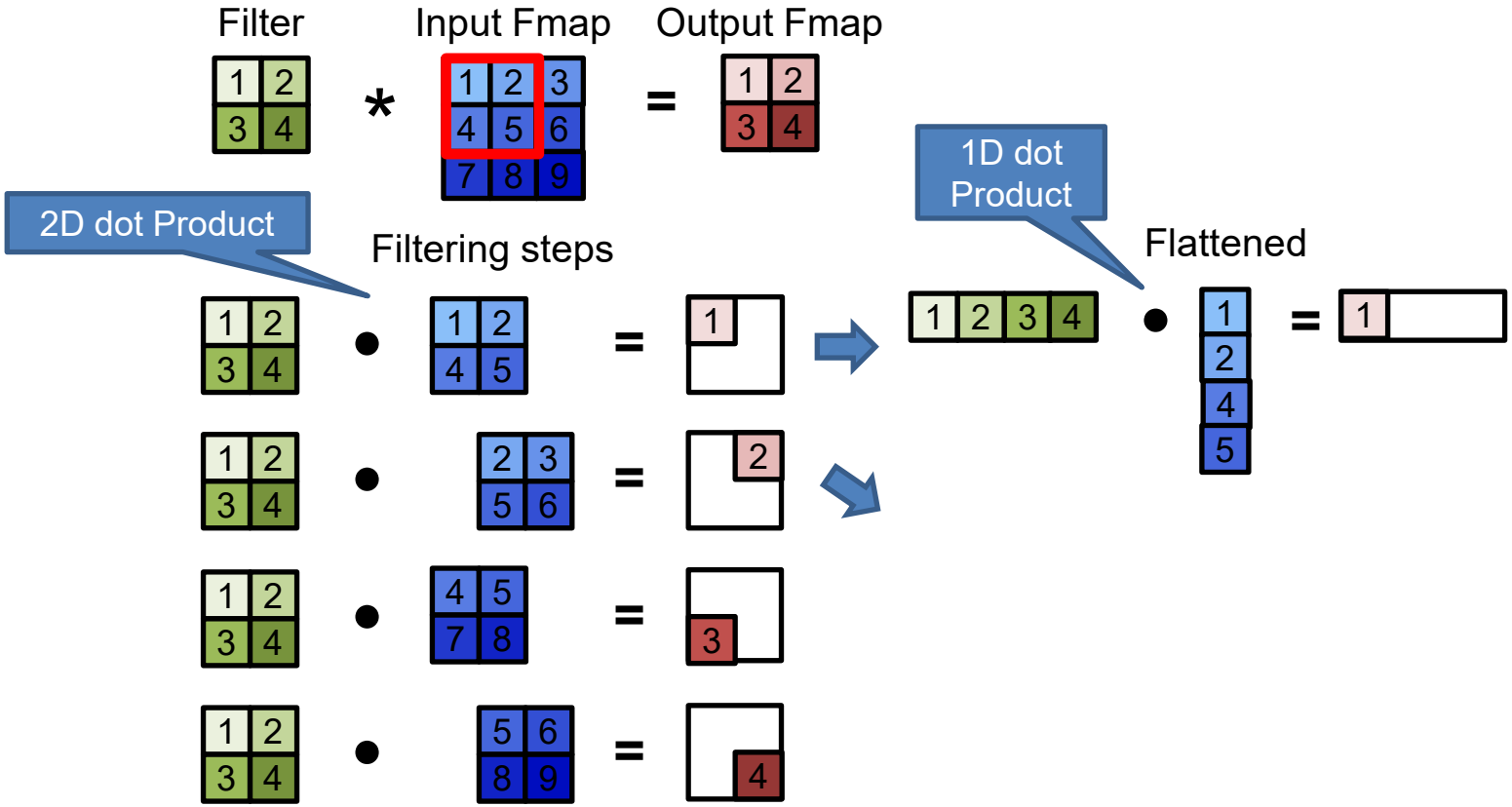

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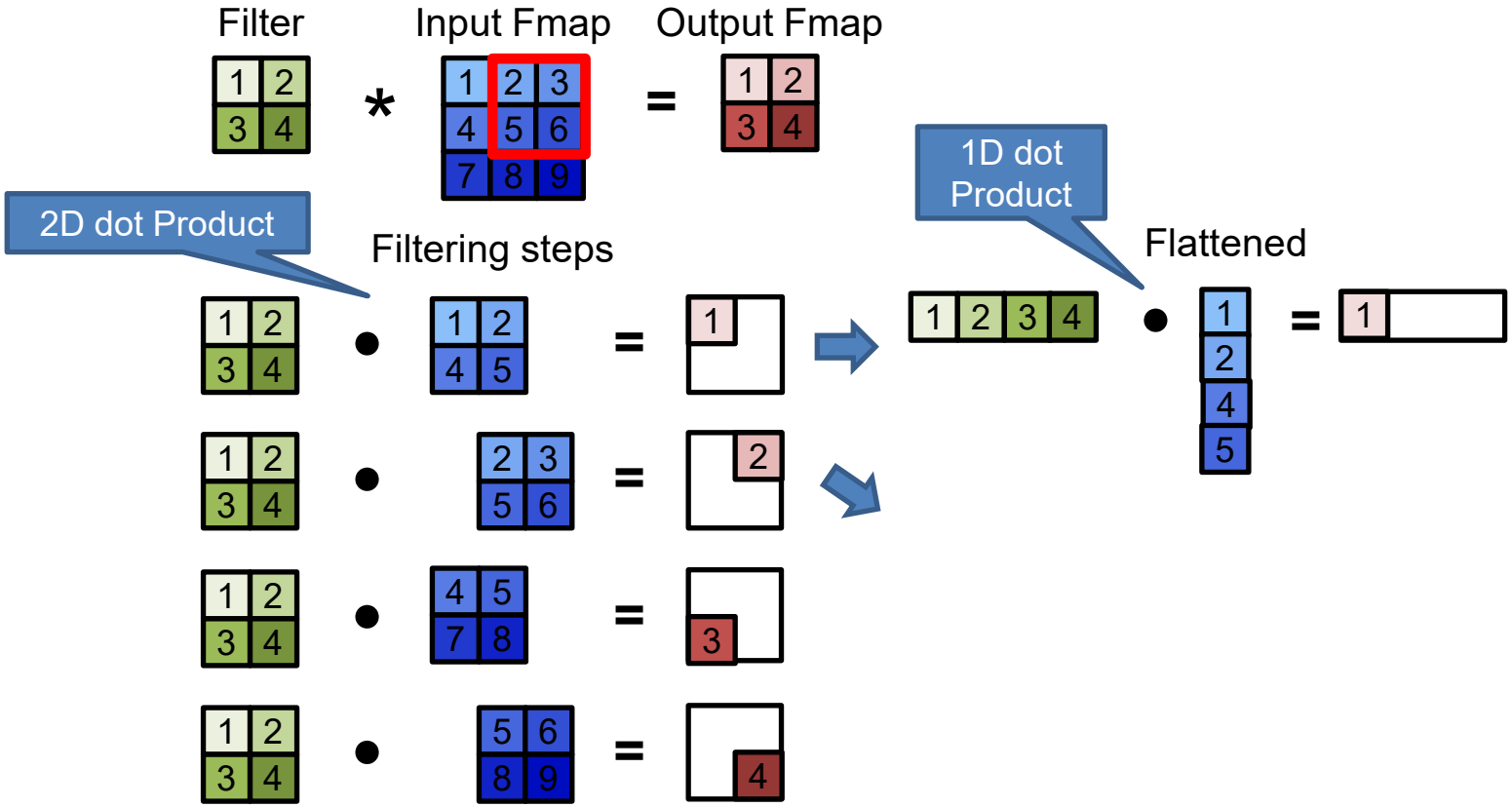

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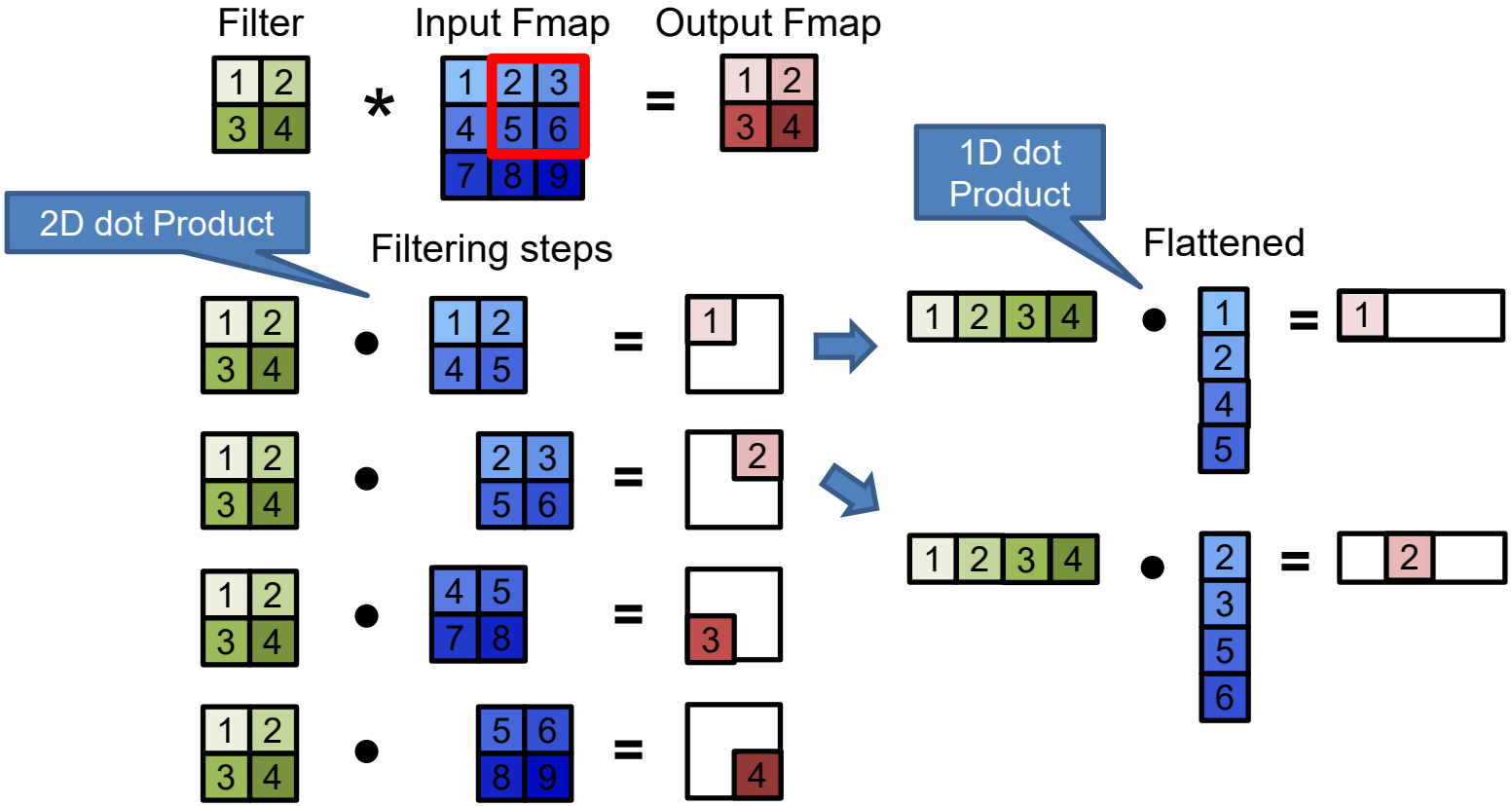

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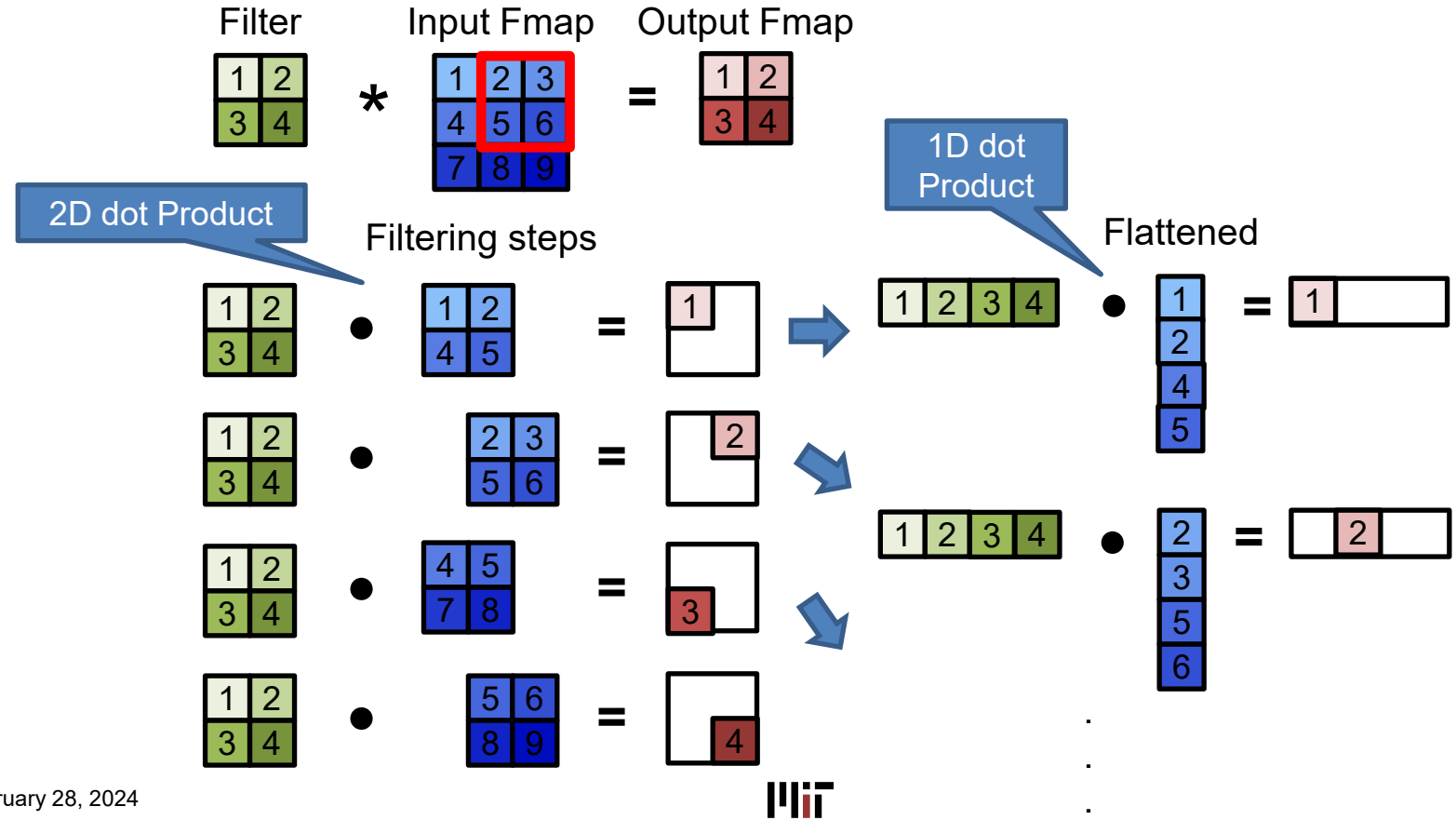

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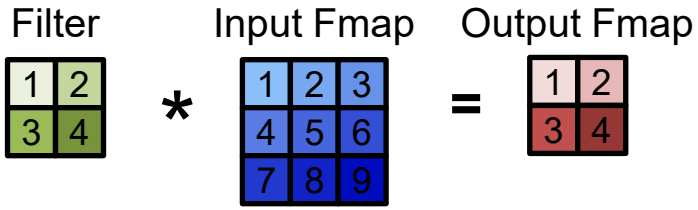
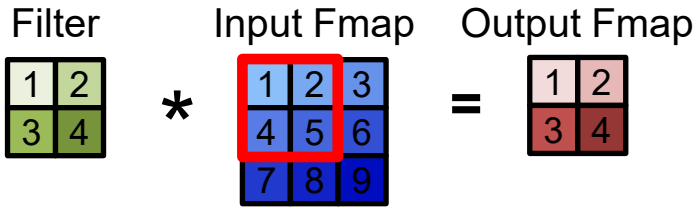


Convolution:

Flattened

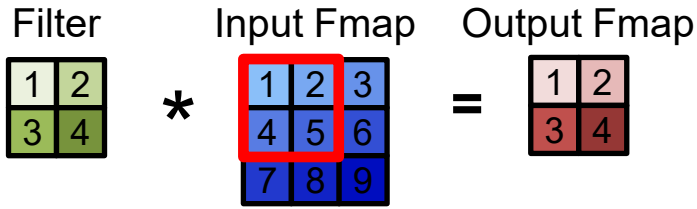
Convolution (CONV) Layer



Convolution:

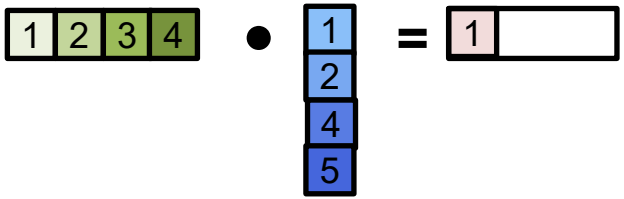
Flattened

Convolution (CONV) Layer



Convolution:

Flattened



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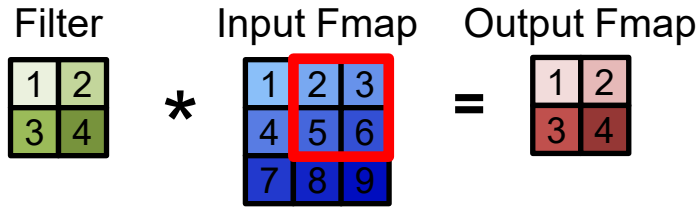
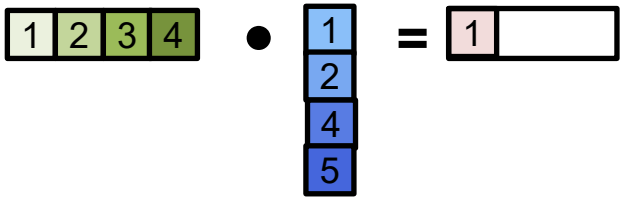
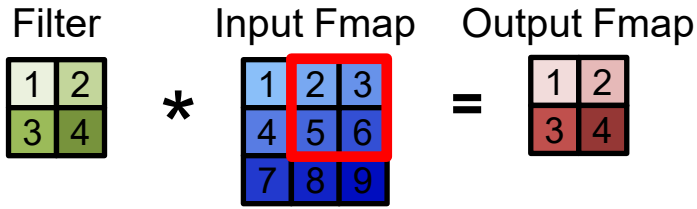


Convolution:

Flattened

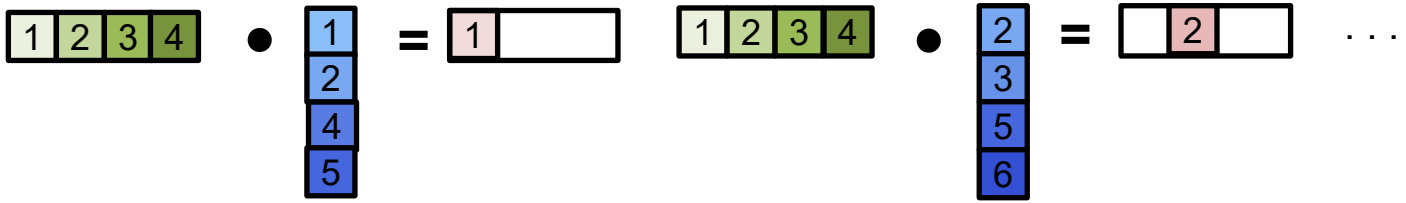


Convolution (CONV) Layer



Convolution:

Flattened



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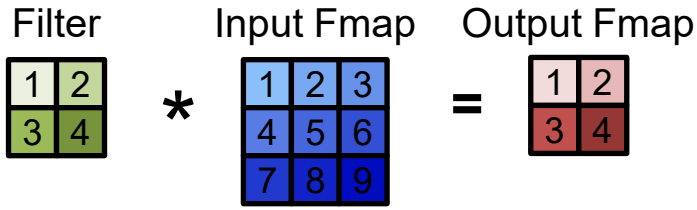
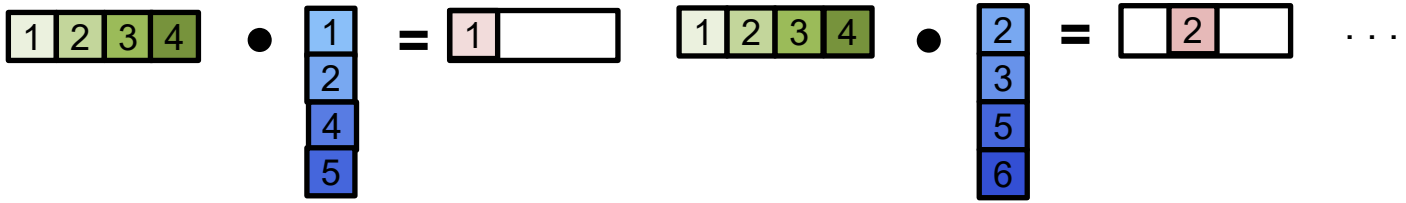


Convolution:

Flattened



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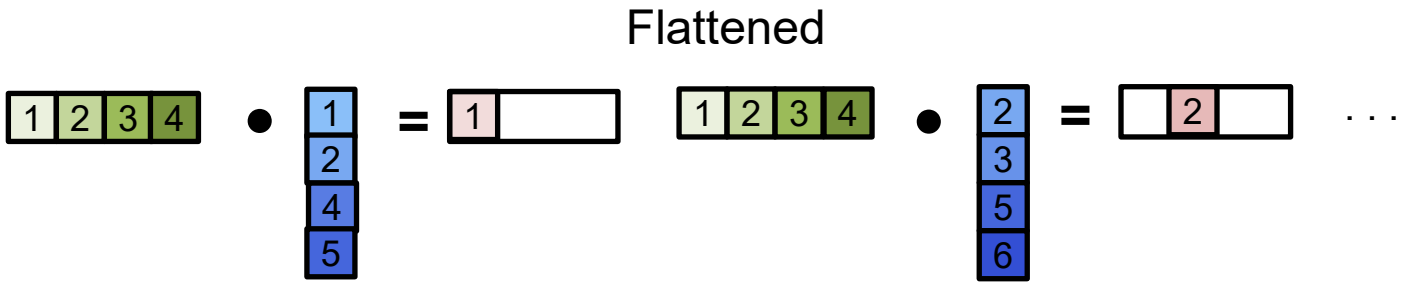

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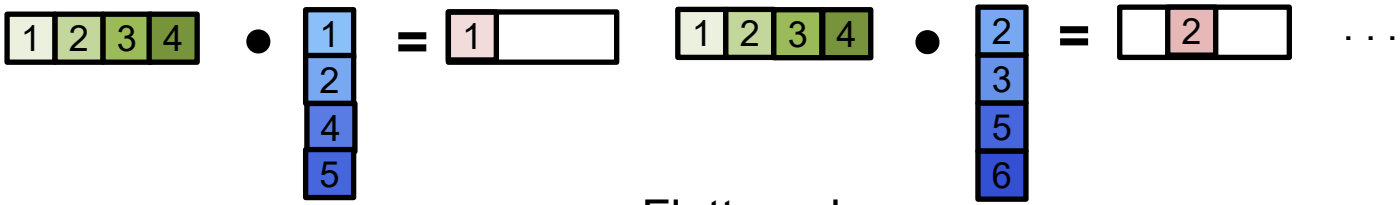
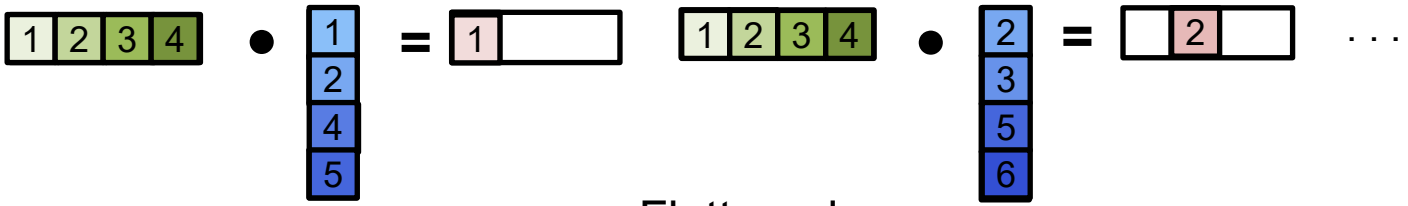


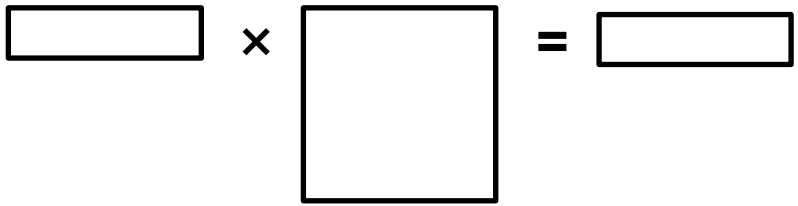
Flattened

Convolution (CONV) Layer

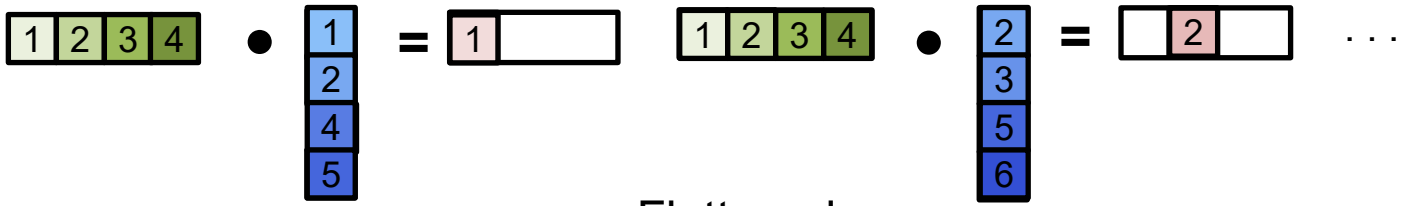


Flattened

Matrix Multiply (by Toeplitz Matrix)

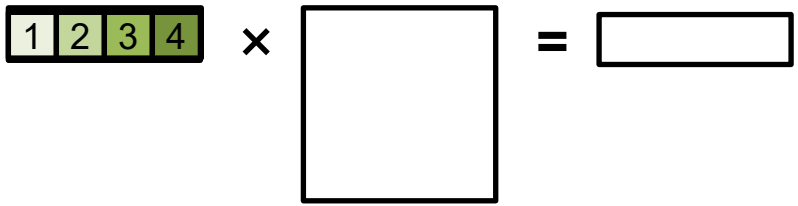


Convolution (CONV) Layer

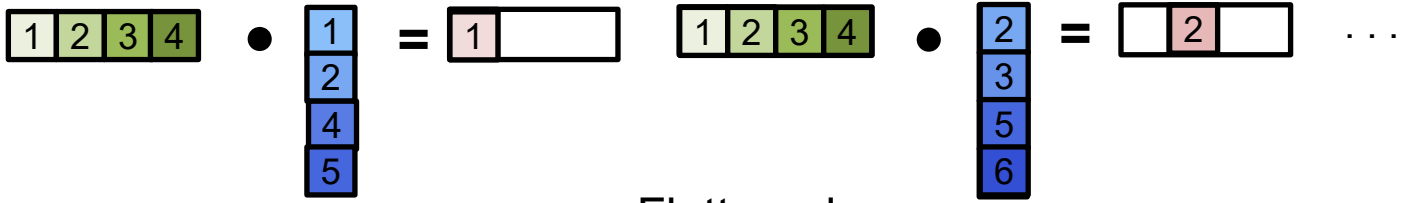


Flattened

Matrix Multiply (by Toeplitz Matrix)

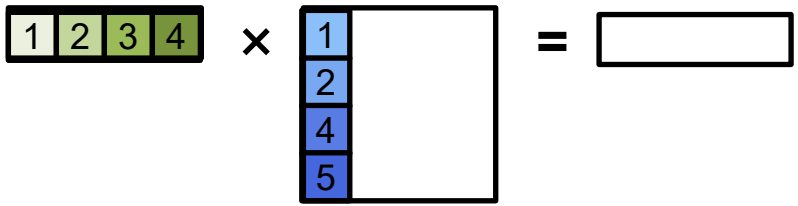


Convolution (CONV) Layer

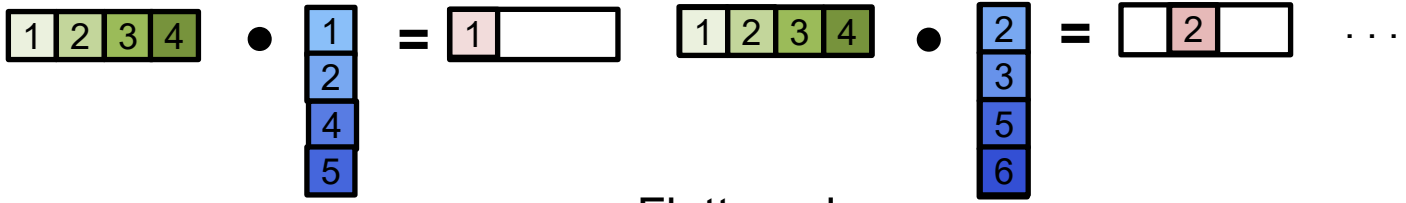


Flattened

Matrix Multiply (by Toeplitz Matrix)

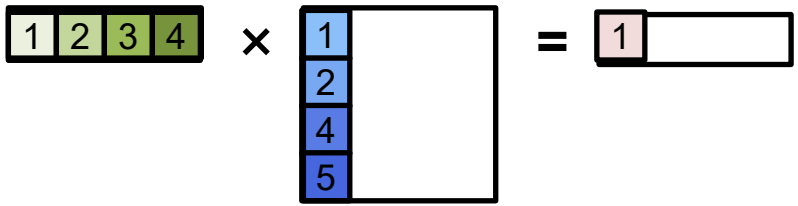


Convolution (CONV) Layer

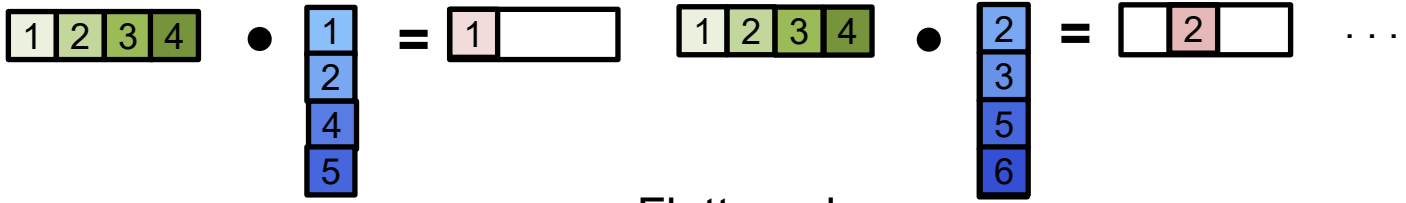


Flattened

Matrix Multiply (by Toeplitz Matrix)

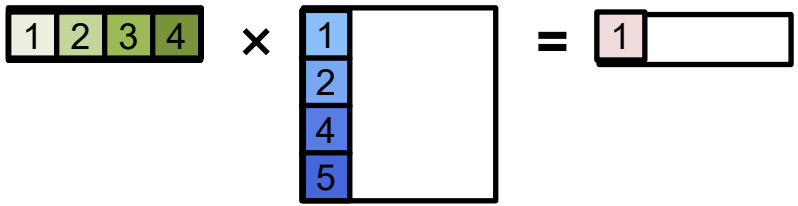


Convolution (CONV) Layer

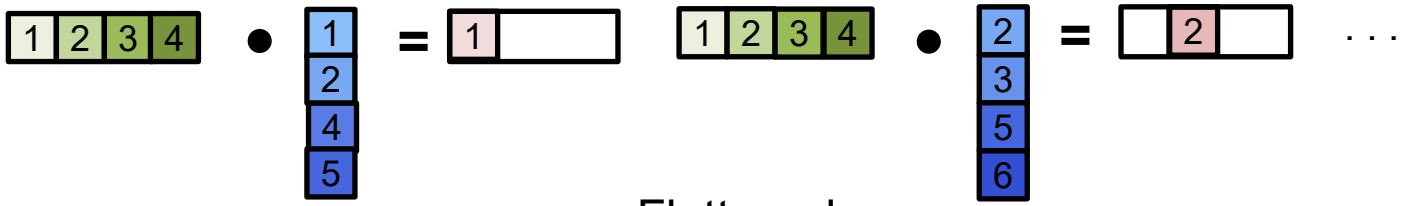


Flattened

Matrix Multiply (by Toeplitz Matrix)



Convolution (CONV) Layer



Flattened

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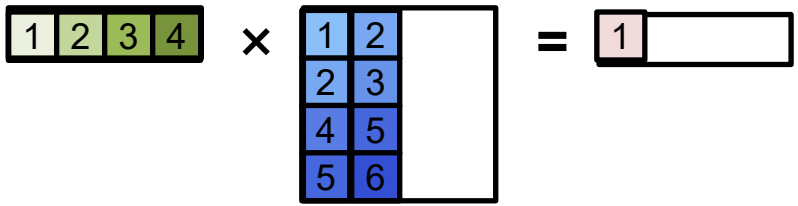
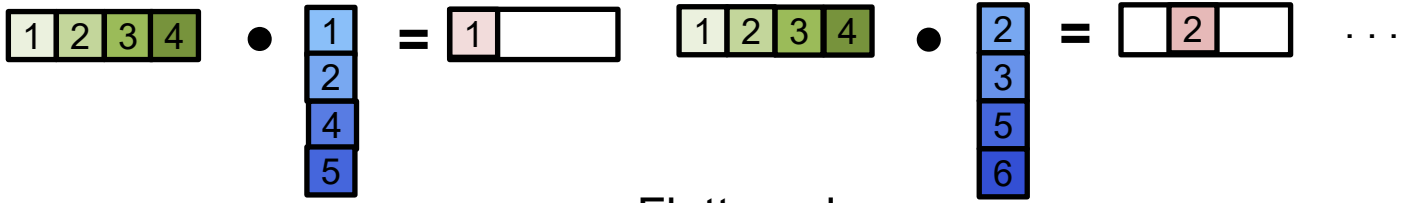
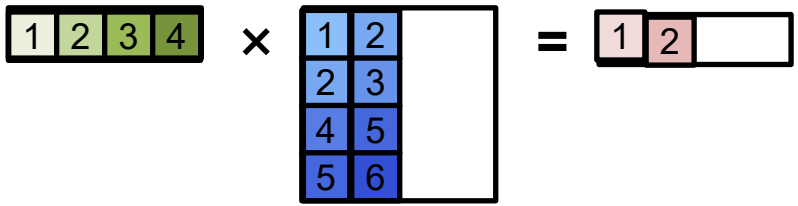


Convolution (CONV) Layer

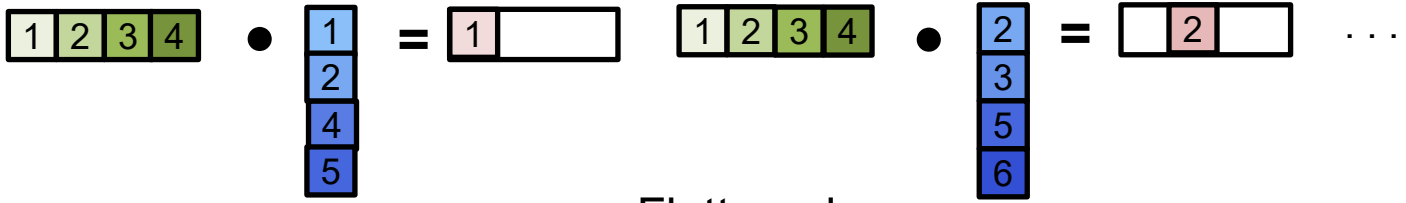


Flattened

Matrix Multiply (by Toeplitz Matrix)



Convolution (CONV) Layer



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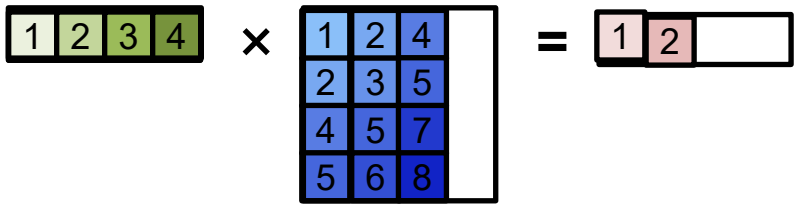
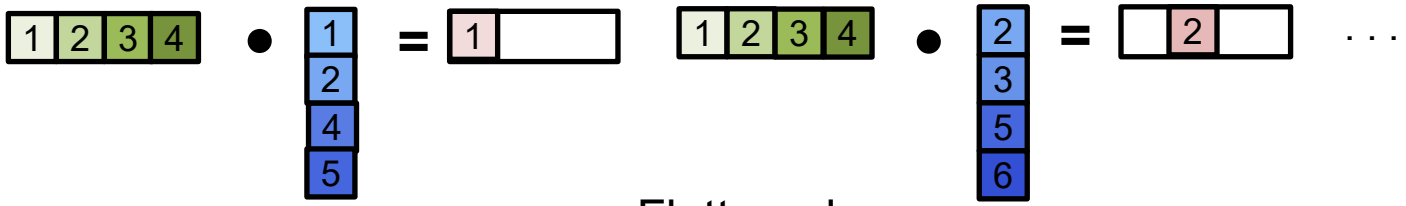
Matrix Multiply (by Toeplitz Matrix)

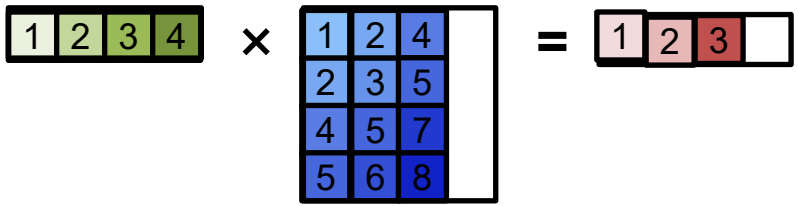


Convolution (CONV) Layer

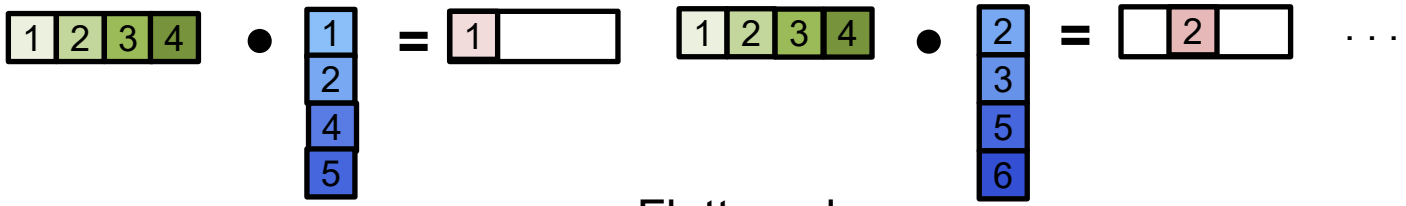


Flattened

Matrix Multiply (by Toeplitz Matrix)

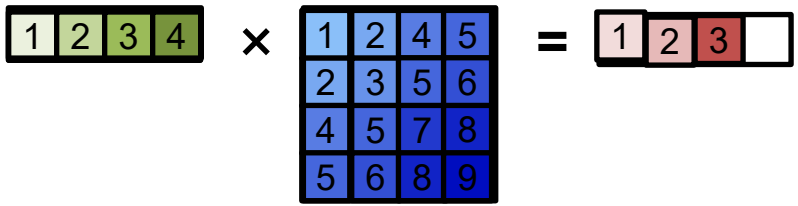


Convolution (CONV) Layer

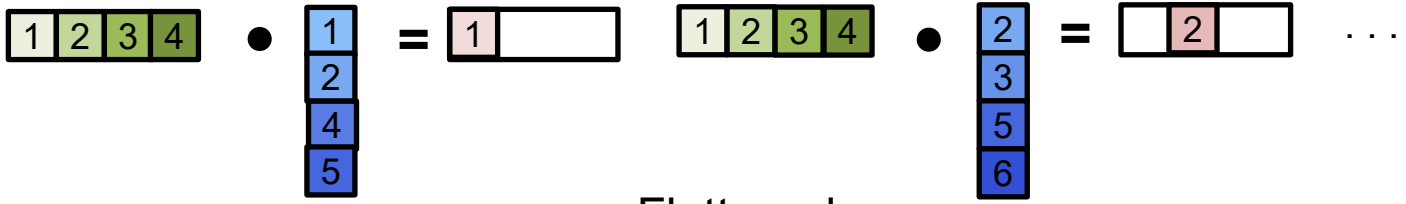


Flattened

Matrix Multiply (by Toeplitz Matrix)

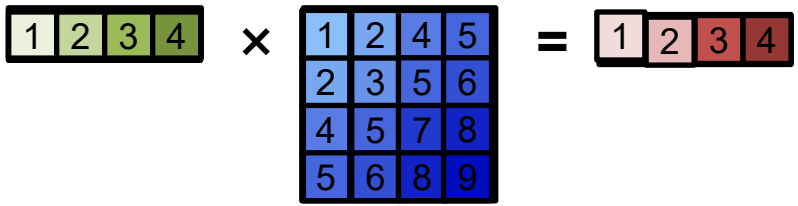


Convolution (CONV) Layer

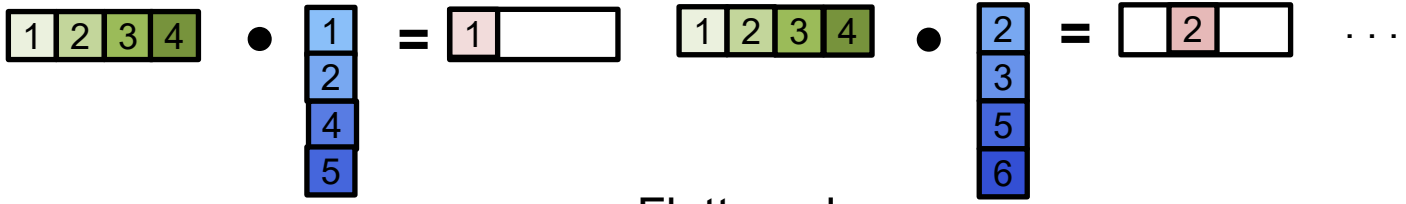


Flattened

Matrix Multiply (by Toeplitz Matrix)

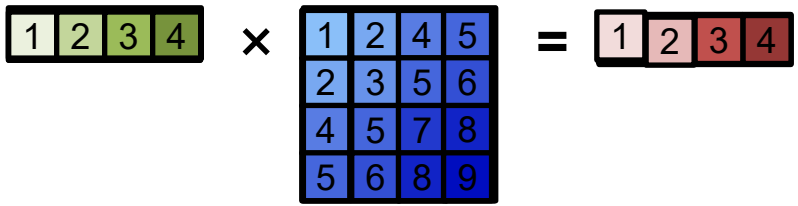


Convolution (CONV) Layer

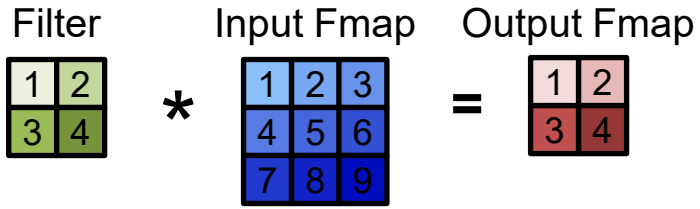


Flattened

Matrix Multiply (by Toeplitz Matrix)

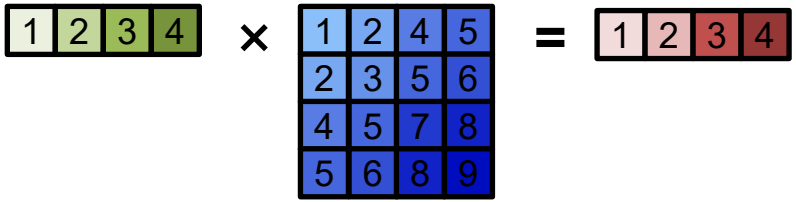


Convolution (CONV) Layer



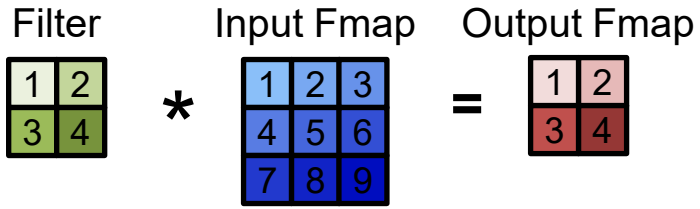
Convolution:

Matrix Multiply (by Toeplitz Matrix)



Convert to matrix multiply using the **Toeplitz Matrix**

Convolution (CONV) Layer



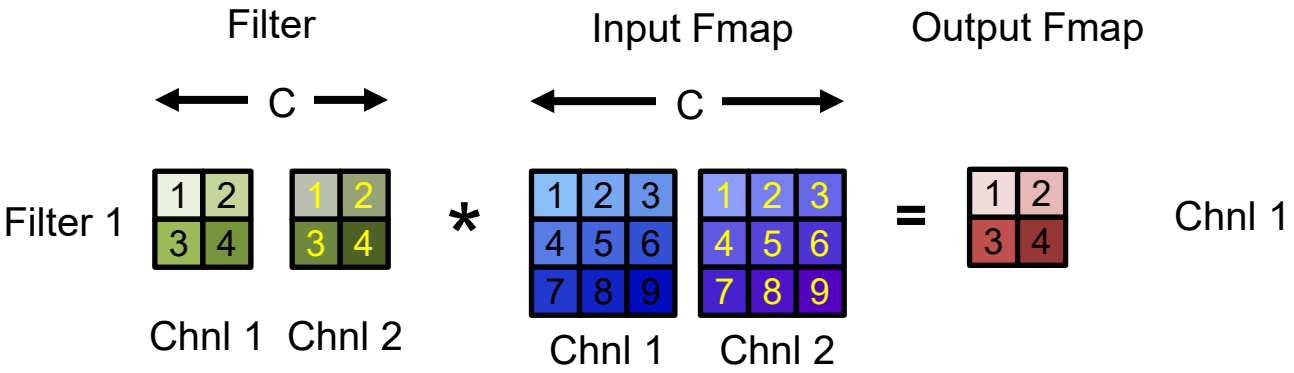
Convolution:

Matrix Multiply (by Toeplitz Matrix)

Data is repeated

Convolution (CONV) Layer

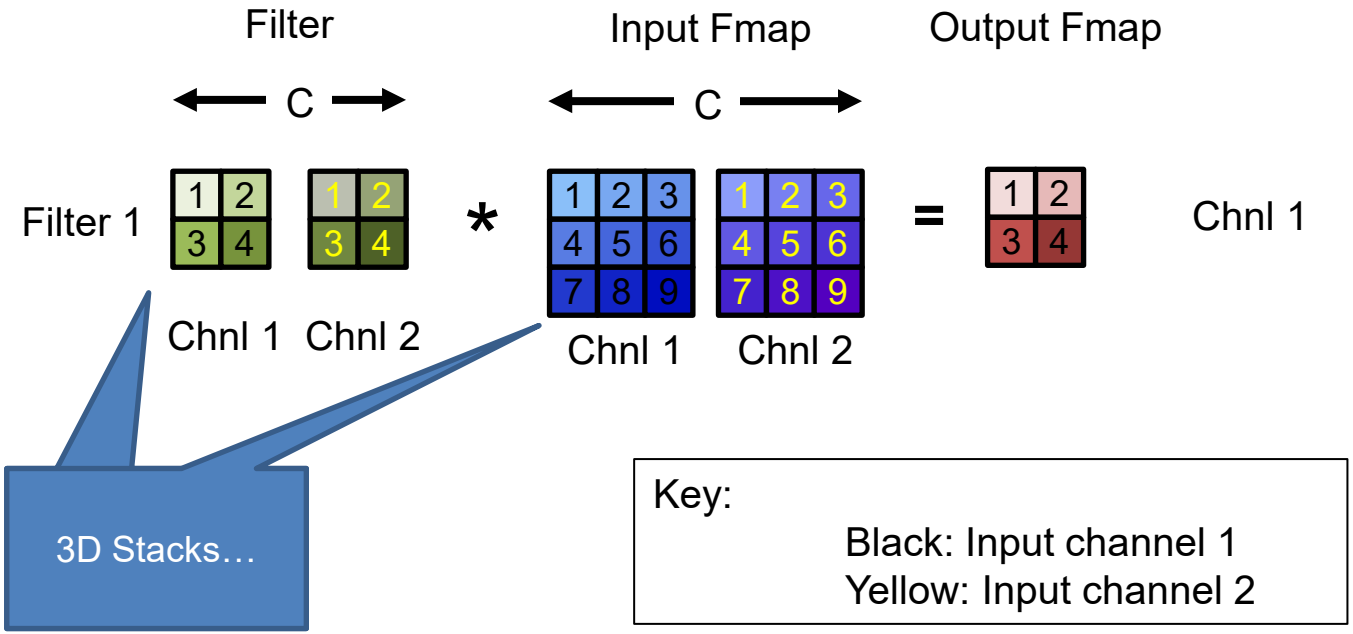
- Multiple Input Channels



Key:
Black: Input channel 1
Yellow: Input channel 2

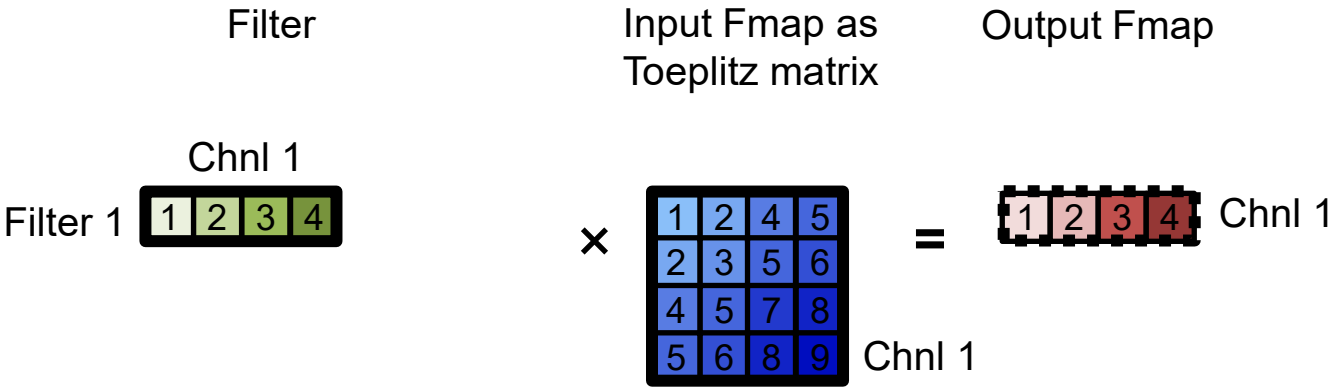
Convolution (CONV) Layer

- Multiple Input Channels



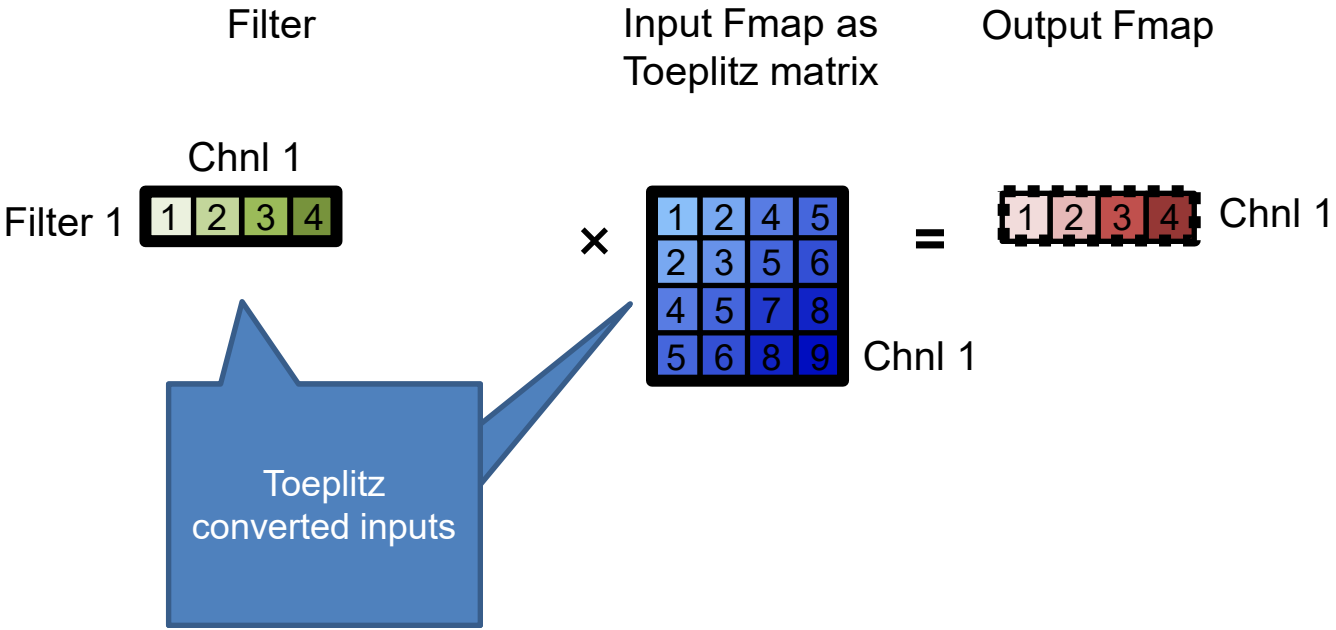
Convolution (CONV) Layer

- Multiple Input Channels



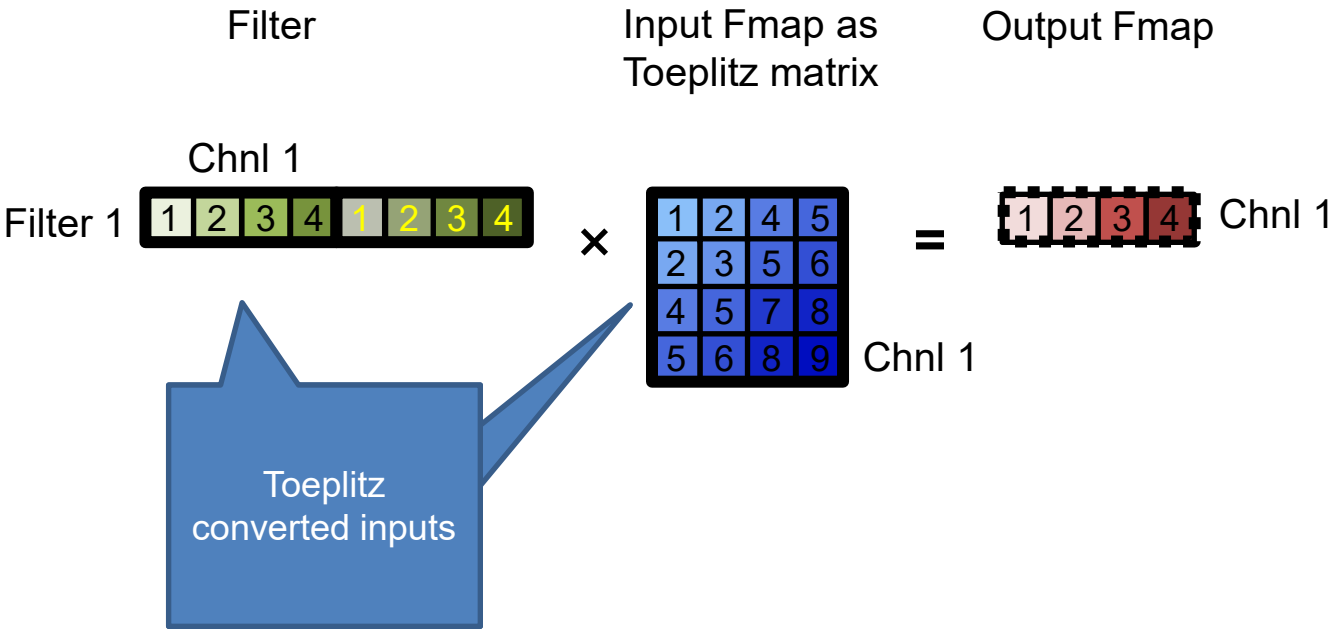
Convolution (CONV) Layer

- Multiple Input Channels



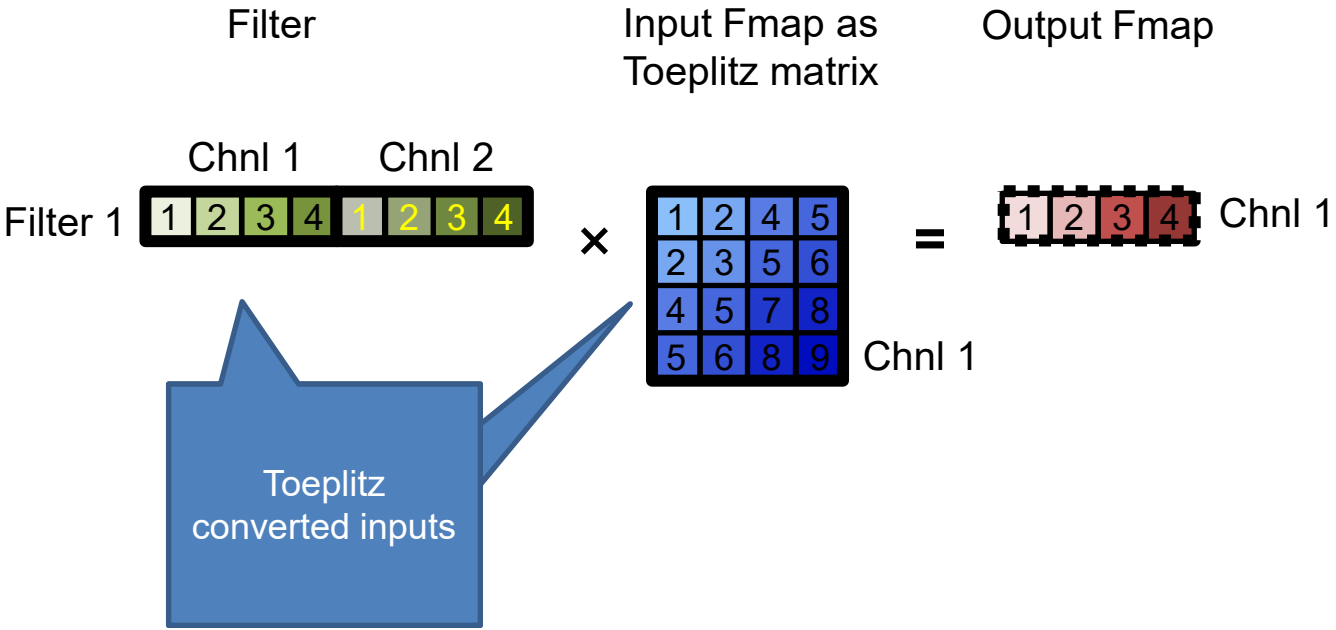
Convolution (CONV) Layer

- Multiple Input Channels



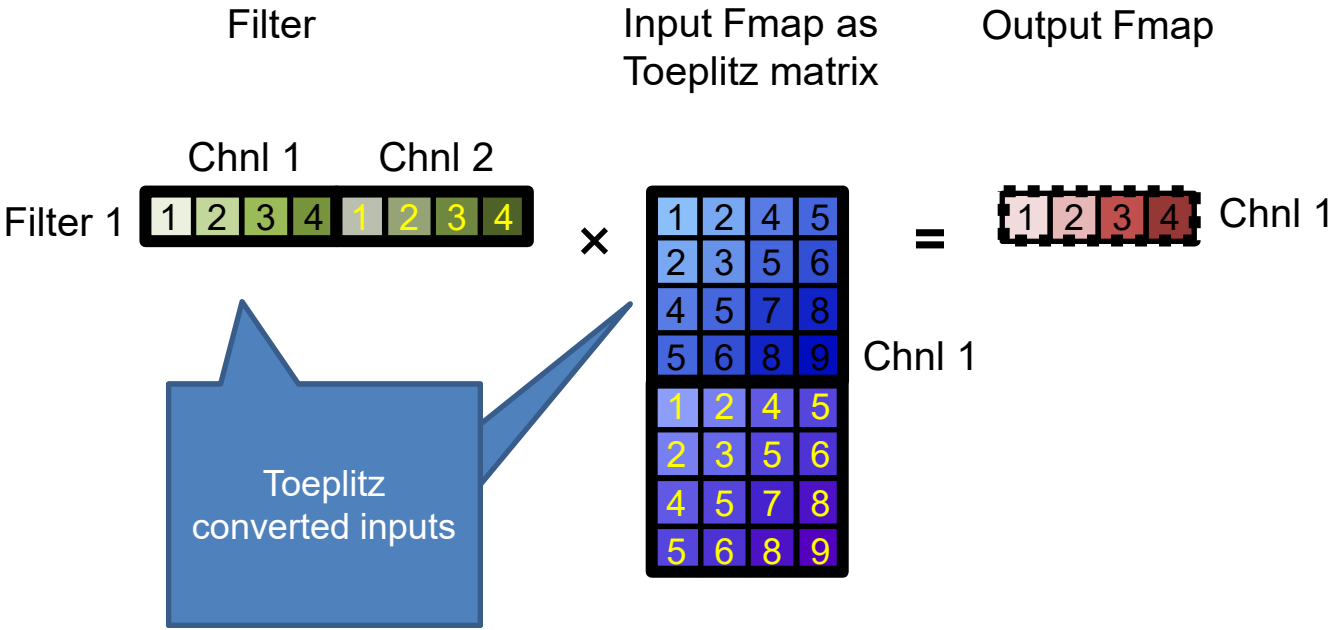
Convolution (CONV) Layer

- Multiple Input Channels



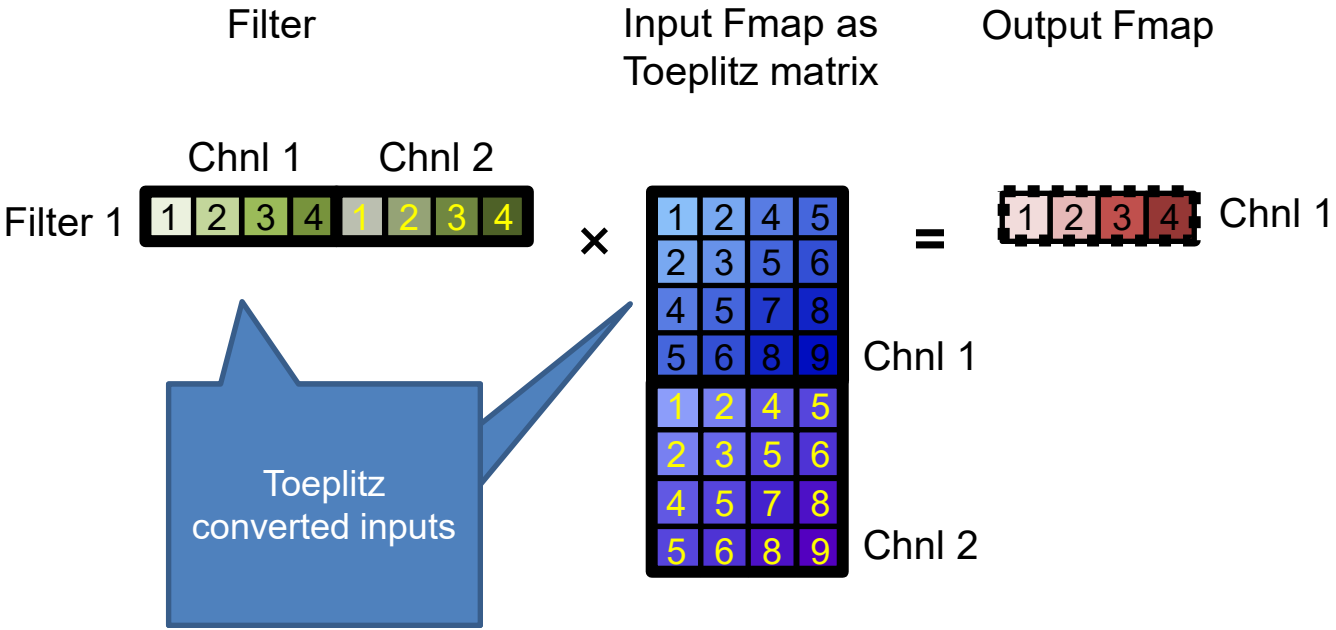
Convolution (CONV) Layer

- Multiple Input Channels



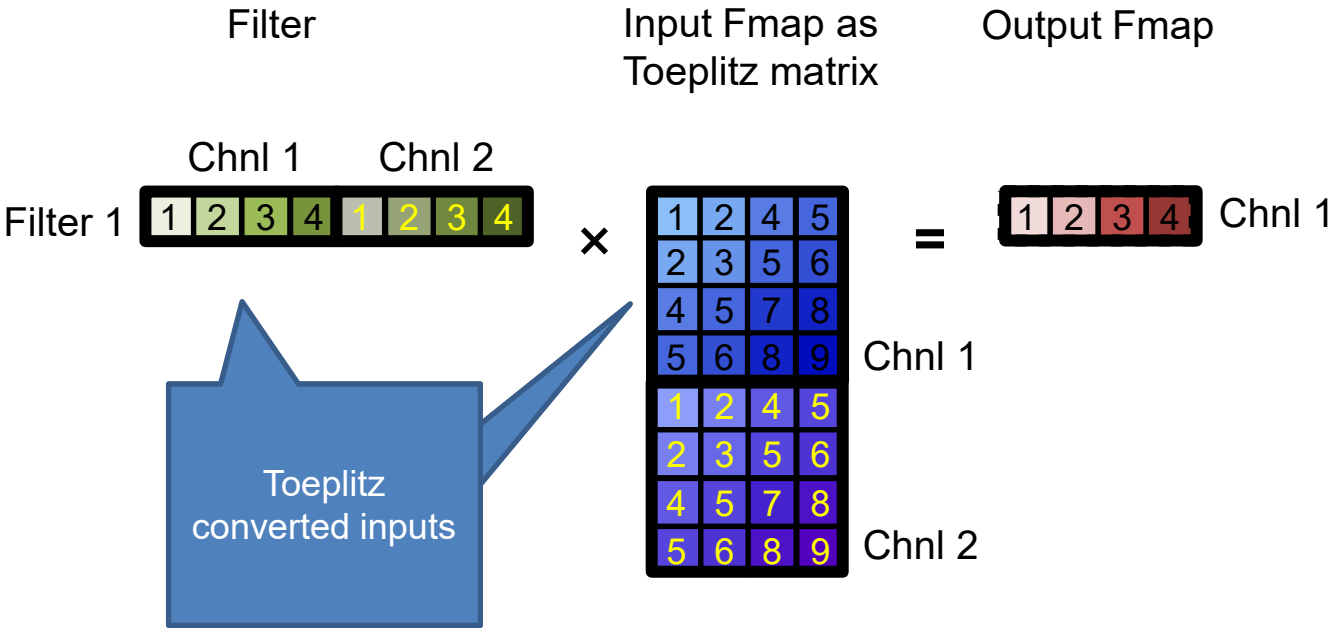
Convolution (CONV) Layer

- Multiple Input Channels



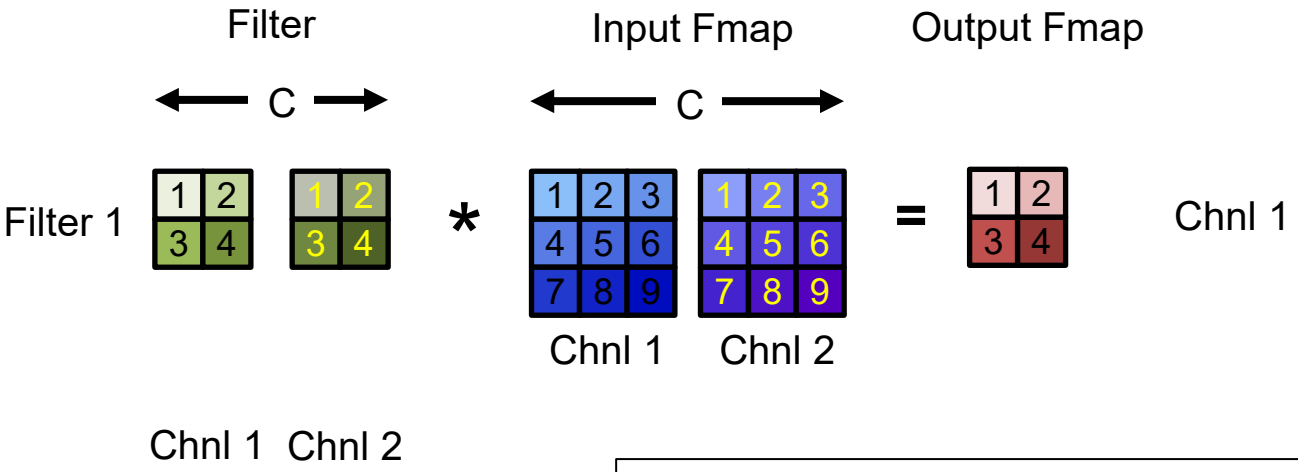
Convolution (CONV) Layer

- Multiple Input Channels



Convolution (CONV) Layer

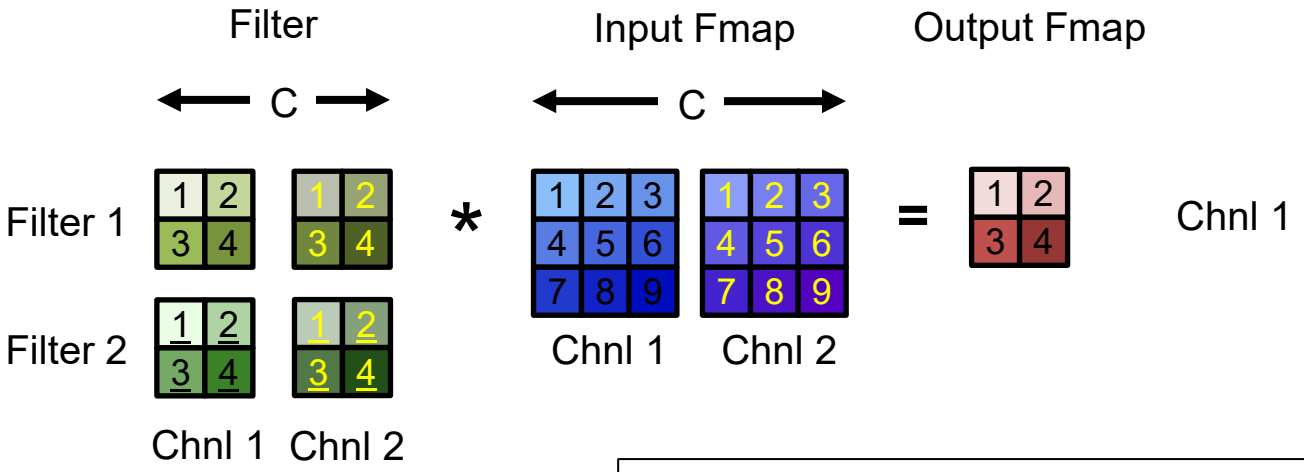
- Multiple Input Channels and Output Channels



Key:
Black: Input channel 1
Yellow: Input channel 2
Underlined: Output channel 2

Convolution (CONV) Layer

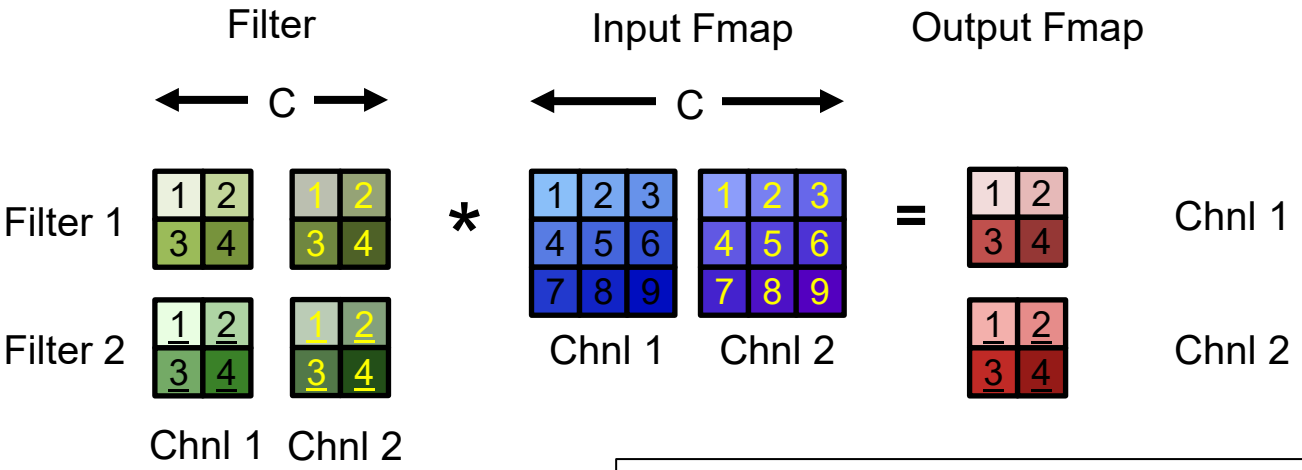
- Multiple Input Channels and Output Channels



Key:
Black: Input channel 1
Yellow: Input channel 2
Underlined: Output channel 2

Convolution (CONV) Layer

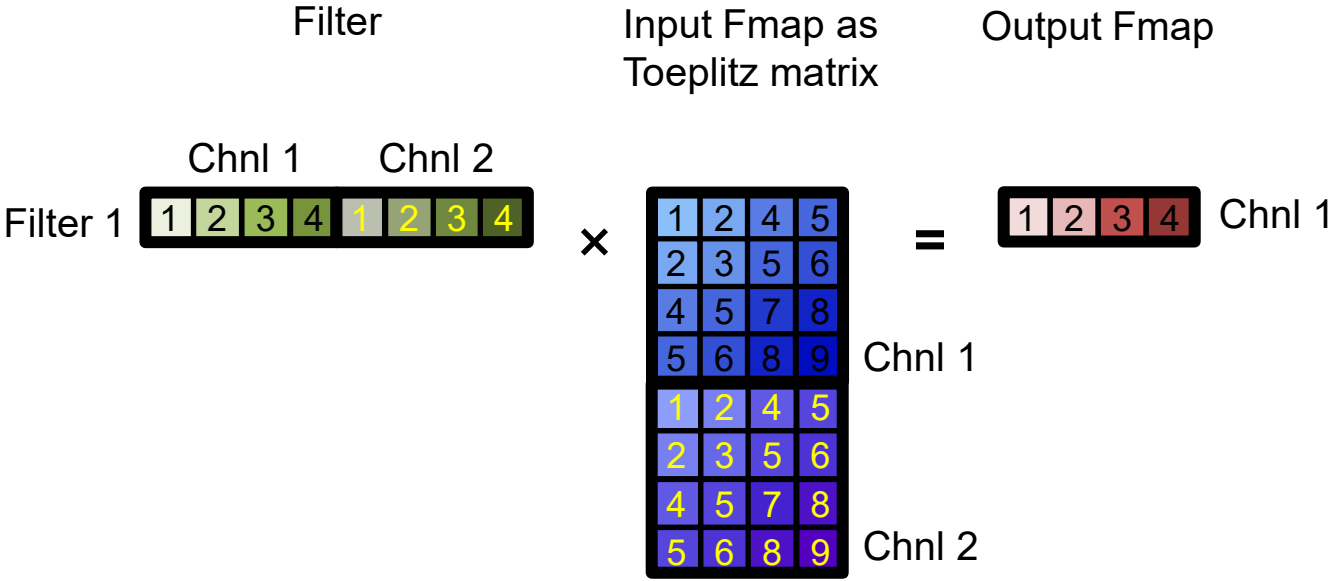
- Multiple Input Channels and Output Channels



Key:
Black: Input channel 1
Yellow: Input channel 2
Underlined: Output channel 2

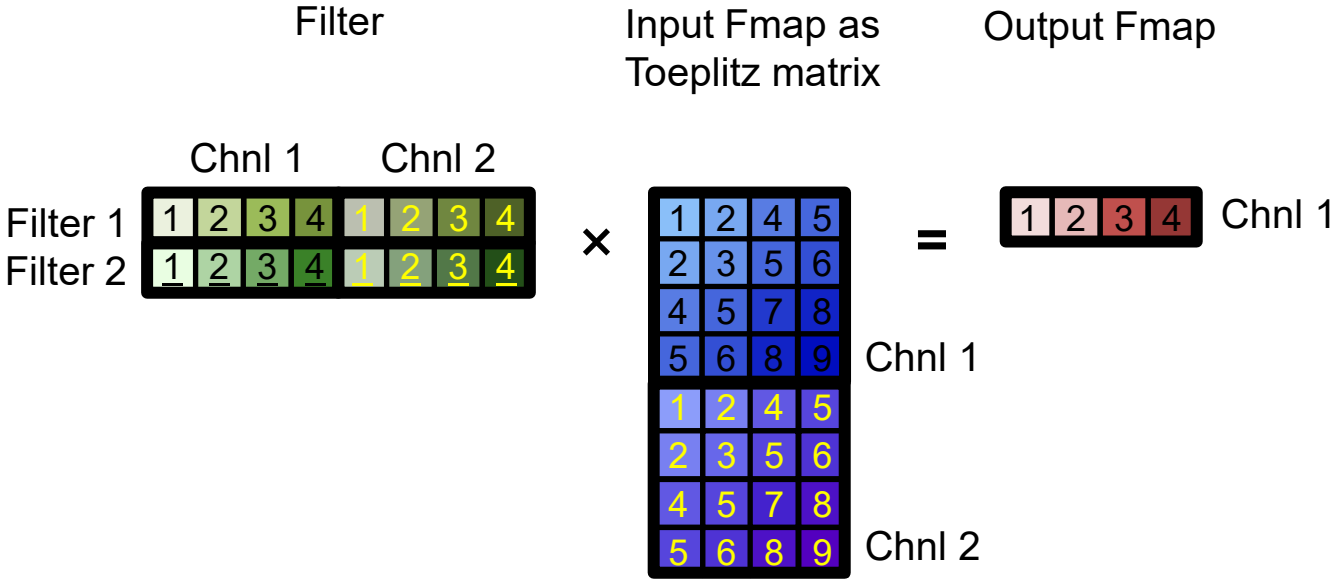
Convolution (CONV) Layer

- Multiple Input Channels and Output Channels



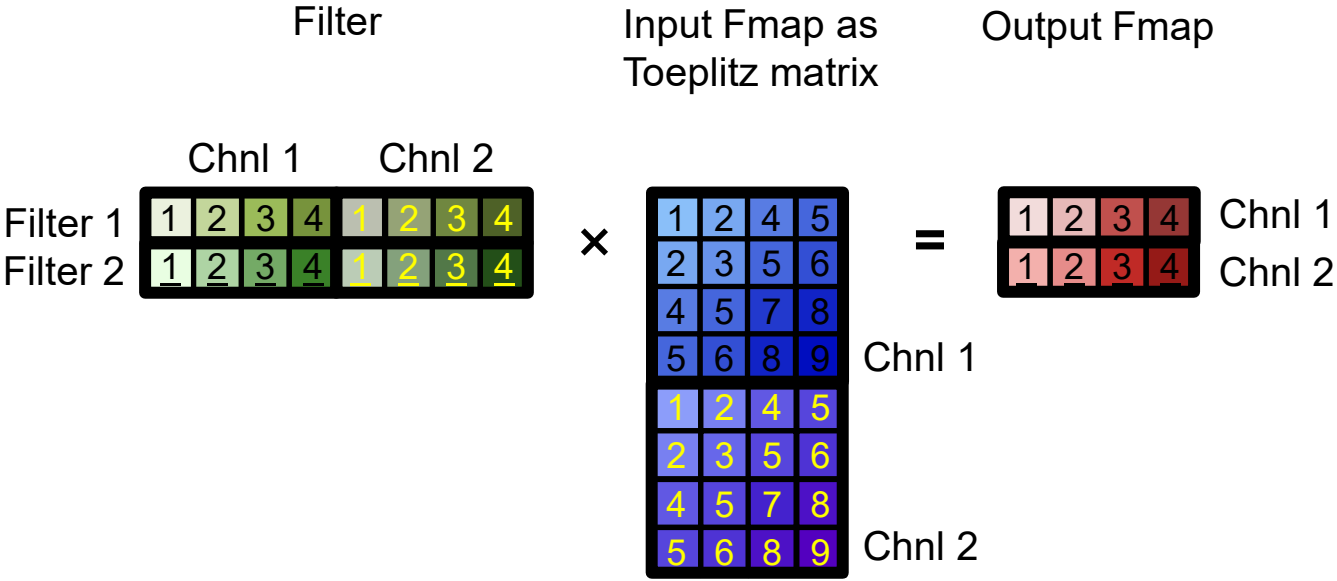
Convolution (CONV) Layer

- Multiple Input Channels and Output Channels



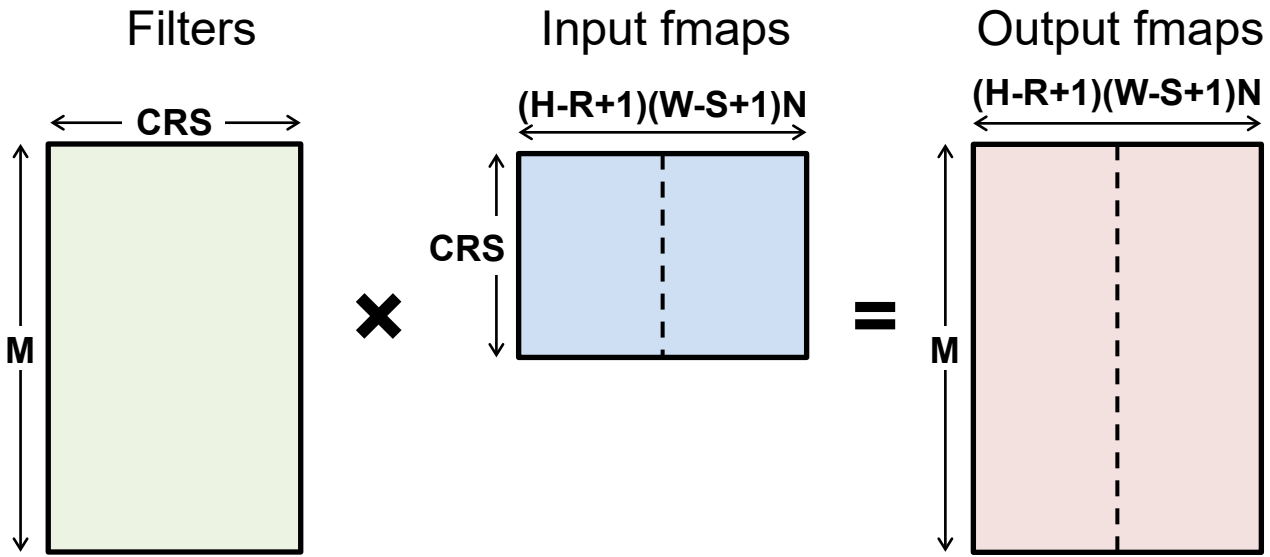
Convolution (CONV) Layer

- Multiple Input Channels and Output Channels



Convolution (CONV) Layer

- Dimensions of matrices for matrix multiply in convolution layers with batch size N



N=2 in example

1-D Toeplitz Convolution Einsum

$$O_{n,m,p,q} = I_{n,c,Up+r,Uq+s} \times F_{m,c,r,s}$$

Simplify to 1-D with N=1, C=1, M=1, U=1

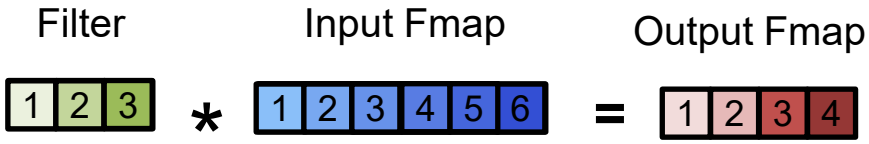
$$O_q = I_{q+s} \times F_s$$

Break into two steps

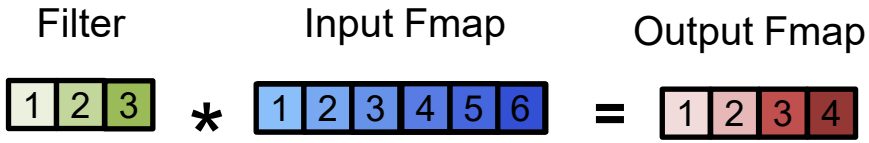
$$T_{q,s} = I_{q+s}$$

$$O_q = T_{q,s} \times F_s$$

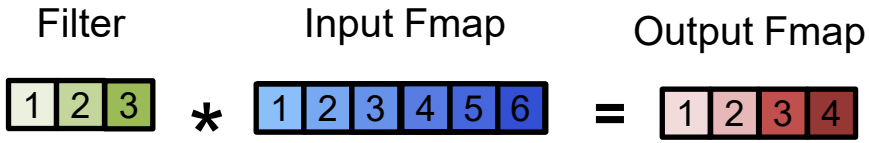
1-D Toeplitz Convolution Einsum



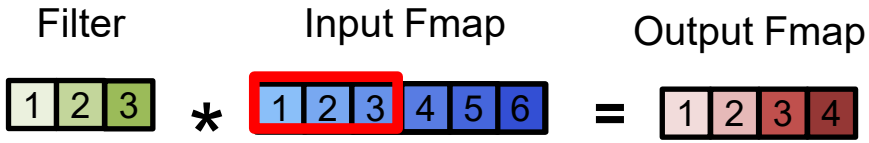
1-D Toeplitz Convolution Einsum



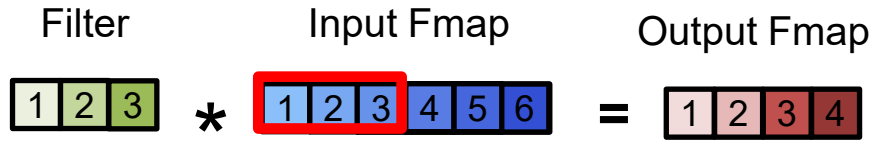
1-D Toeplitz Convolution Einsum



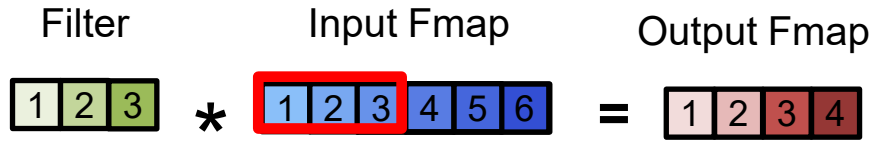
1-D Toeplitz Convolution Einsum

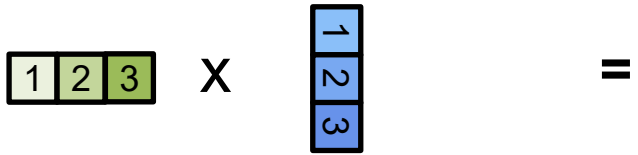


1-D Toeplitz Convolution Einsum

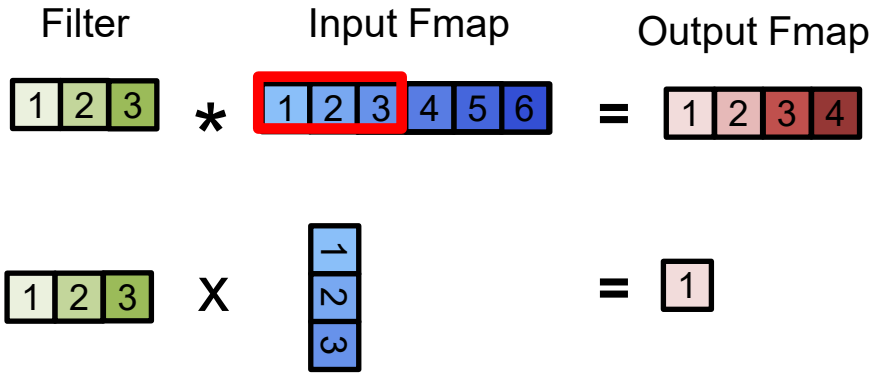


1-D Toeplitz Convolution Einsum

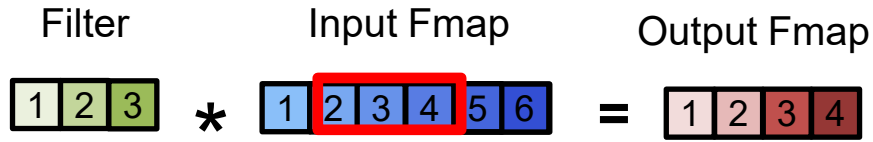


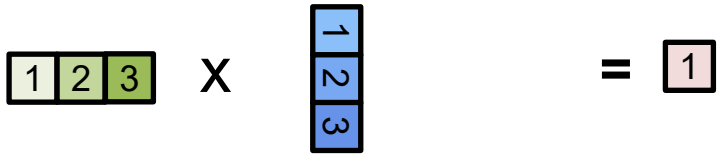


1-D Toeplitz Convolution Einsum

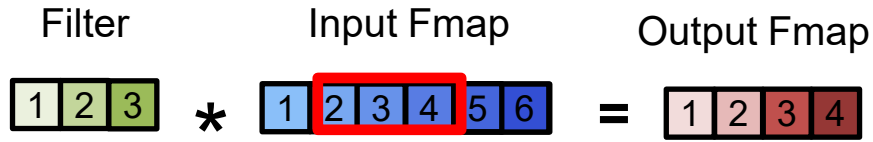


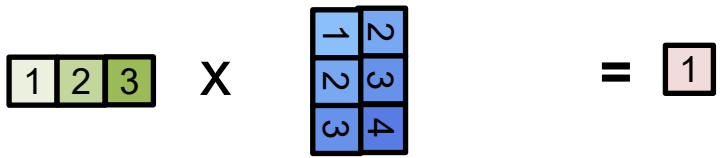
1-D Toeplitz Convolution Einsum



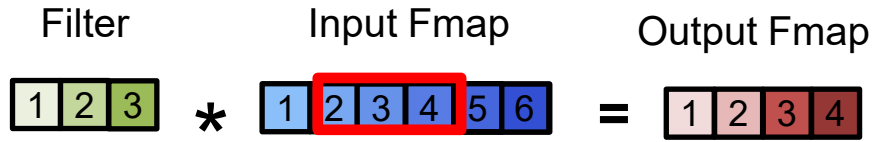


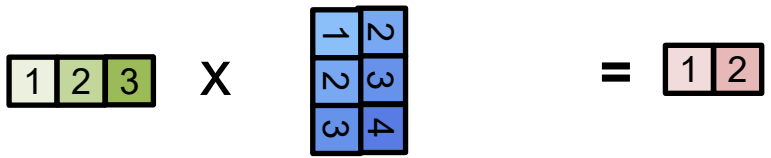
1-D Toeplitz Convolution Einsum



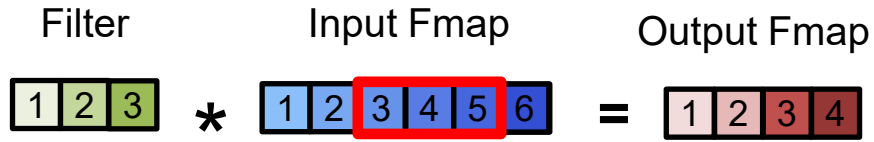


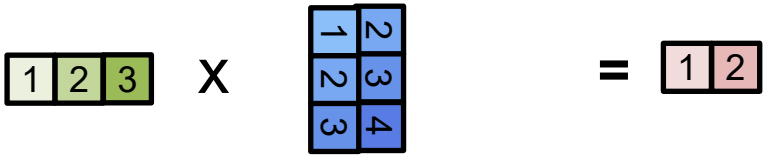
1-D Toeplitz Convolution Einsum



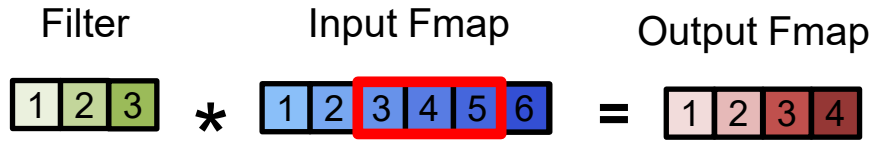


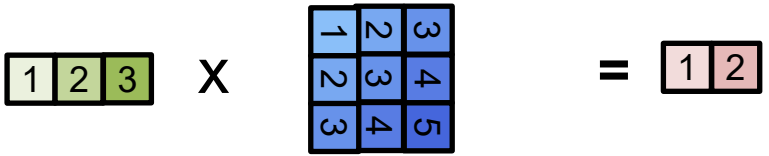
1-D Toeplitz Convolution Einsum



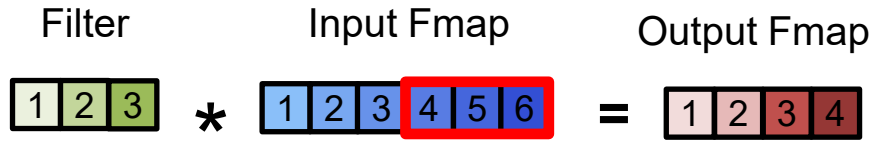


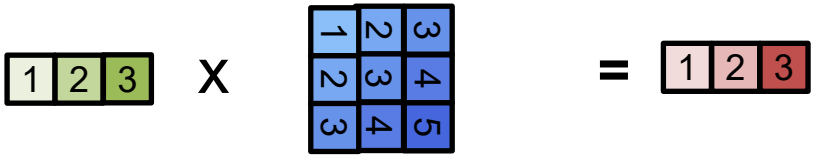
1-D Toeplitz Convolution Einsum



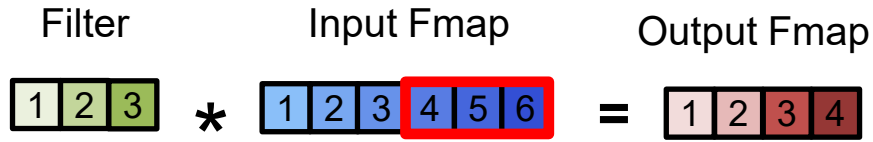


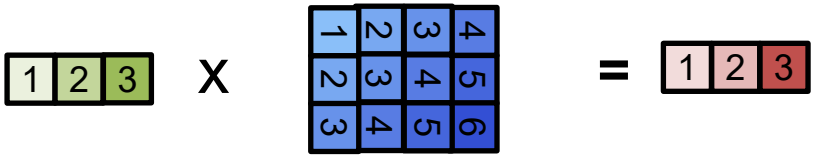
1-D Toeplitz Convolution Einsum



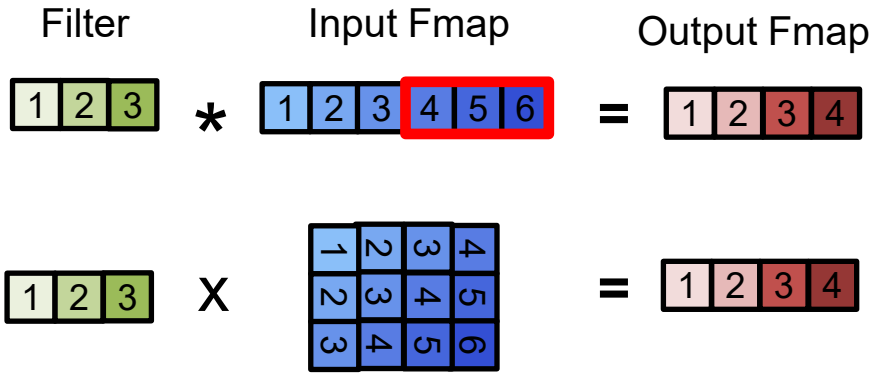


1-D Toeplitz Convolution Einsum

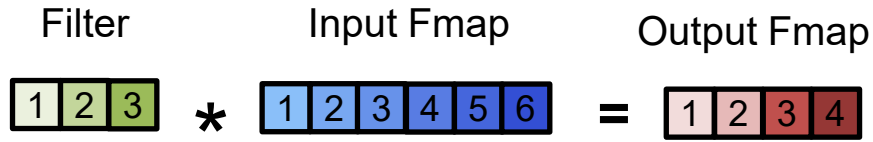


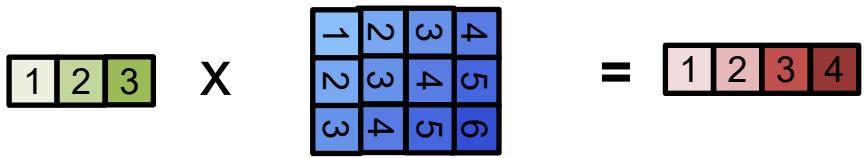


1-D Toeplitz Convolution Einsum

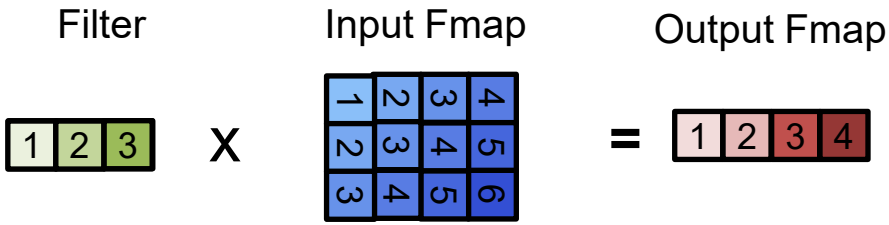


1-D Toeplitz Convolution Einsum

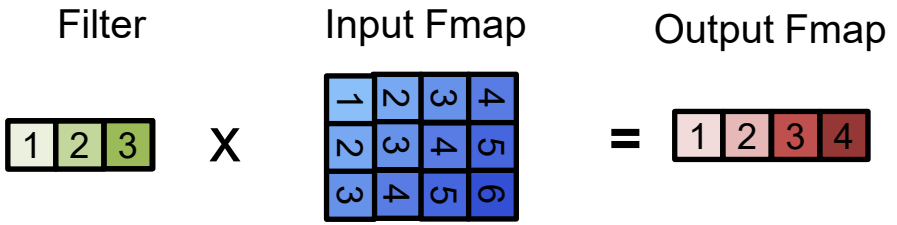




1-D Toeplitz Convolution Einsum

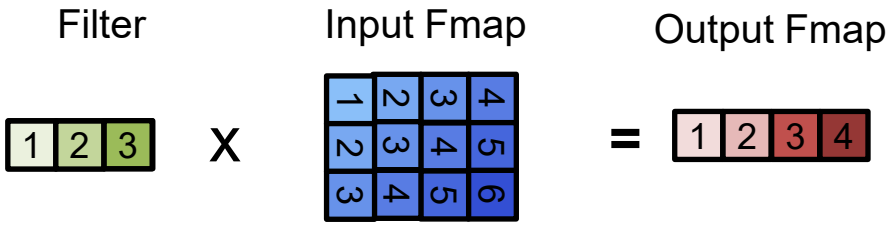


1-D Toeplitz Convolution Einsum

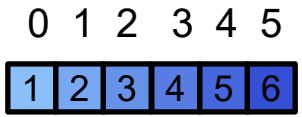


$$I_{q,s} = I_{q+s}$$

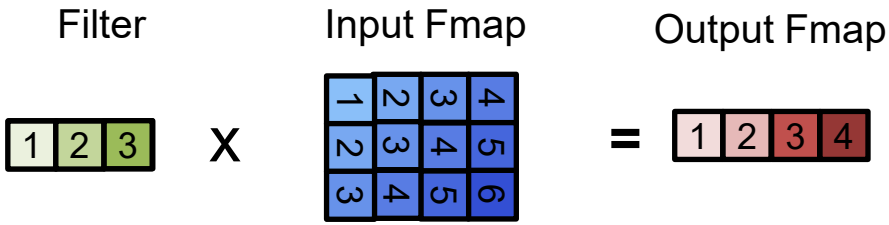
1-D Toeplitz Convolution Einsum



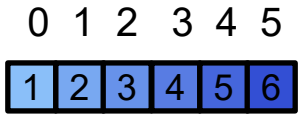
$$I_{q,s} = I_{q+s}$$

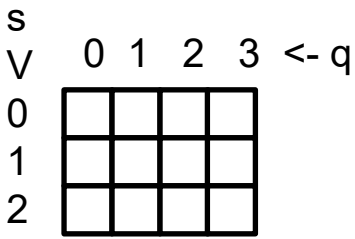


1-D Toeplitz Convolution Einsum

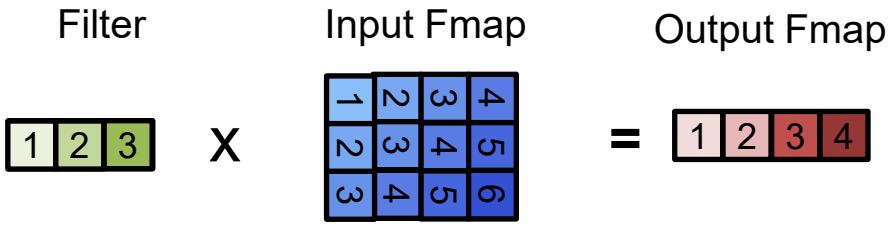


$$I_{q,s} = I_{q+s}$$



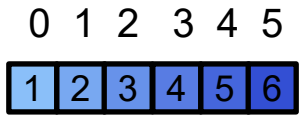


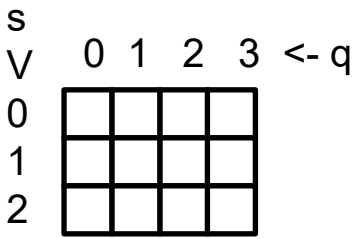
1-D Toeplitz Convolution Einsum



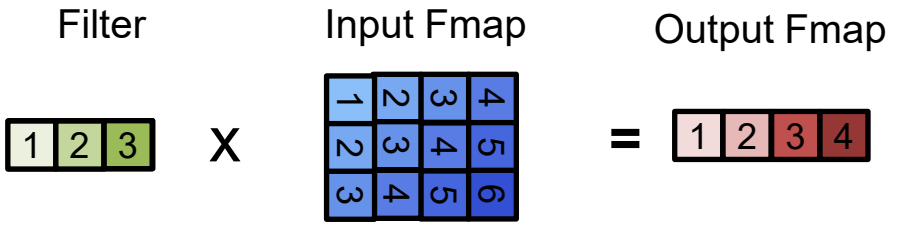
$$I_{q,s} = I_{q+s}$$

q	s	q+s
0	0	0



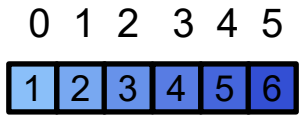


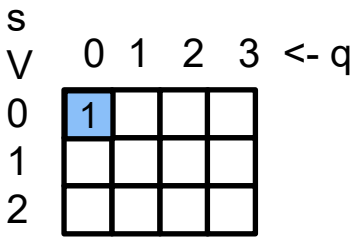
1-D Toeplitz Convolution Einsum



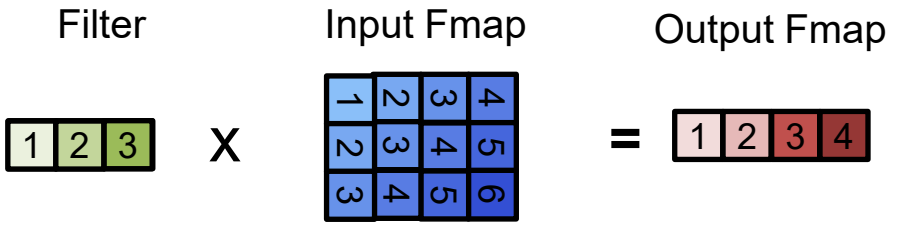
$$I_{q,s} = I_{q+s}$$

q	s	q+s
0	0	0



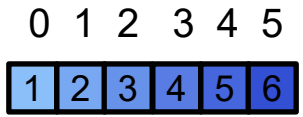


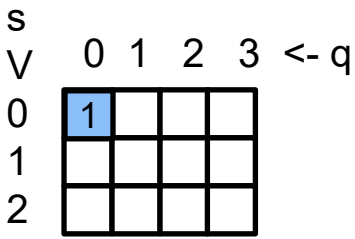
1-D Toeplitz Convolution Einsum



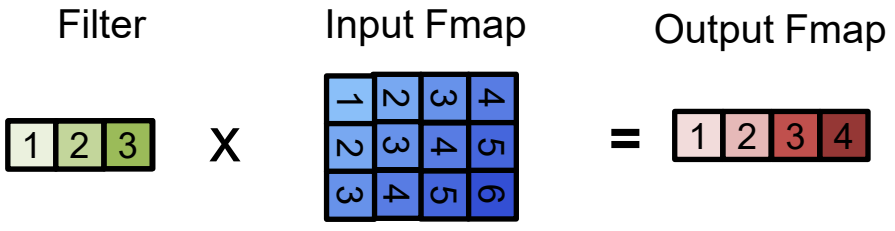
$$I_{q,s} = I_{q+s}$$

q	s	q+s
0	0	0
0	1	1



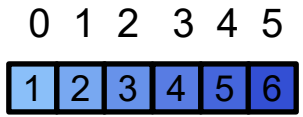


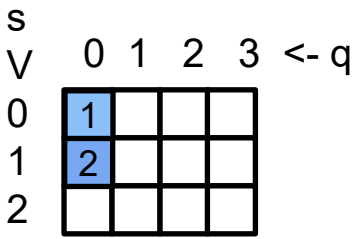
1-D Toeplitz Convolution Einsum



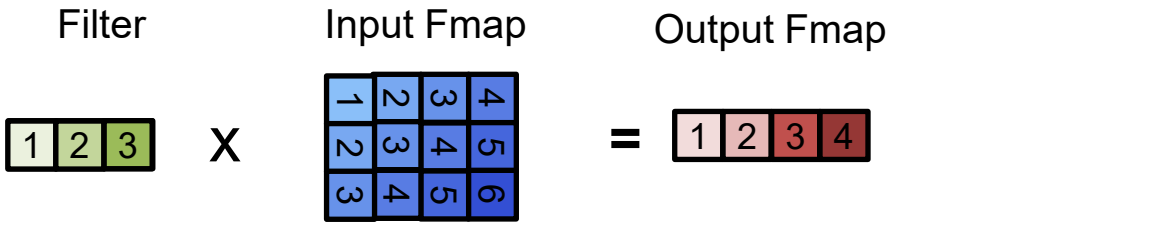
$$I_{q,s} = I_{q+s}$$

q	s	q+s
0	0	0
0	1	1

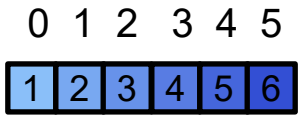




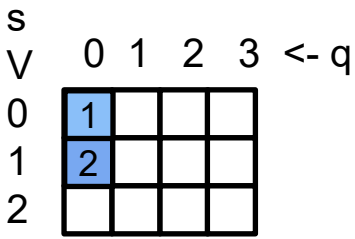
1-D Toeplitz Convolution Einsum



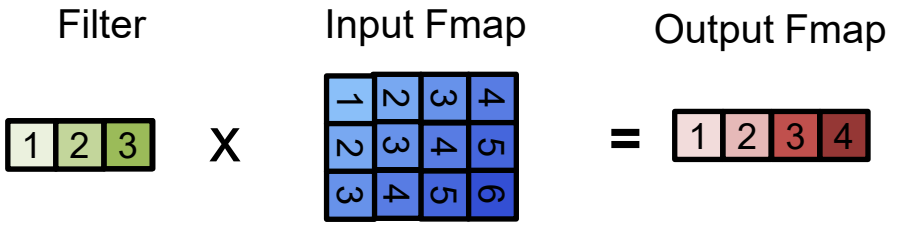
$$I_{q,s} = I_{q+s}$$



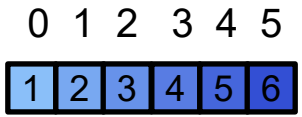
q	s	q+s
0	0	0
0	1	1
0	2	2



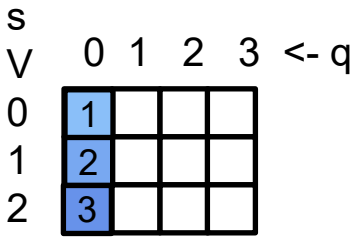
1-D Toeplitz Convolution Einsum



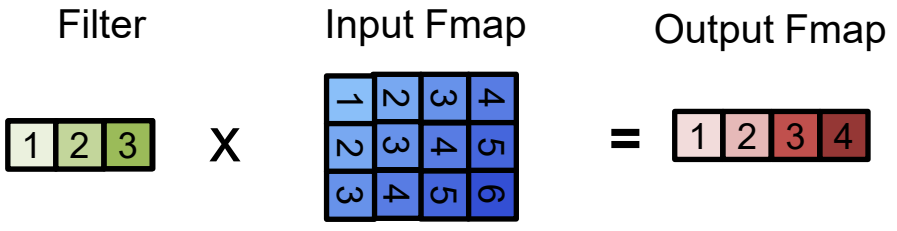
$$I_{q,s} = I_{q+s}$$



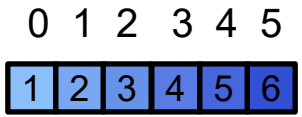
q	s	q+s
0	0	0
0	1	1
0	2	2



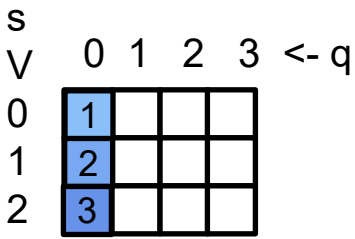
1-D Toeplitz Convolution Einsum



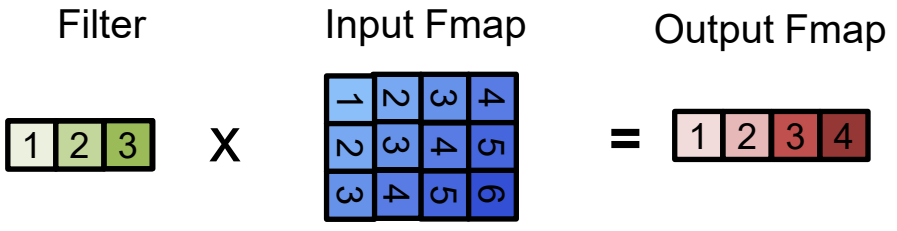
$$I_{q,s} = I_{q+s}$$



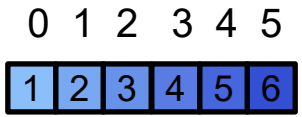
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1



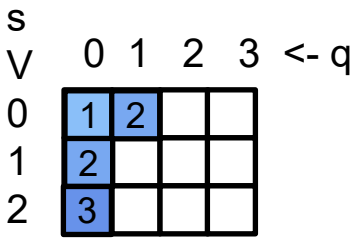
1-D Toeplitz Convolution Einsum



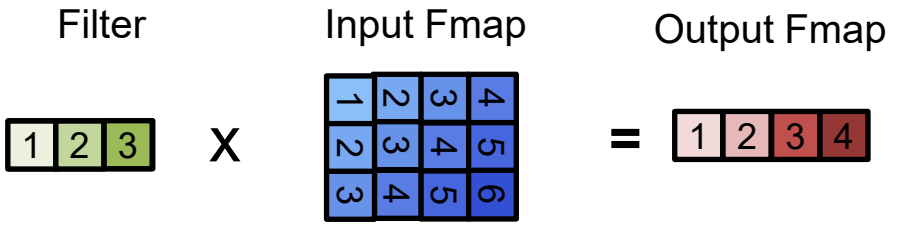
$$I_{q,s} = I_{q+s}$$



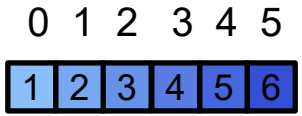
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1

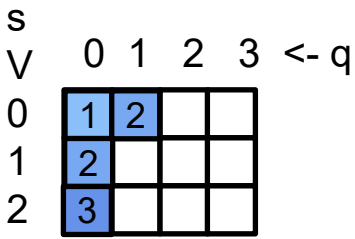


1-D Toeplitz Convolution Einsum



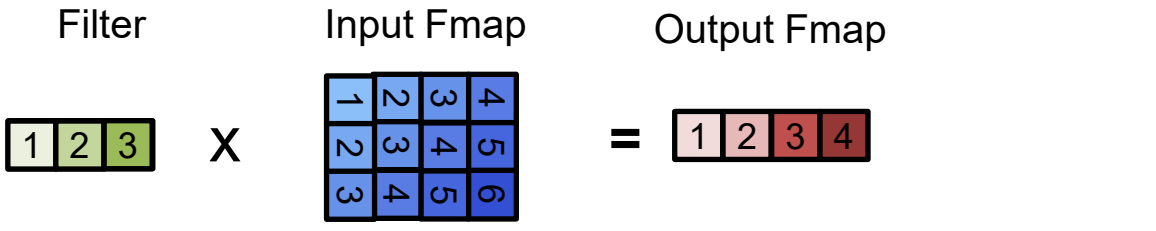
$$I_{q,s} = I_{q+s}$$



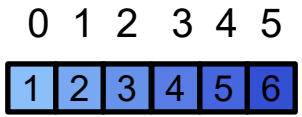


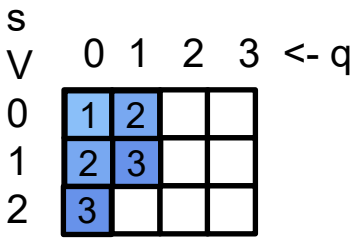
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2

1-D Toeplitz Convolution Einsum



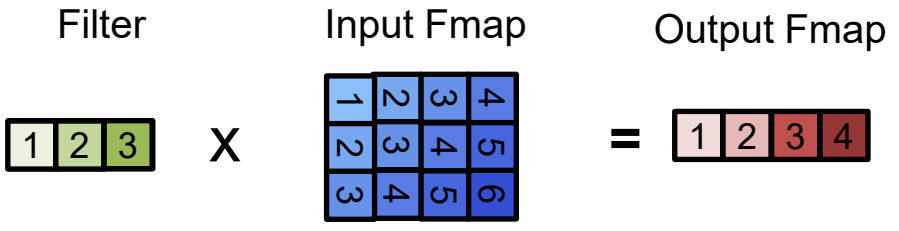
$$I_{q,s} = I_{q+s}$$



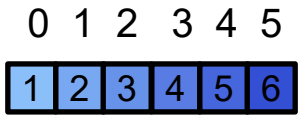


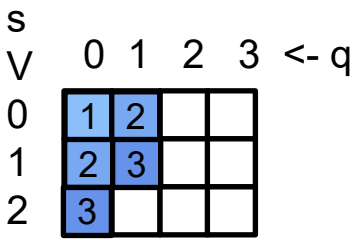
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2

1-D Toeplitz Convolution Einsum



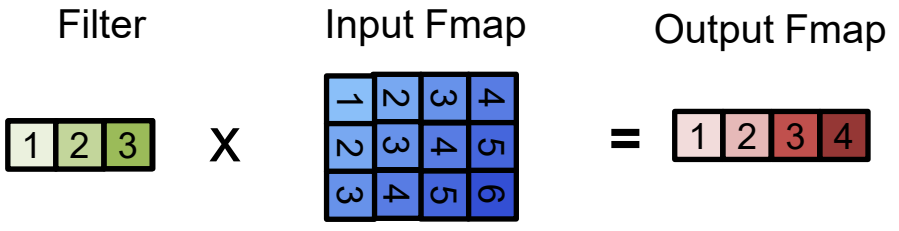
$$I_{q,s} = I_{q+s}$$



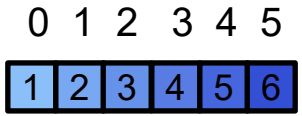


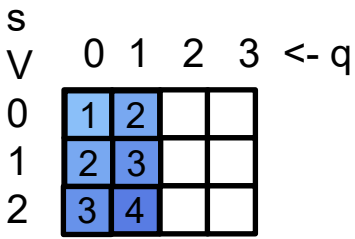
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3

1-D Toeplitz Convolution Einsum



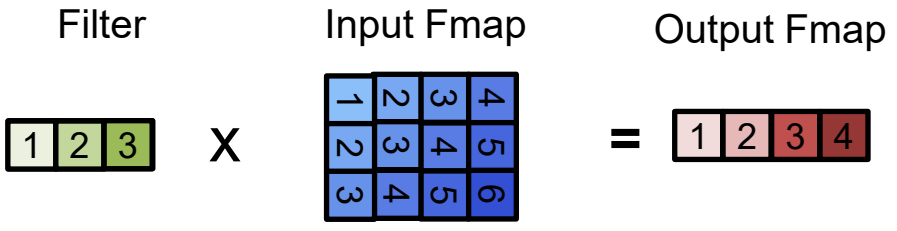
$$I_{q,s} = I_{q+s}$$



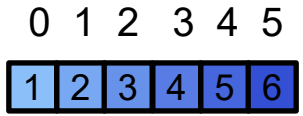


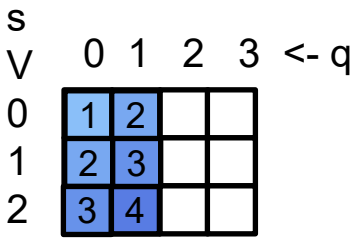
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3

1-D Toeplitz Convolution Einsum



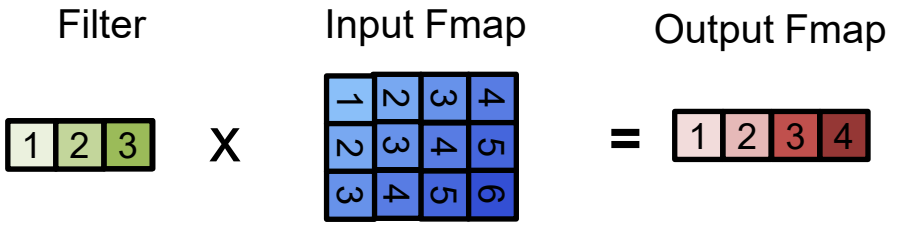
$$I_{q,s} = I_{q+s}$$



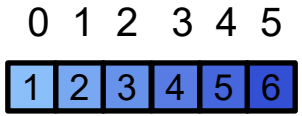


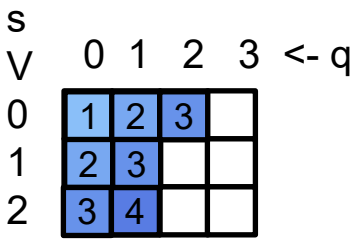
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3
2	0	2

1-D Toeplitz Convolution Einsum



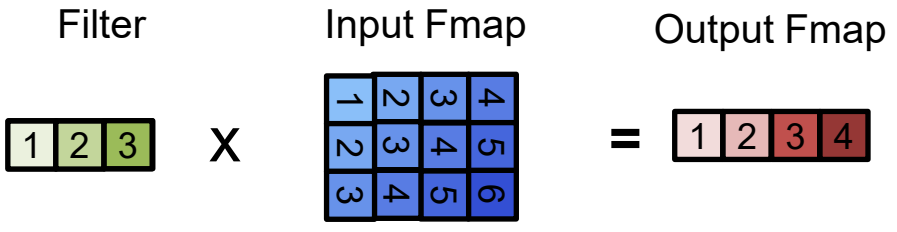
$$I_{q,s} = I_{q+s}$$



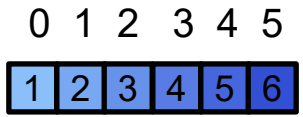


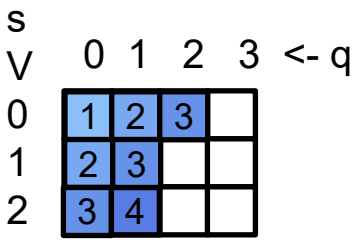
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3
2	0	2

1-D Toeplitz Convolution Einsum



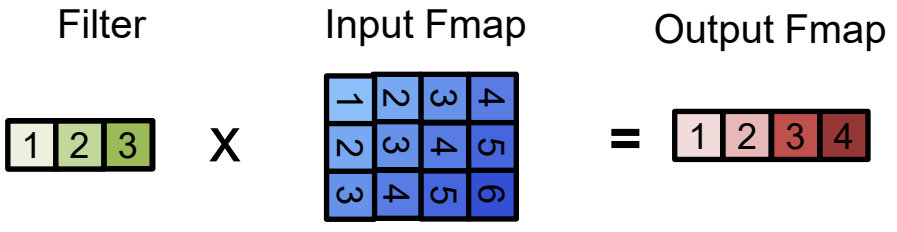
$$I_{q,s} = I_{q+s}$$



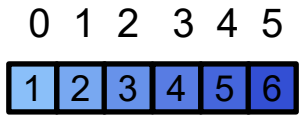


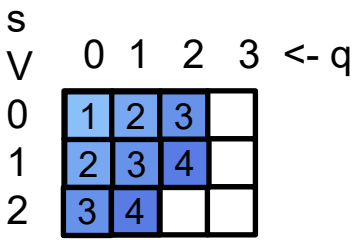
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3
2	0	2
2	1	3

1-D Toeplitz Convolution Einsum



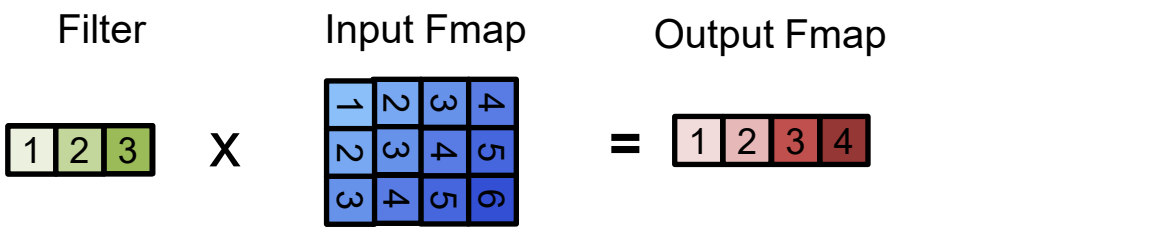
$$I_{q,s} = I_{q+s}$$



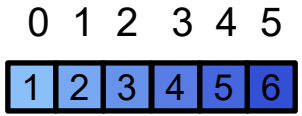


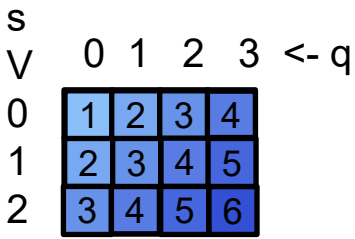
q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3
2	0	2
2	1	3

1-D Toeplitz Convolution Einsum



$$I_{q,s} = I_{q+s}$$





q	s	q+s
0	0	0
0	1	1
0	2	2
1	0	1
1	1	2
1	2	3
2	0	2
2	1	3

2-D Toeplitz Convolution Einsum

$$O_{m,p,q} = I_{c,p+r,q+s} \times F_{m,c,r,s}$$

Break out Toeplitz conversion

$$T_{c,p,q,r,s} = I_{c,p+r,q+s}$$

Flatten ranks

$$T'_{pq,crs} = T_{c,p,q,r,s}$$

$$F'_{m,crs} = F_{m,c,r,s}$$

$$O_{m,pq} = T'_{pq,crs} \times F'_{m,crs}$$

Next Lecture: GPUs

Thank you!