

6.5930/1

Hardware Architectures for Deep Learning

Mapping to Hardware

March 18, 2024

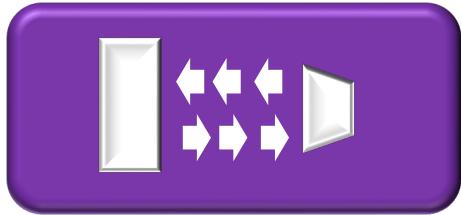
Joel Emer and Vivienne Sze

Massachusetts Institute of Technology
Electrical Engineering & Computer Science

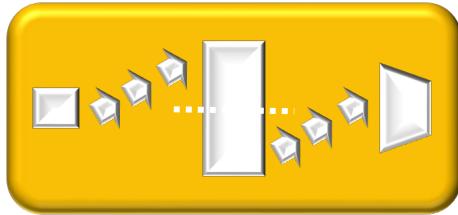


Data Orchestration

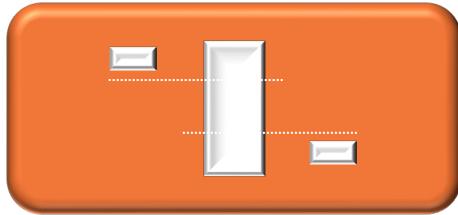
Guiding Principles for Data Orchestration



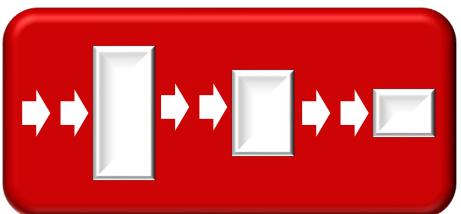
Efficient reuse – small storage physically close to consuming units for reused data



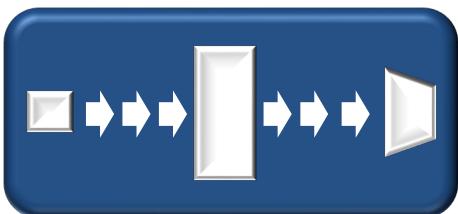
Delivery/use overlap - Next tile should be available when current is done (e.g., double-buffering)



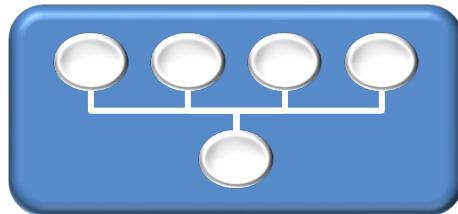
Precise synchronization – Only wait for exactly data you need, respond quickly (e.g., no barriers or remote polling)



Storage usage efficiency – Minimize idle storage waiting for long round trip latency



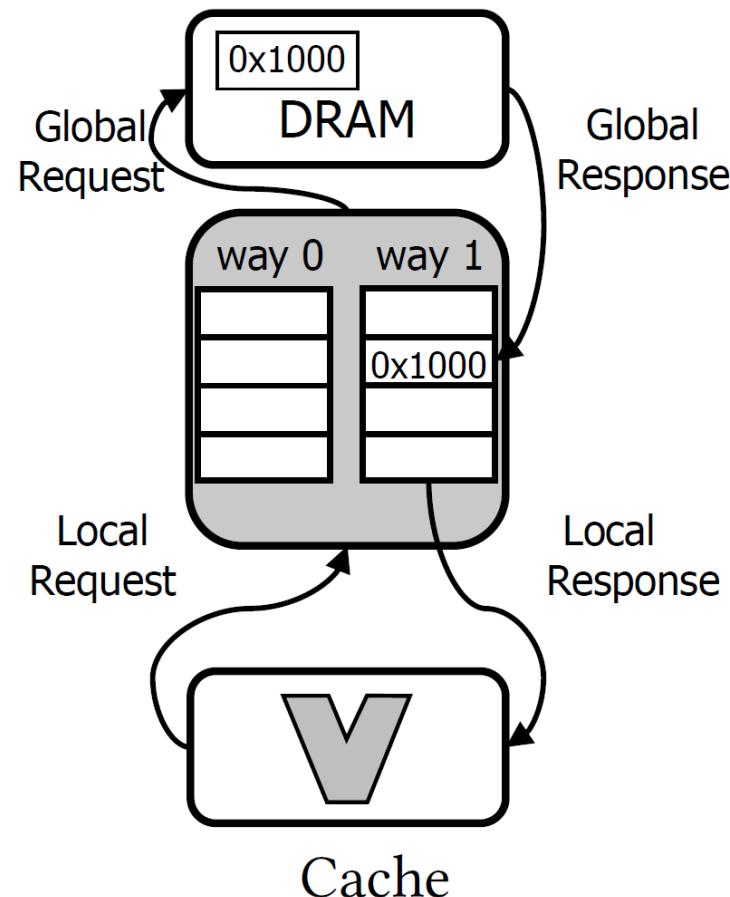
Bandwidth efficiency - Maximize delivery rate by controlling outstanding requests



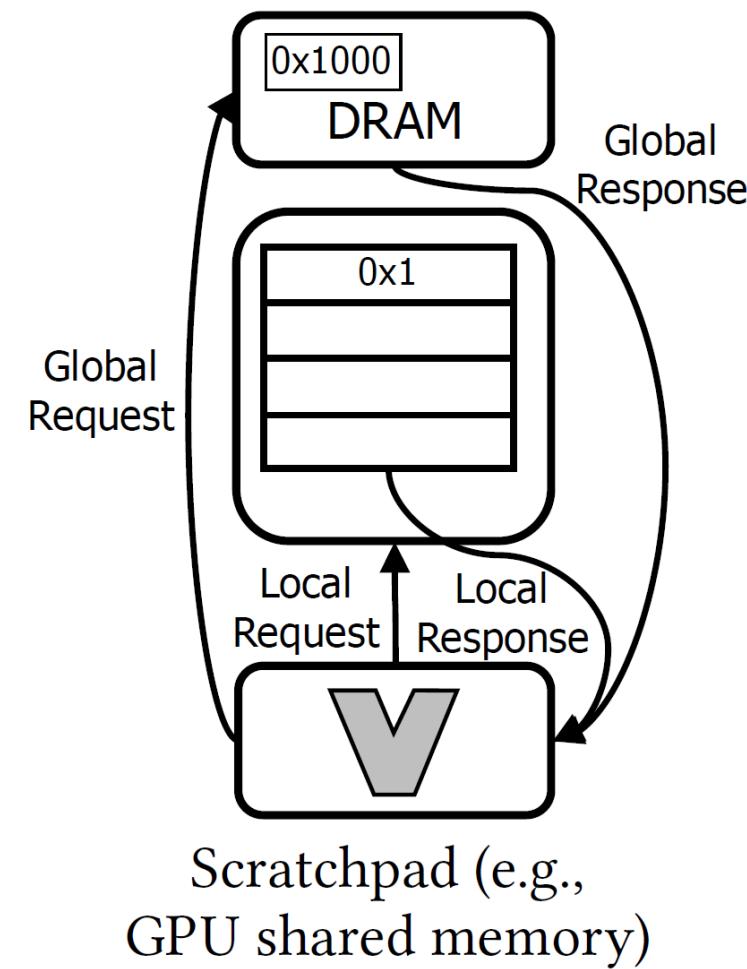
Cross-unit use – amortize data access and communication

Approaches: Implicit versus Explicit

Implicit:

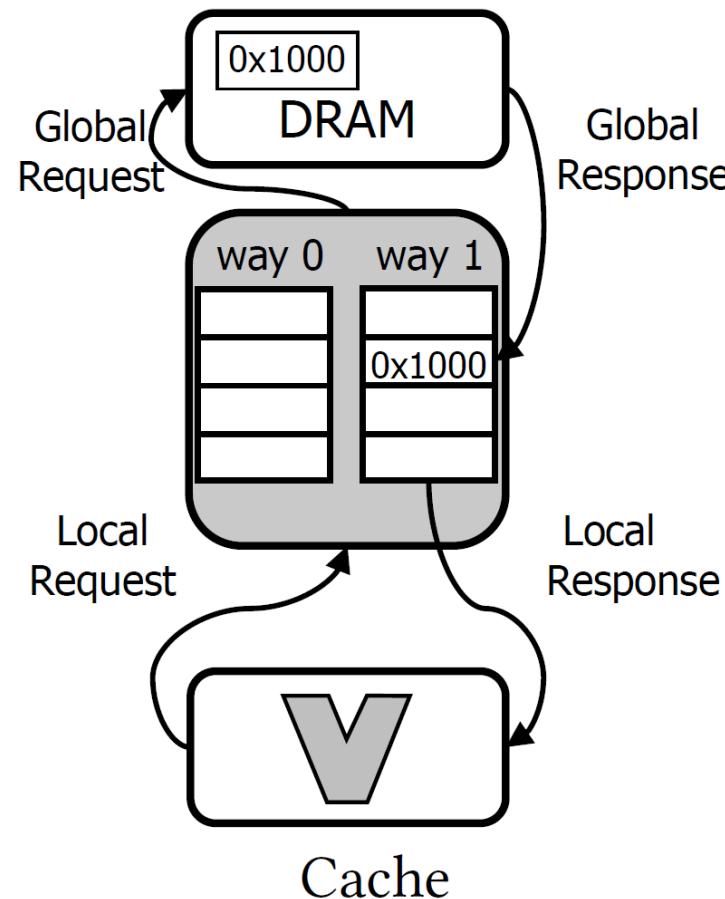


Explicit:

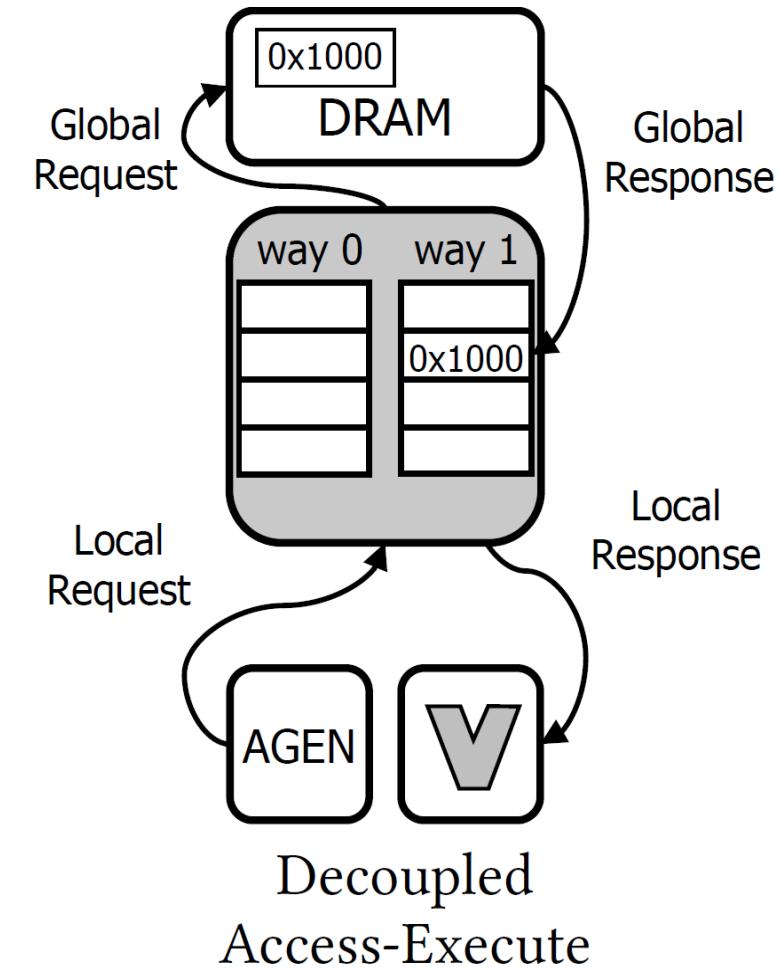


Approaches: Coupled versus Decoupled

Implicit + Coupled

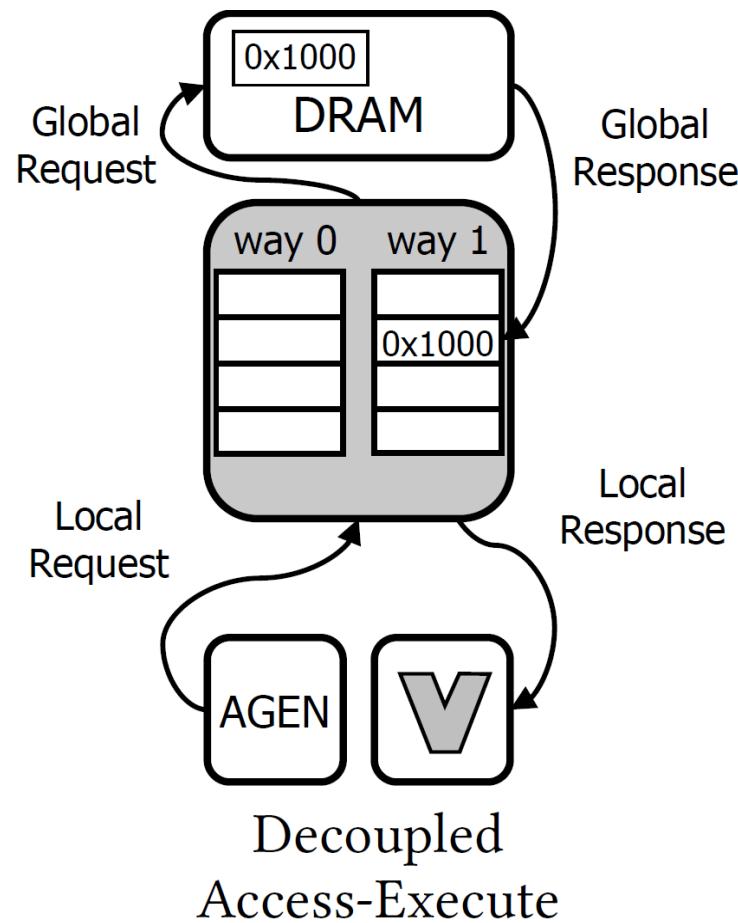


Implicit + *Decoupled*

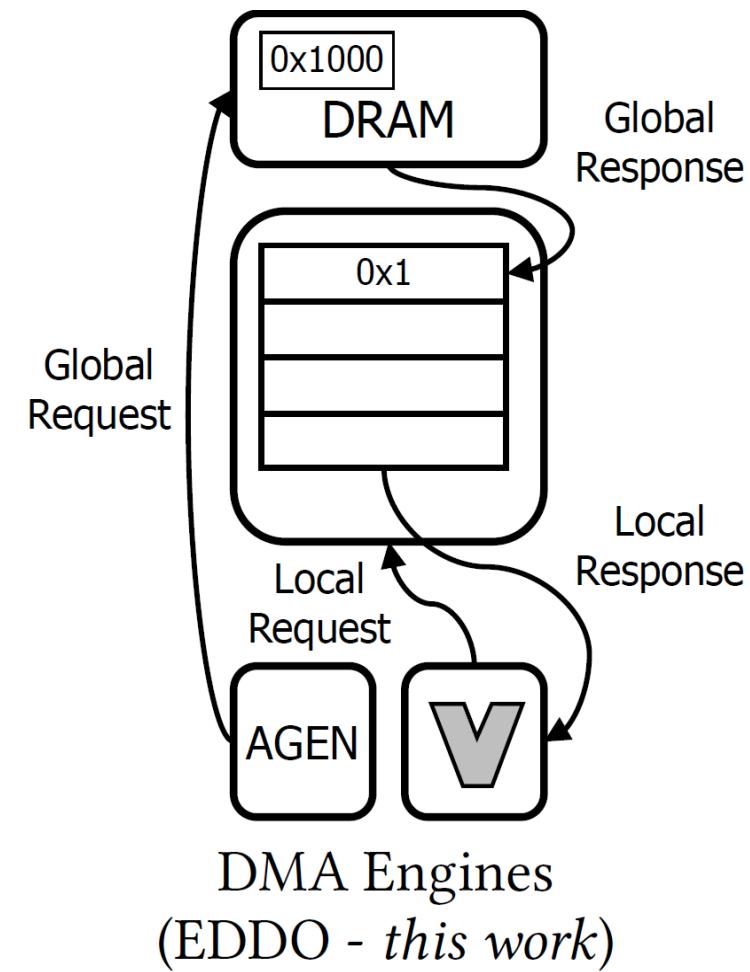


Explicit Decoupled Data Orchestration

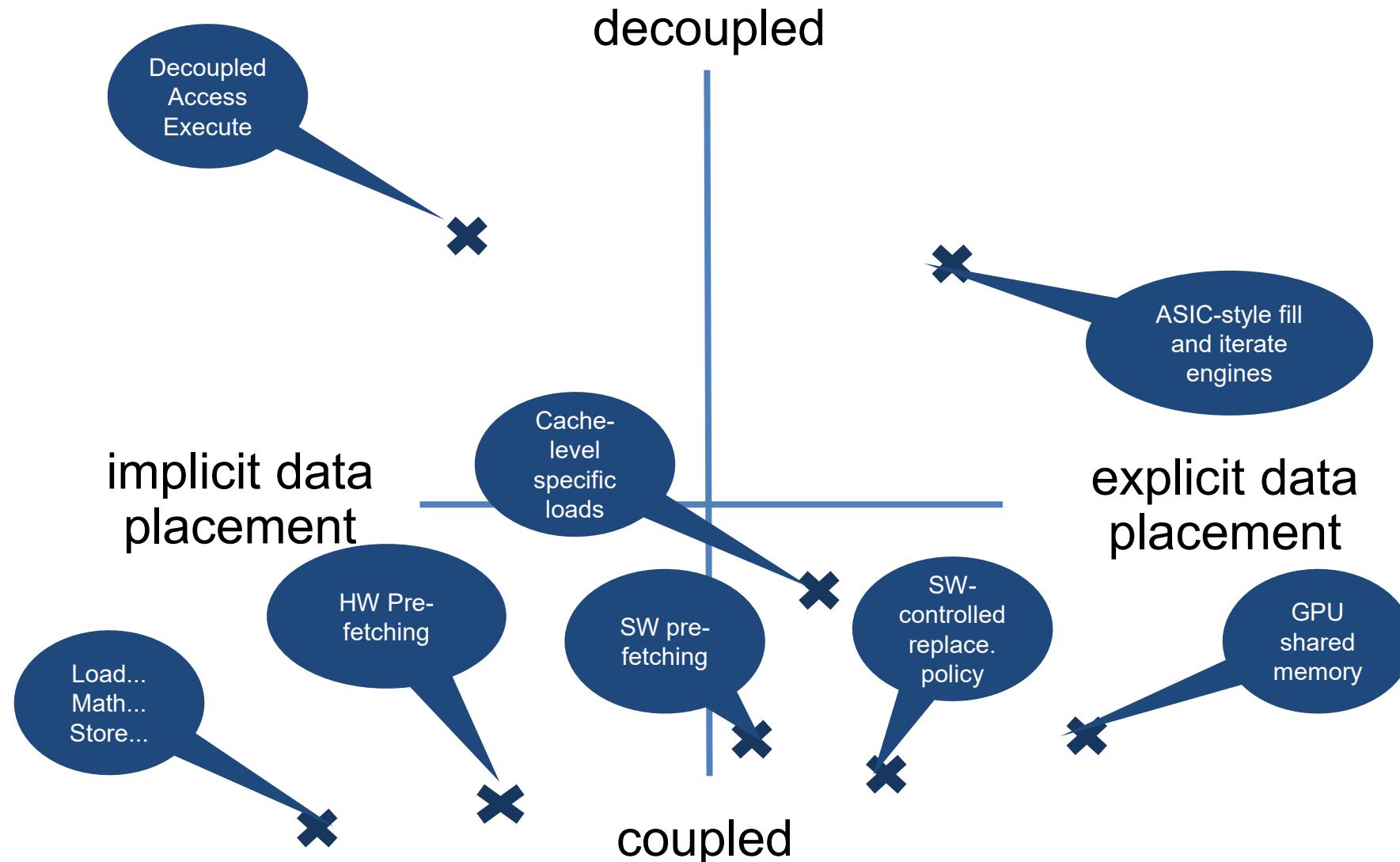
Implicit + Decoupled



Explicit + Decoupled

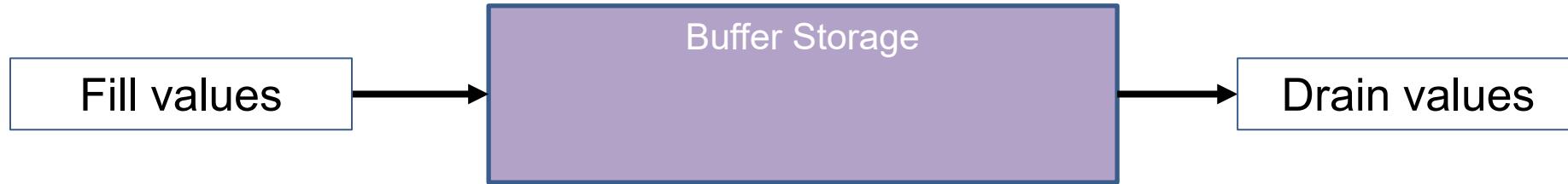


Classifying Orchestration Approaches

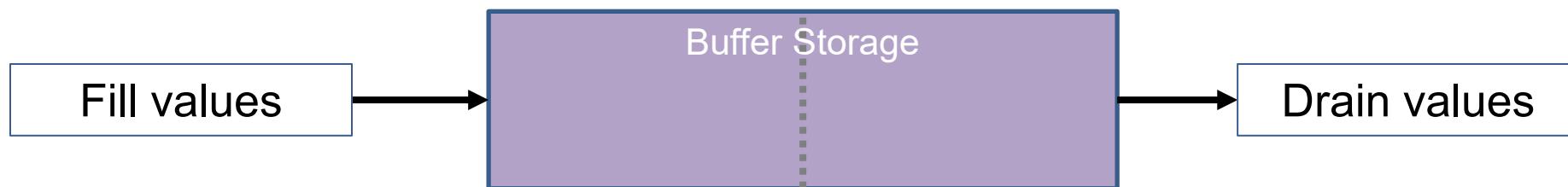


EDDO Strategies

Fill and use:



Double buffer



Rolling use

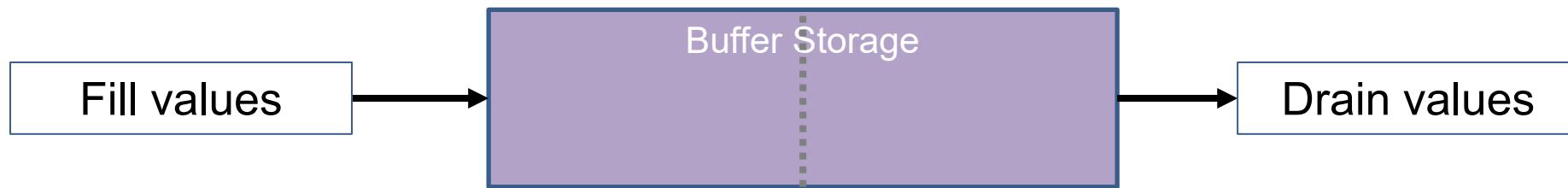


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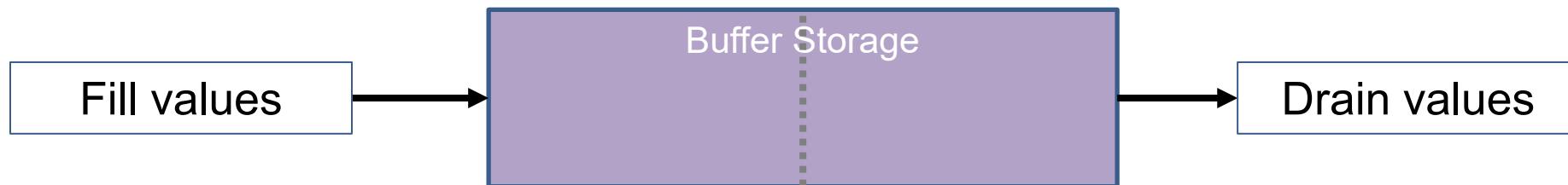


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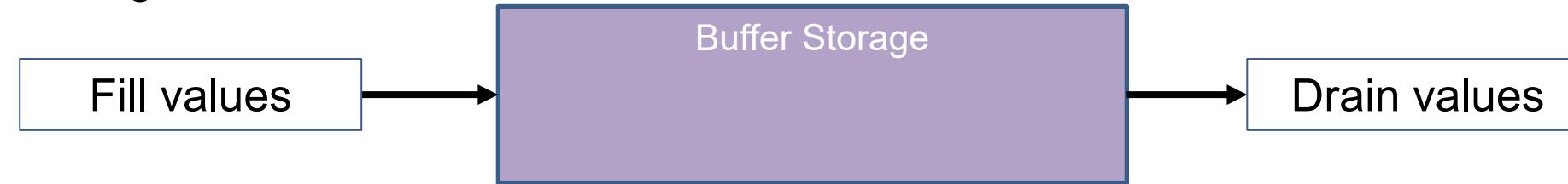
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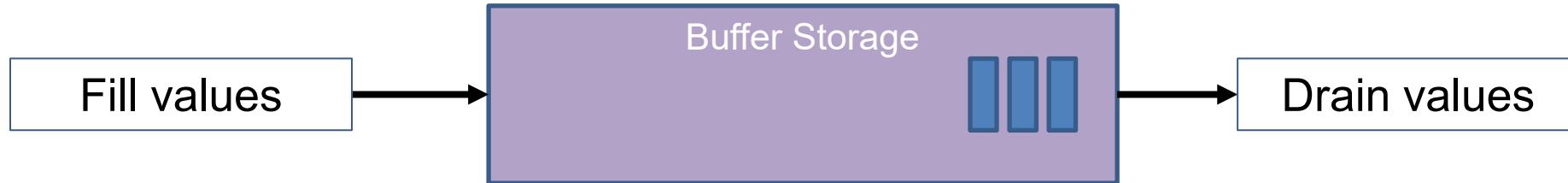


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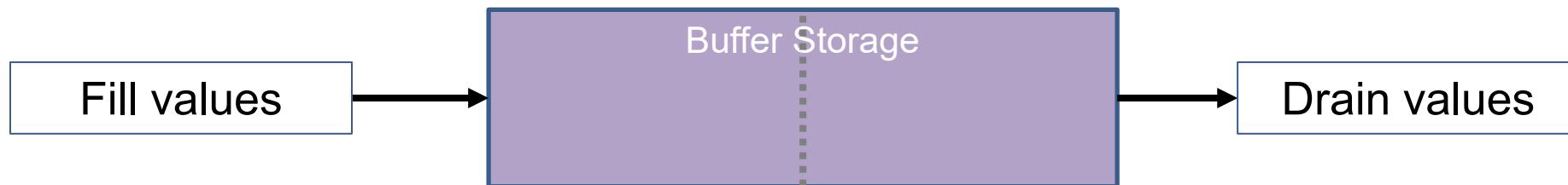


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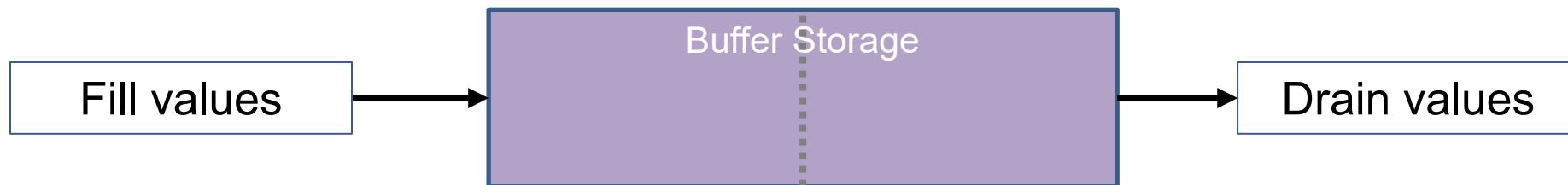


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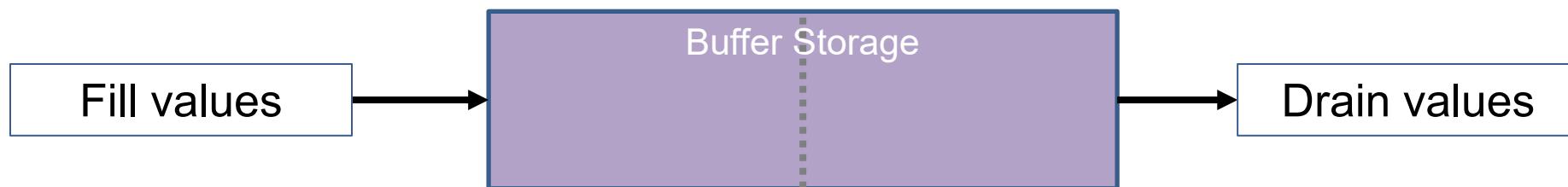


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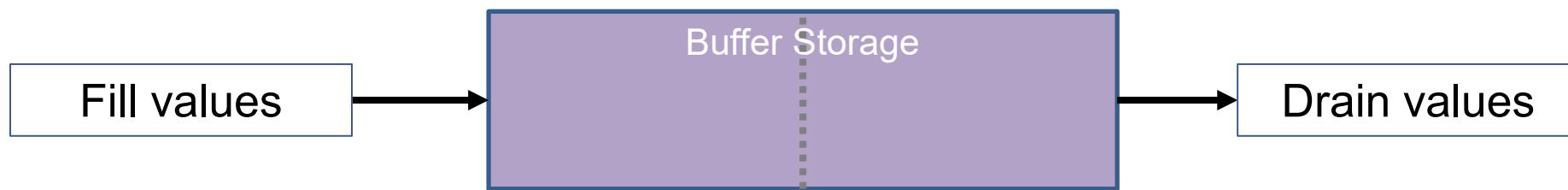


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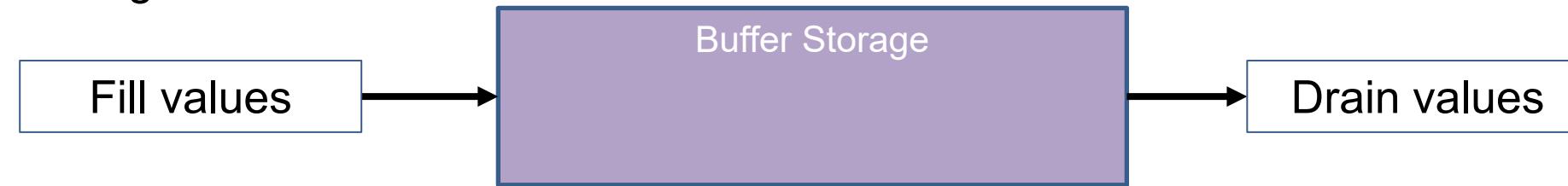
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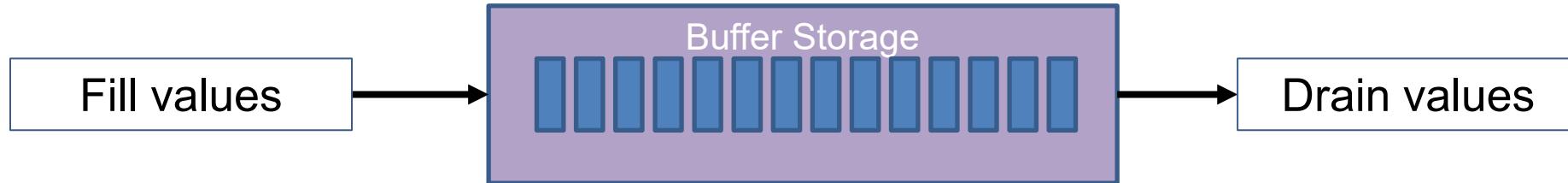


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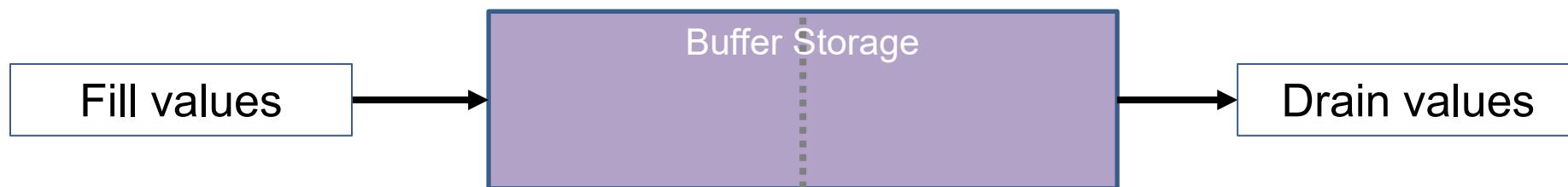


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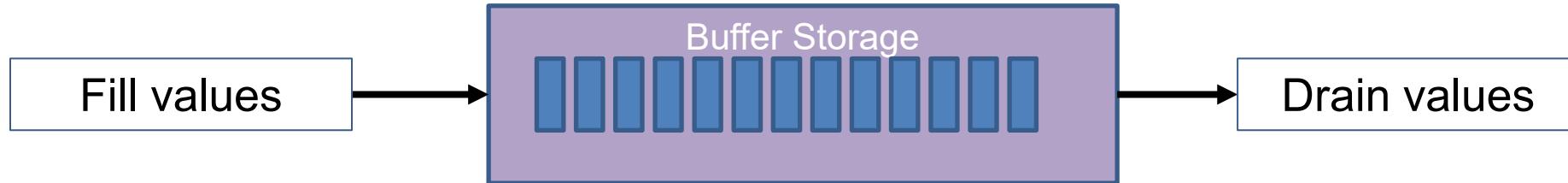


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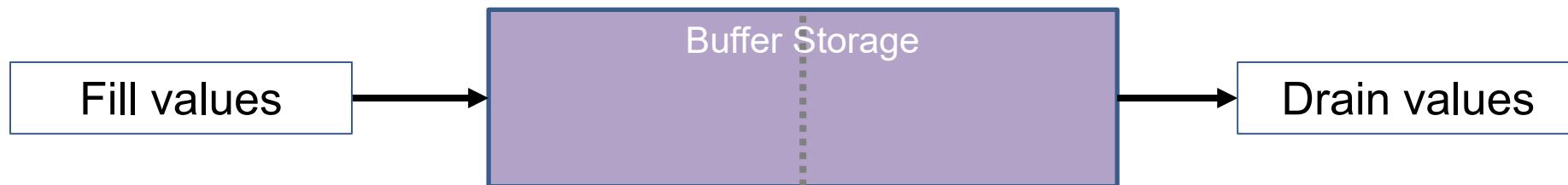


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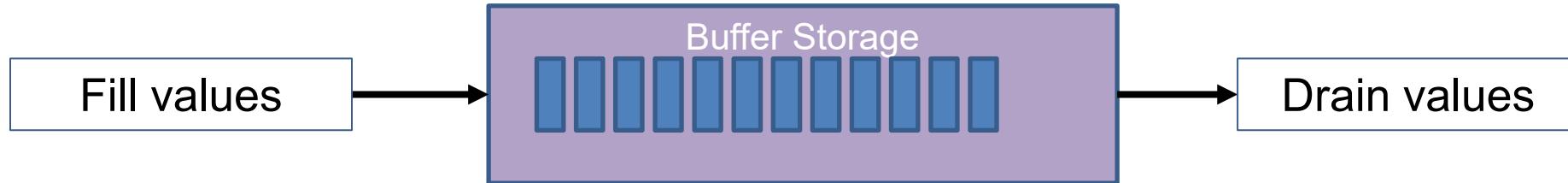


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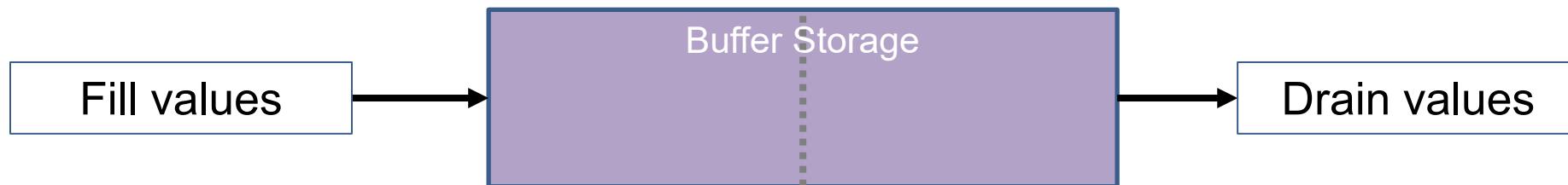


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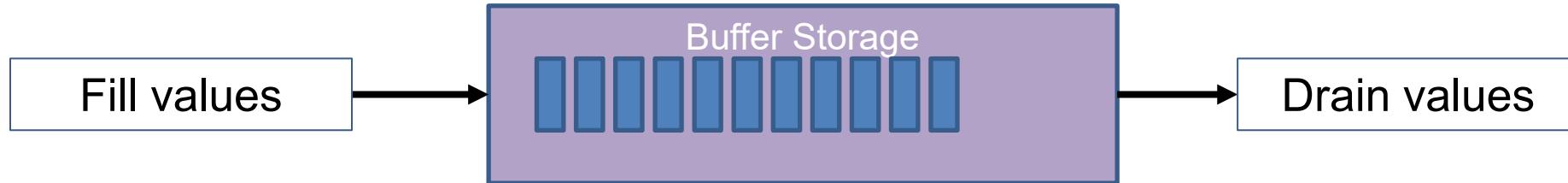


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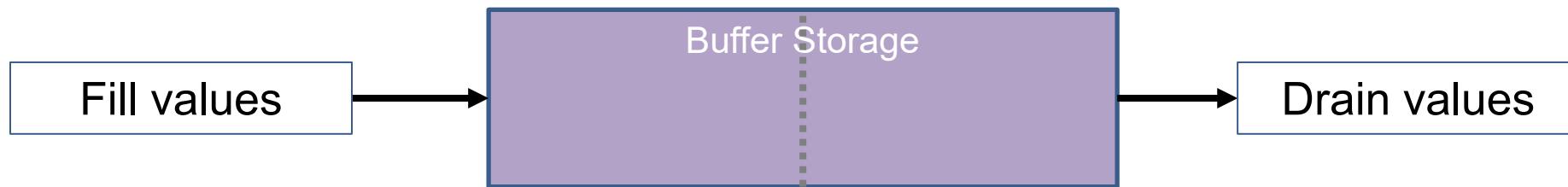


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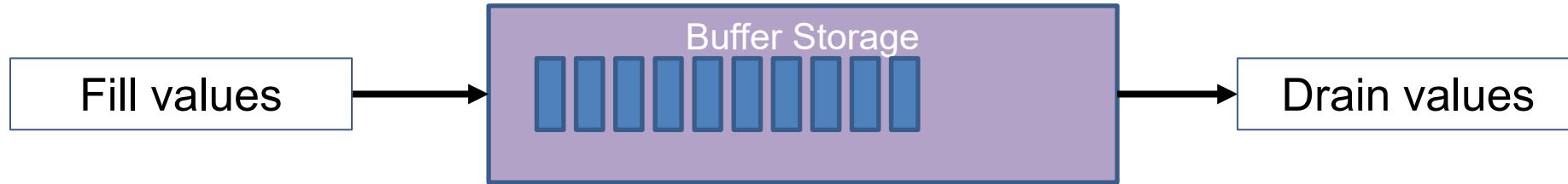


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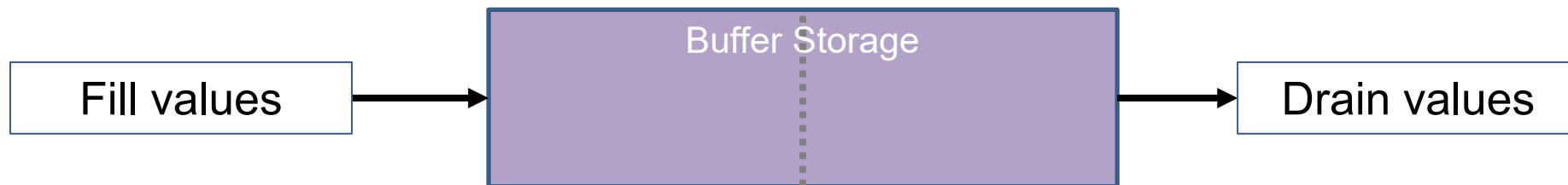


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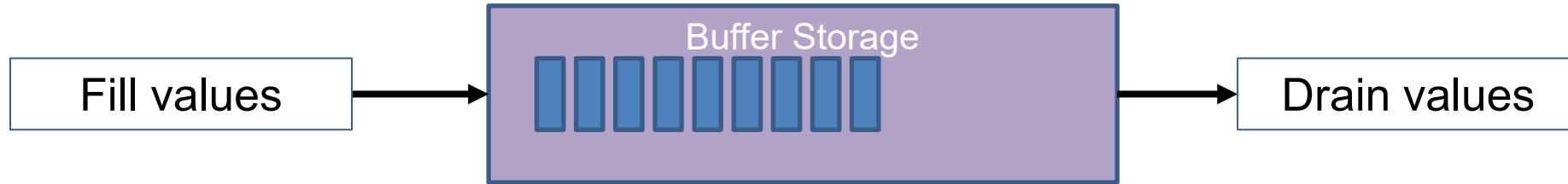


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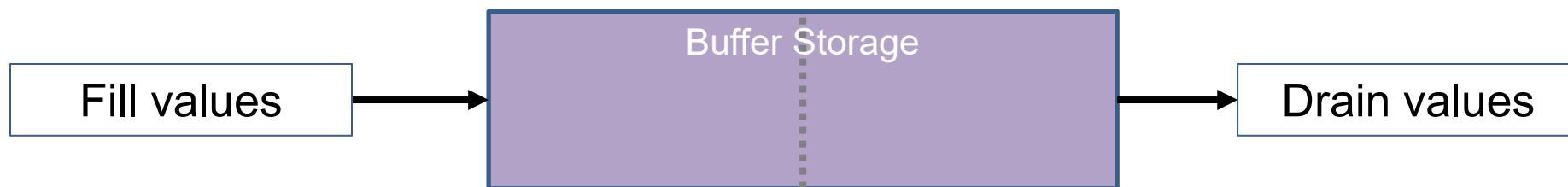


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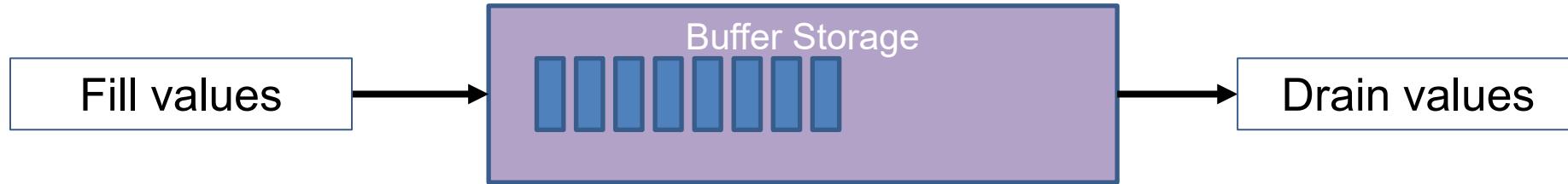


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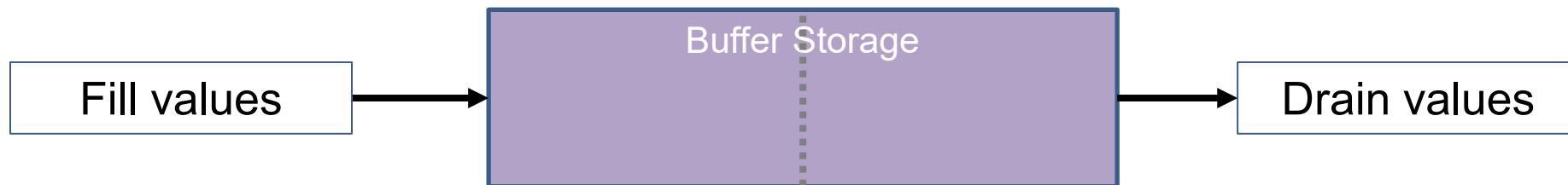


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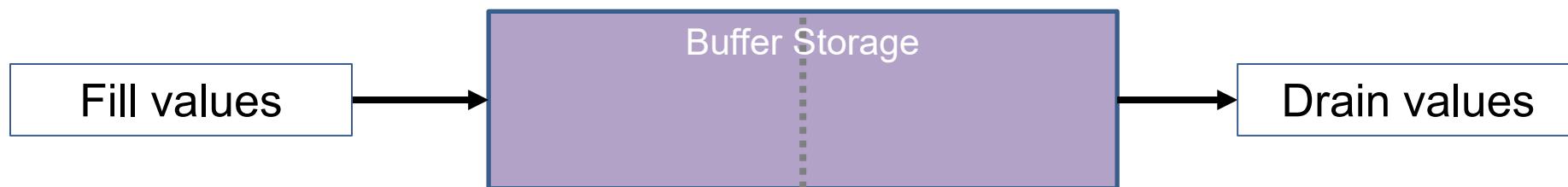


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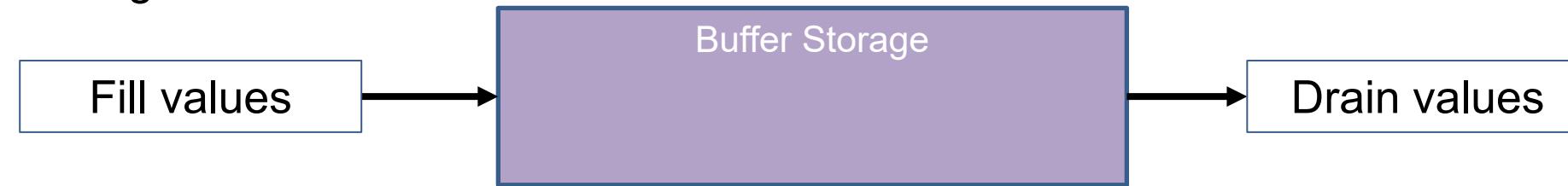
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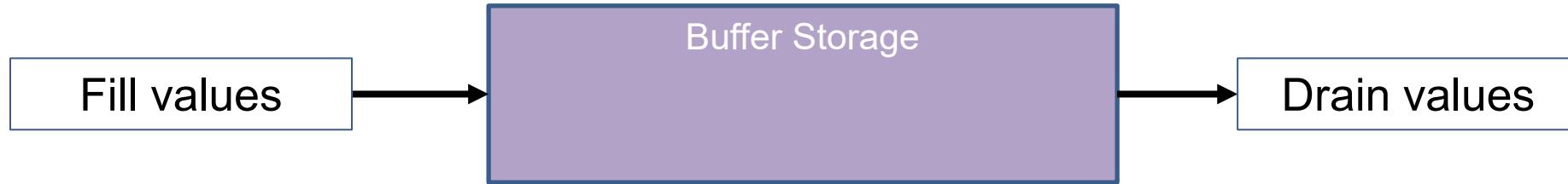


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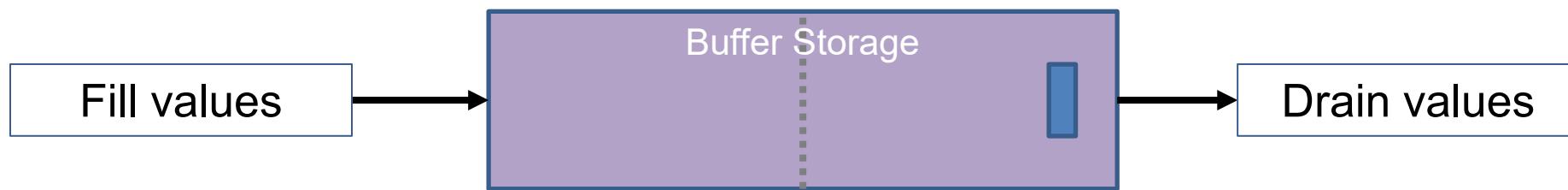


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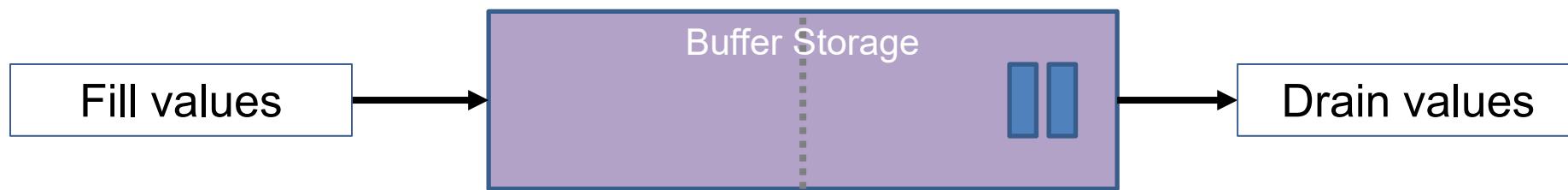


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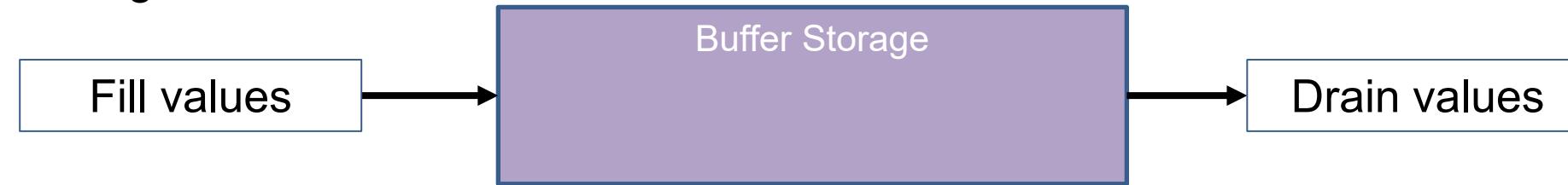
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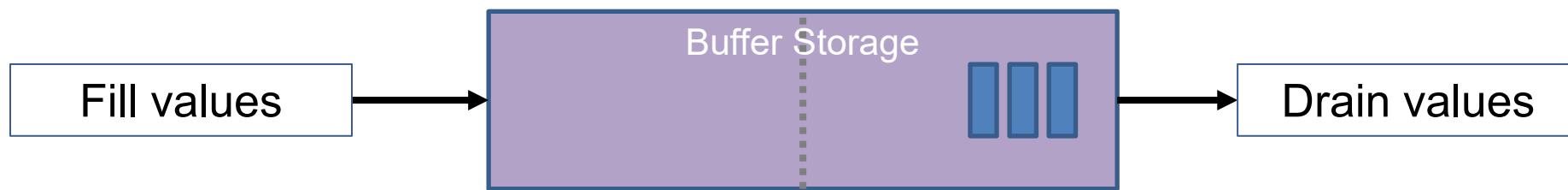


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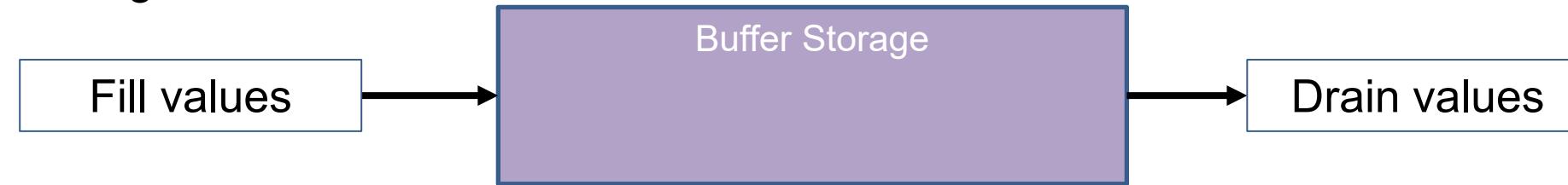
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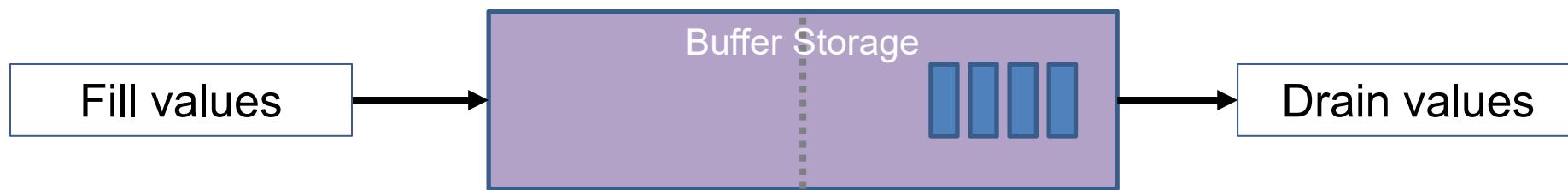


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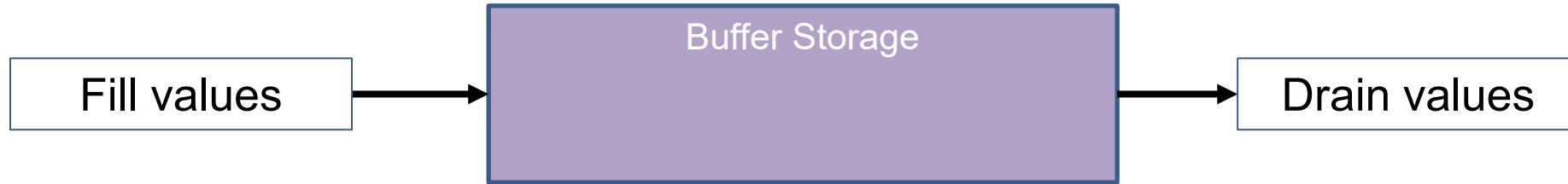


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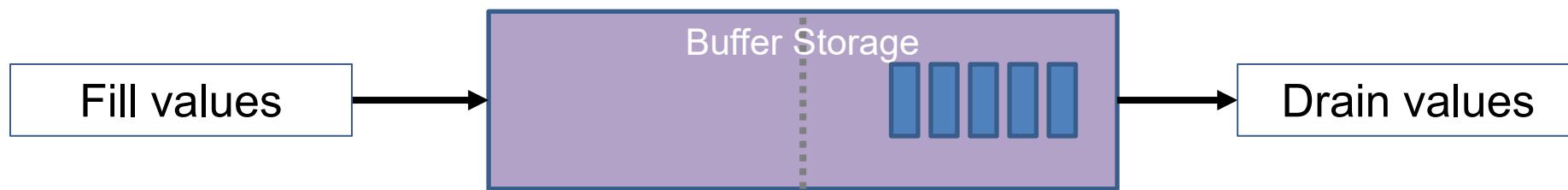


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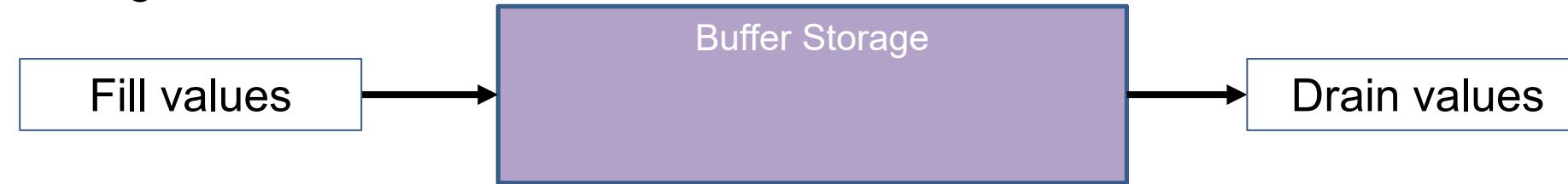
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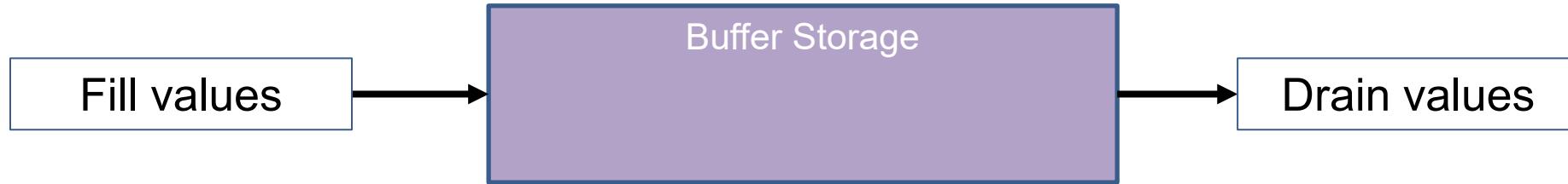


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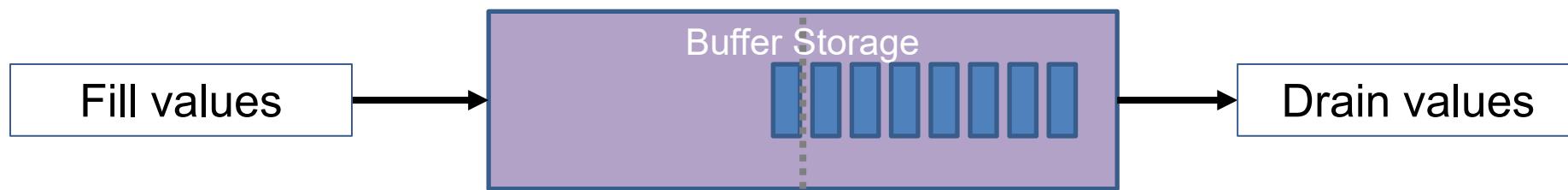


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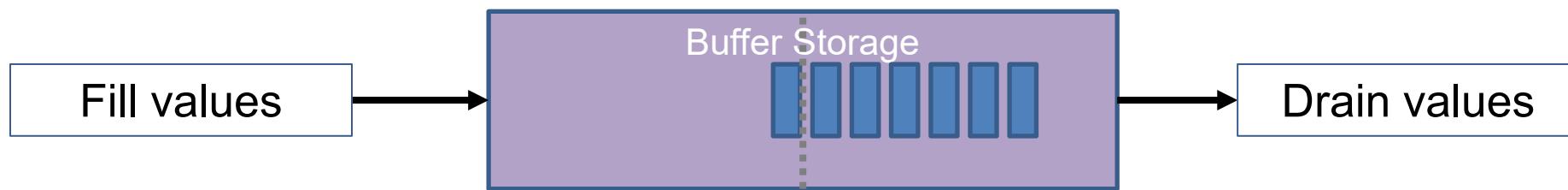


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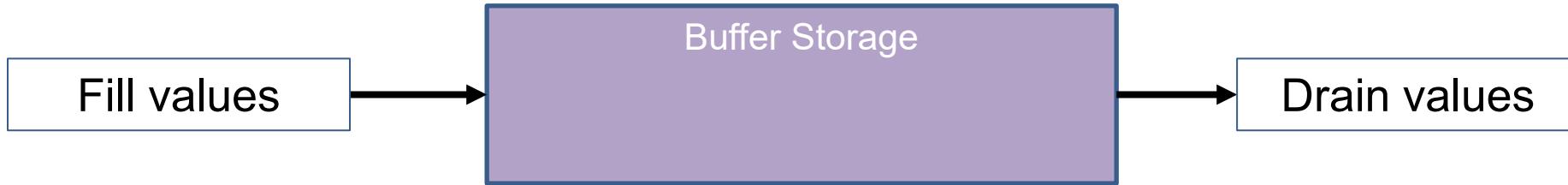


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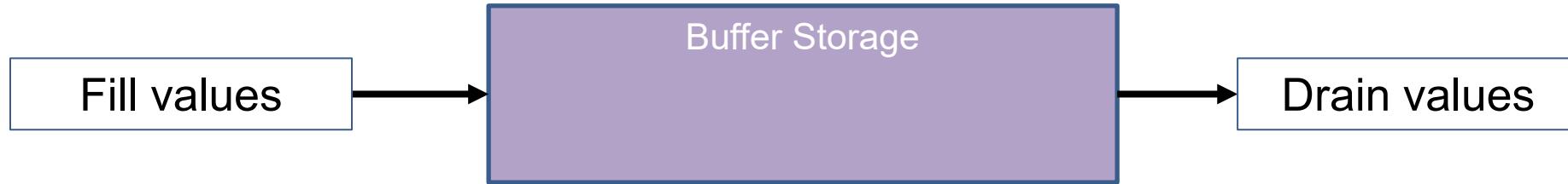


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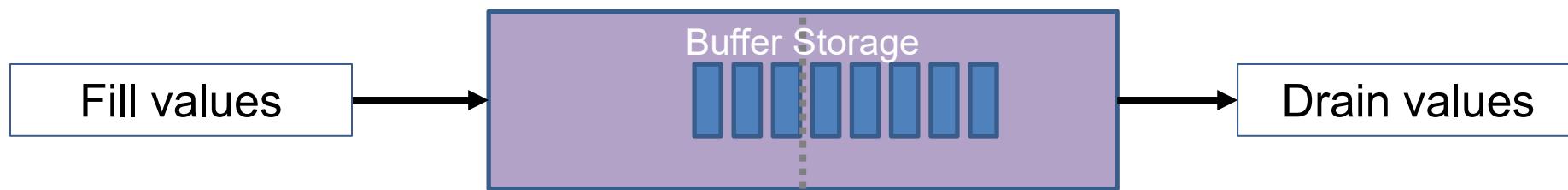


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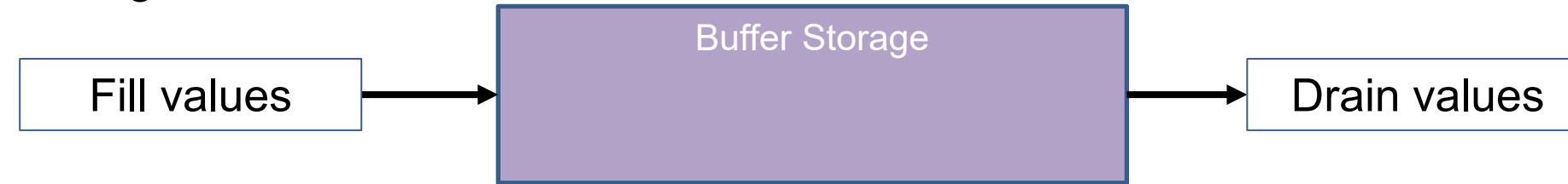
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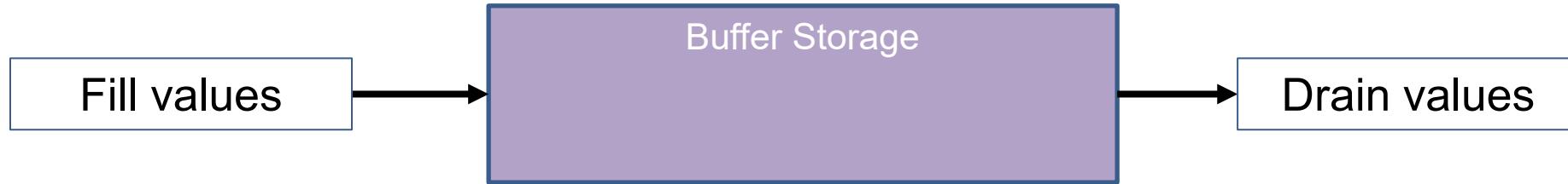


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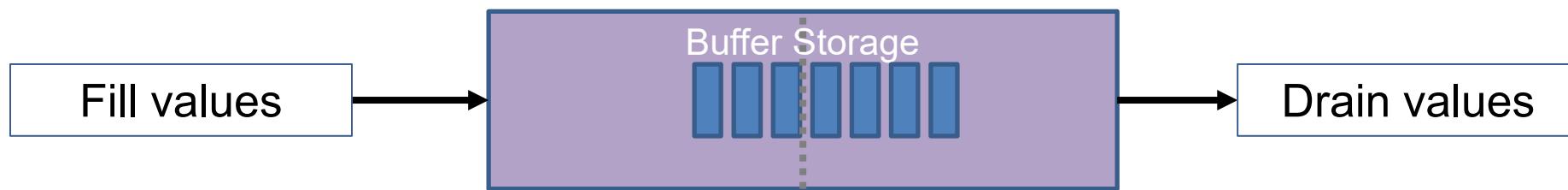


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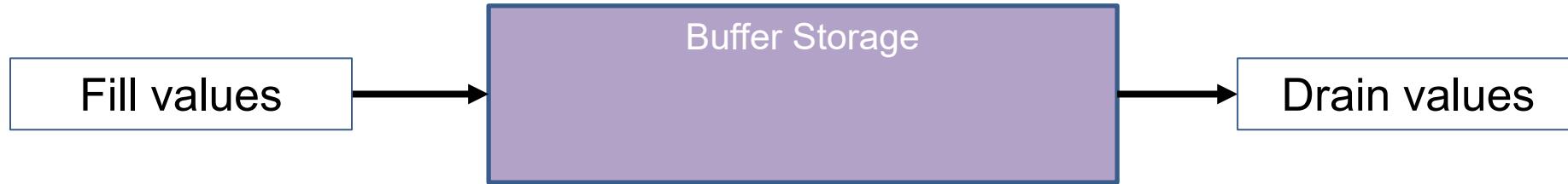


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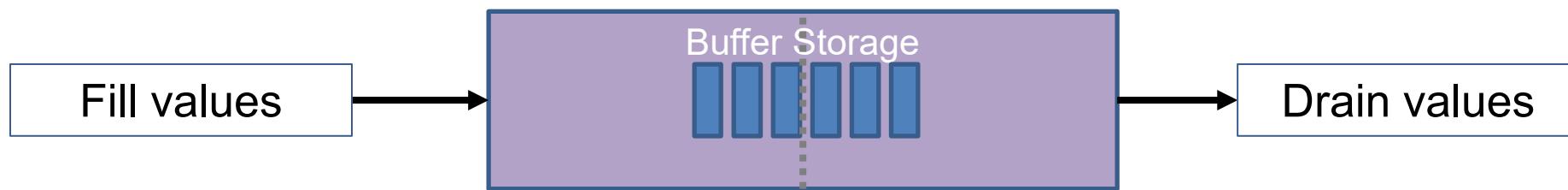


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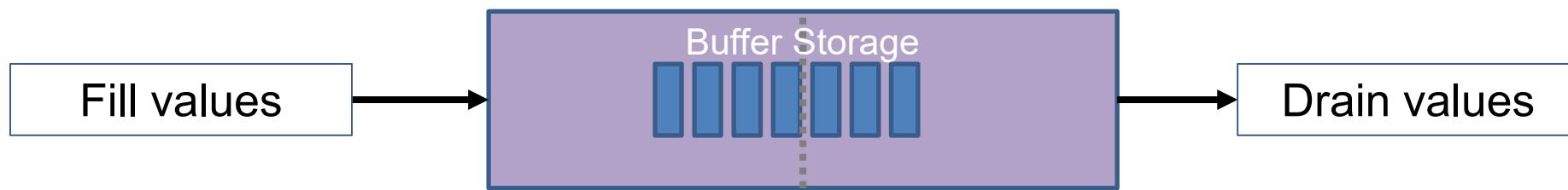


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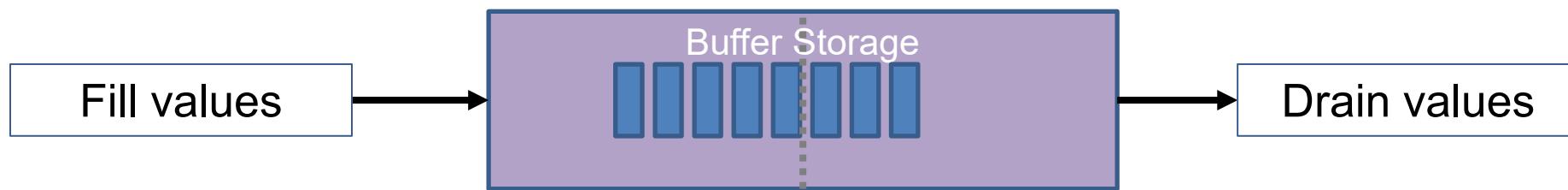


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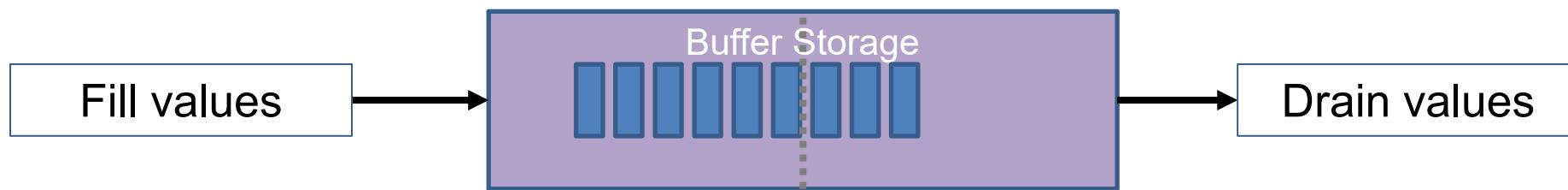


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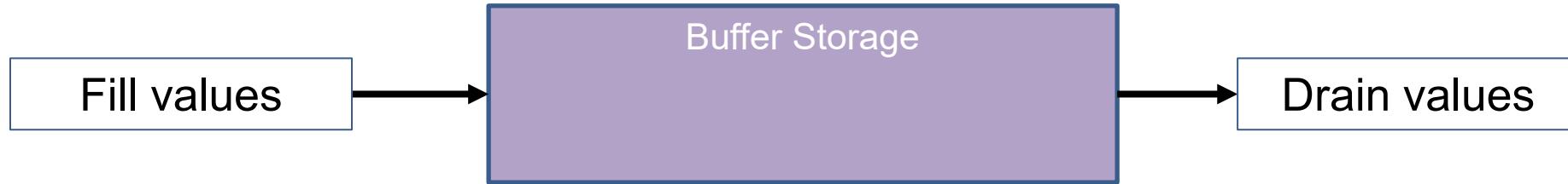


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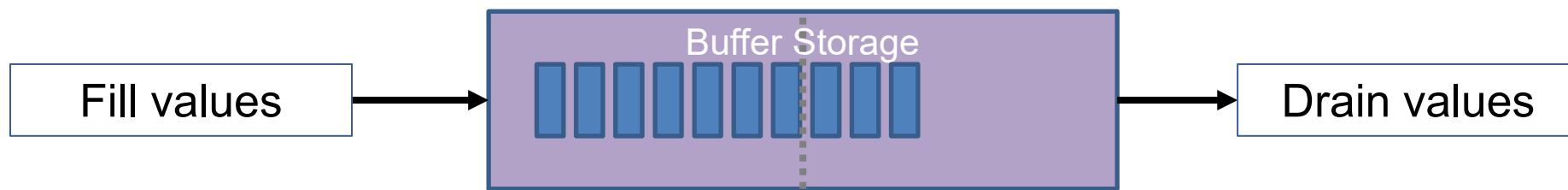


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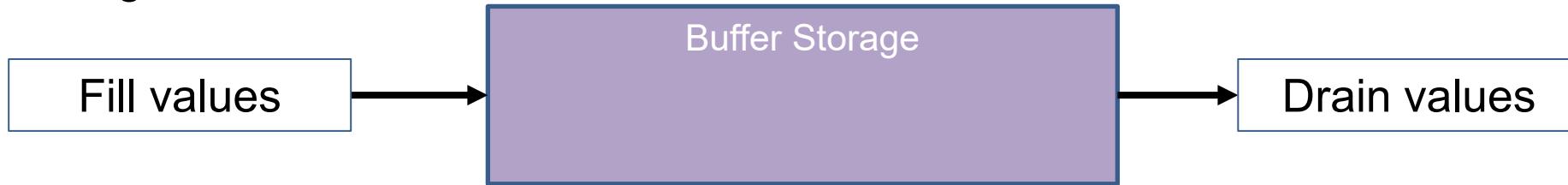
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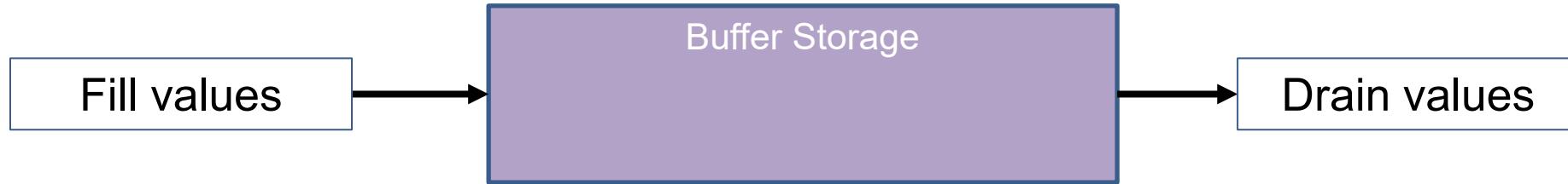


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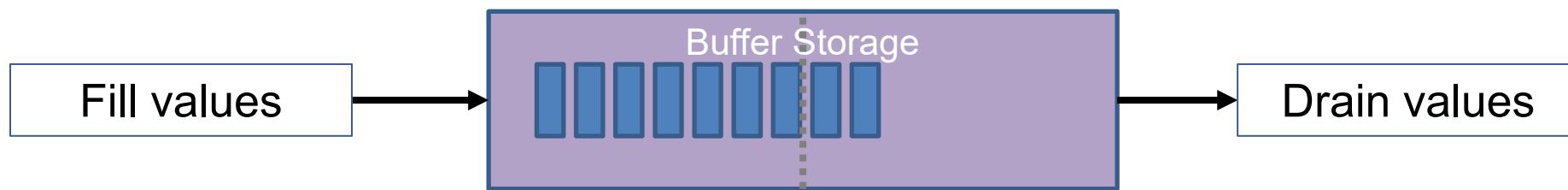


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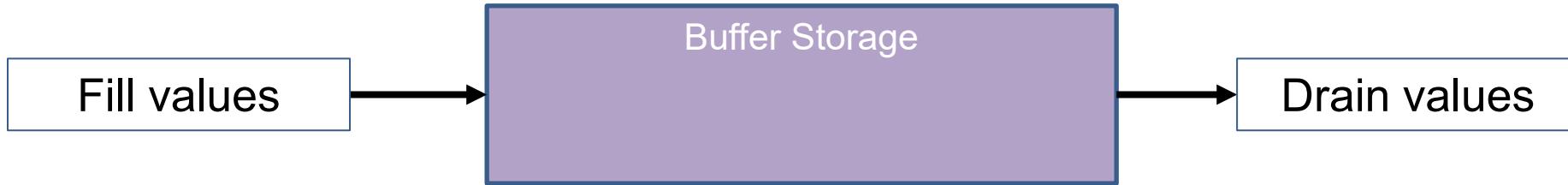


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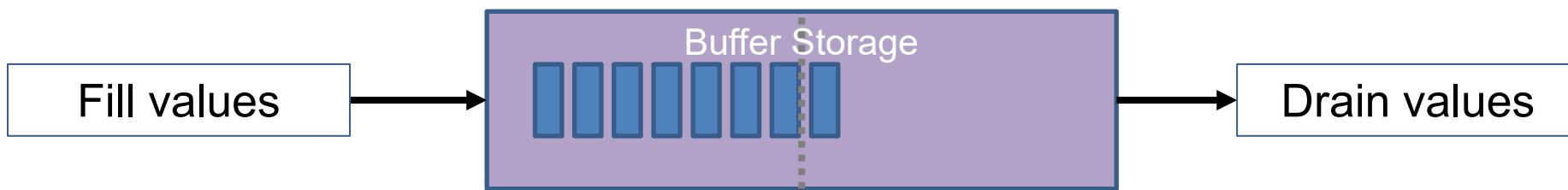


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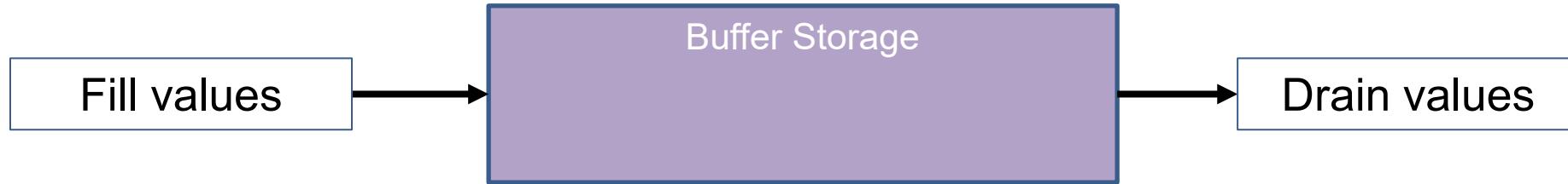


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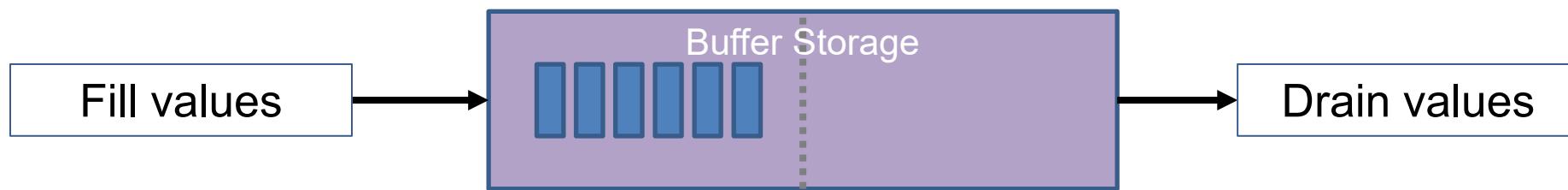


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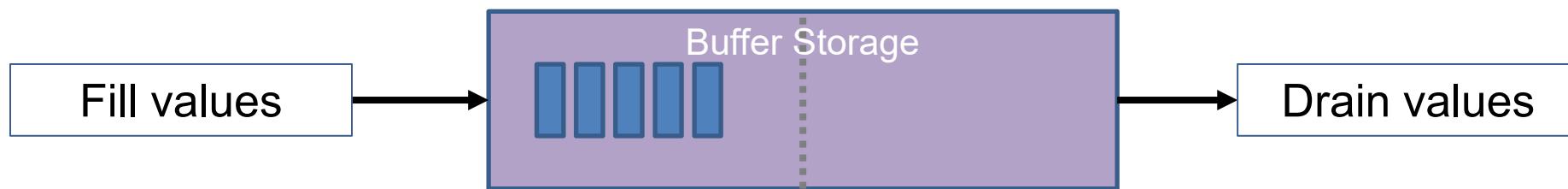


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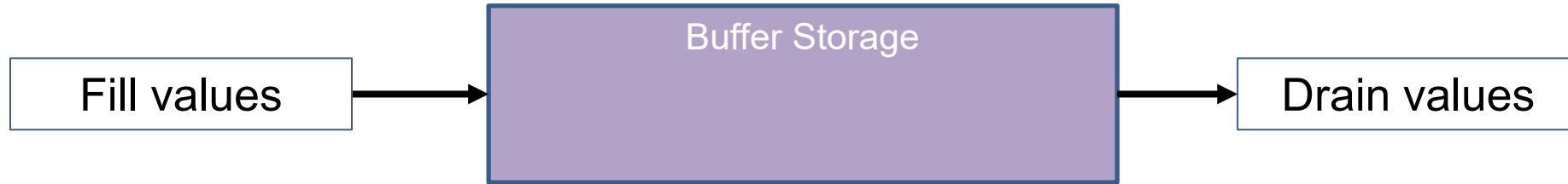


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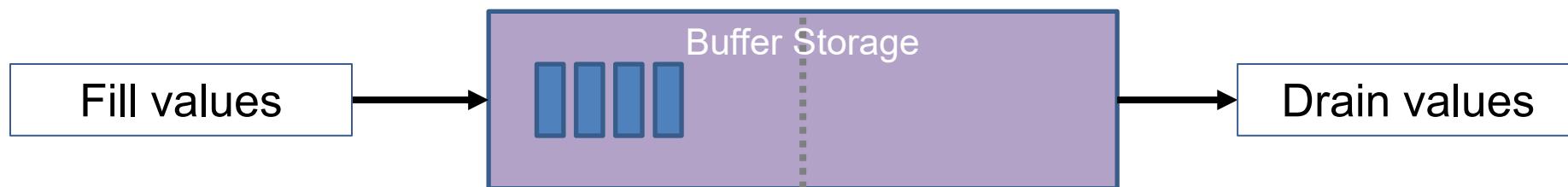


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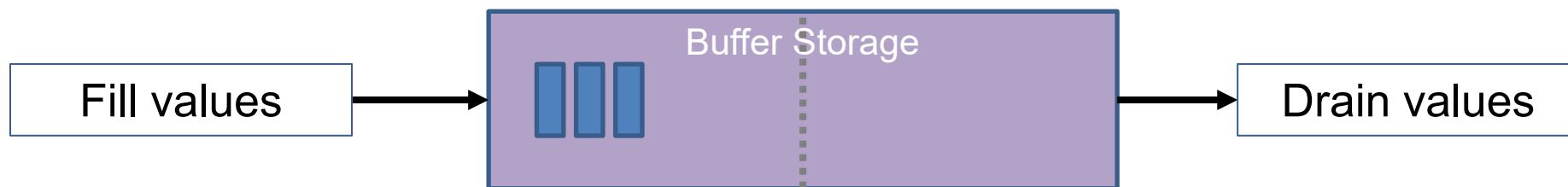


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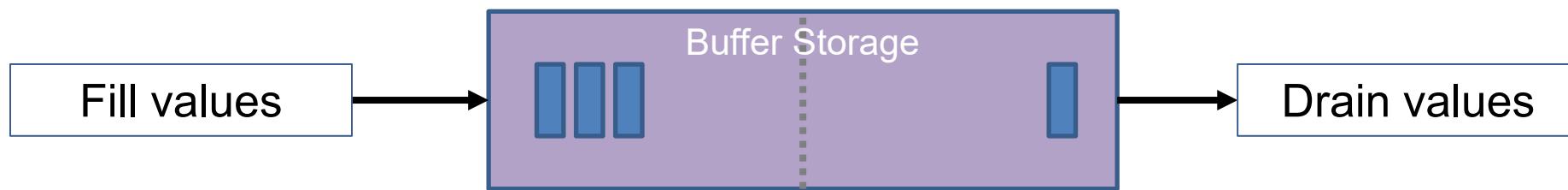


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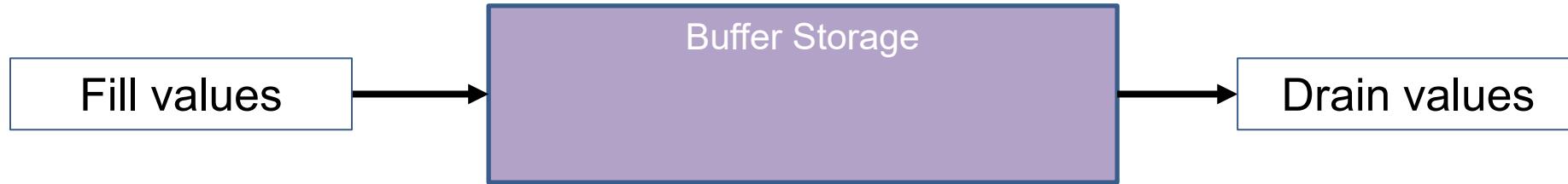


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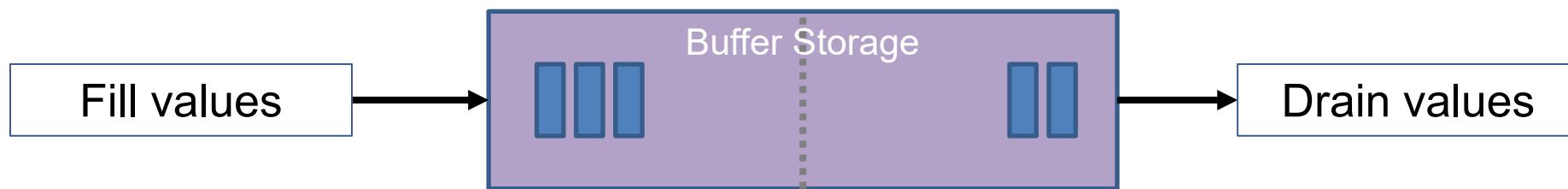


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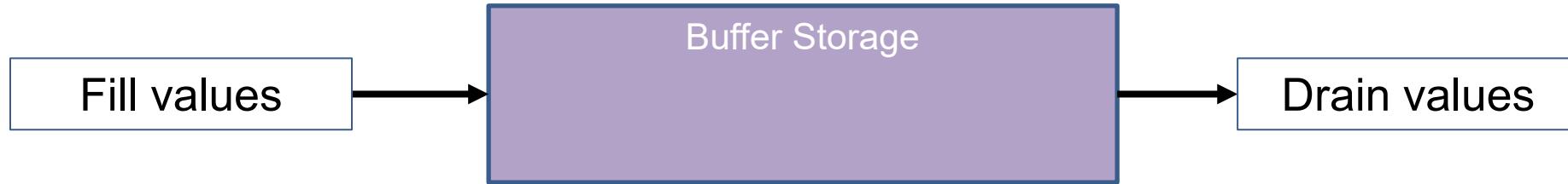


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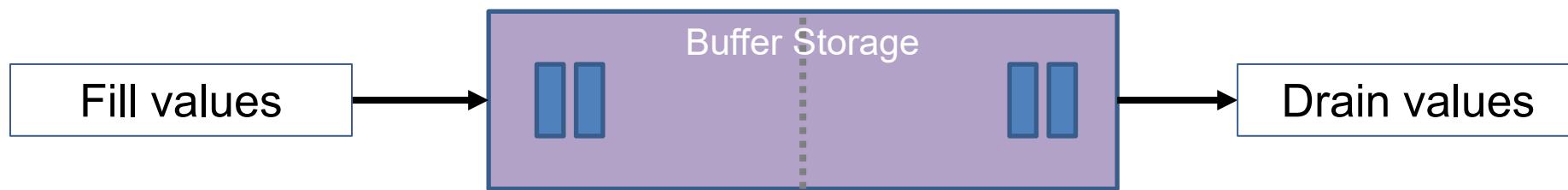


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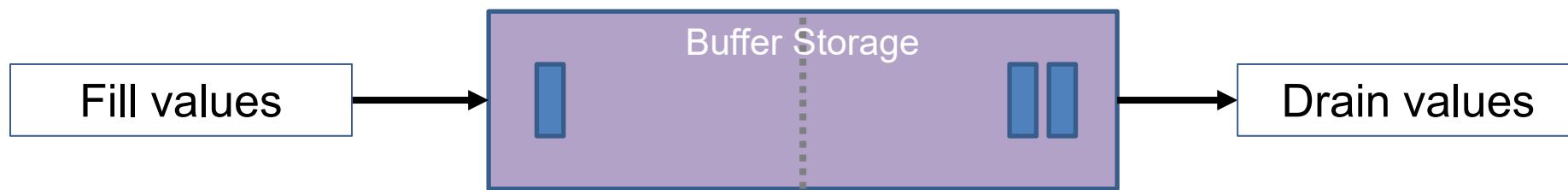


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Rolling use

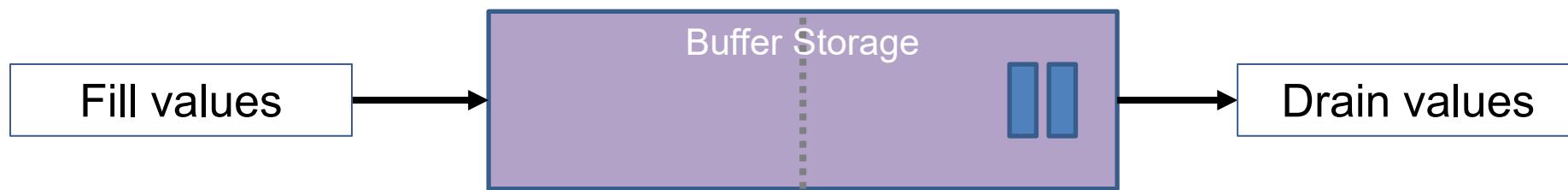


EDDO Strategies

Fill and use:



Double buffer



Rolling use

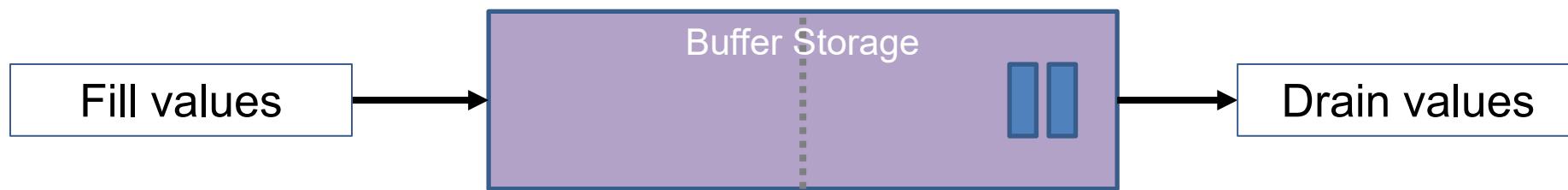


EDDO Strategies

Fill and use:



Double buffer

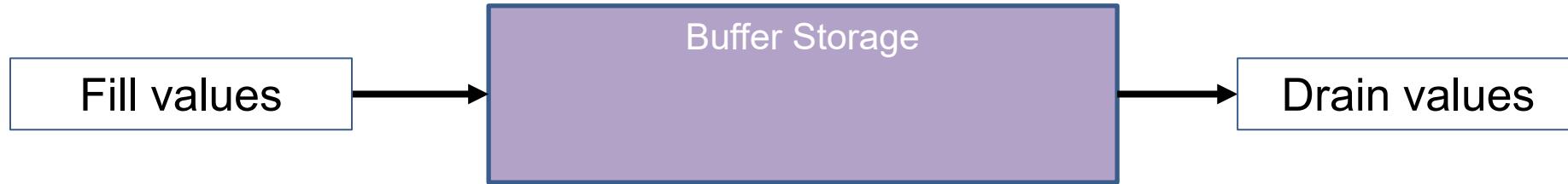


Rolling use

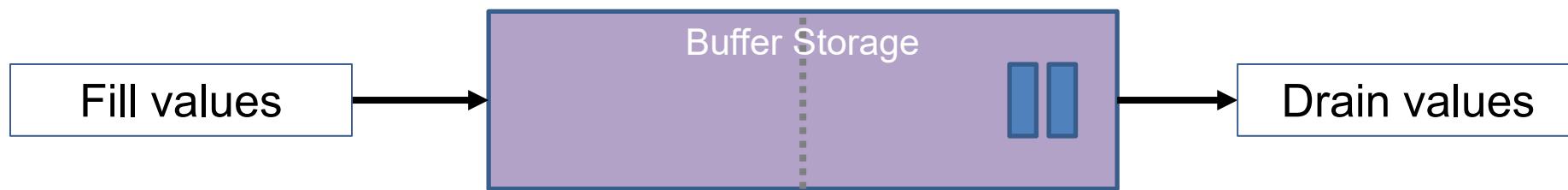


EDDO Strategies

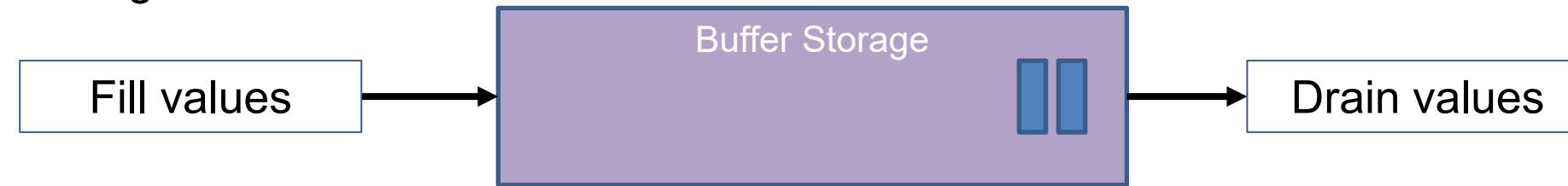
Fill and use:



Double buffer

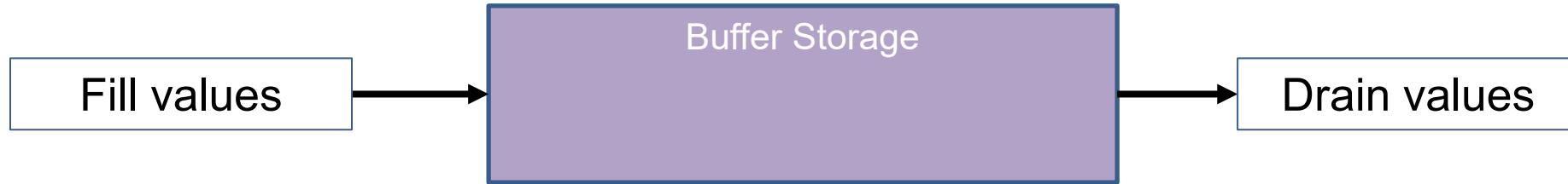


Rolling use

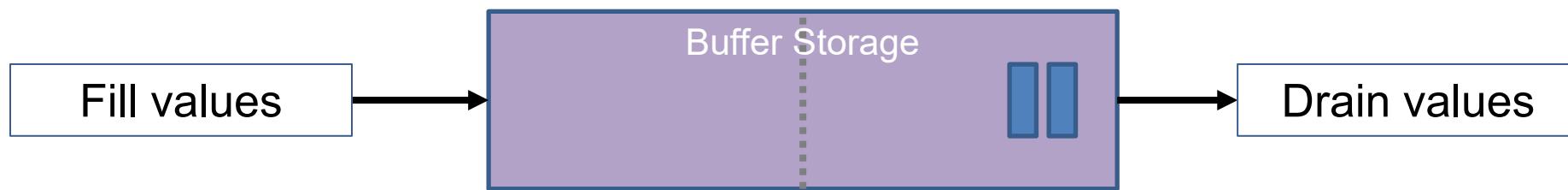


EDDO Strategies

Fill and use:



Double buffer



Rolling use

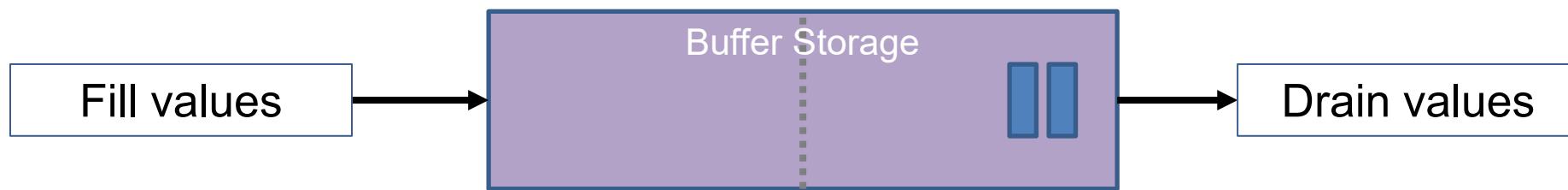


EDDO Strategies

Fill and use:



Double buffer



Rolling use

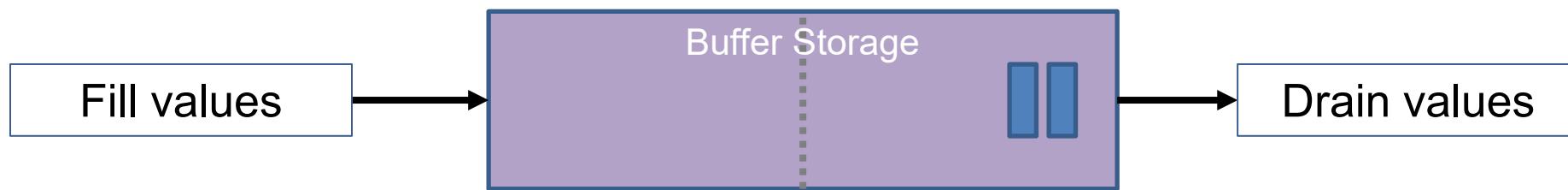


EDDO Strategies

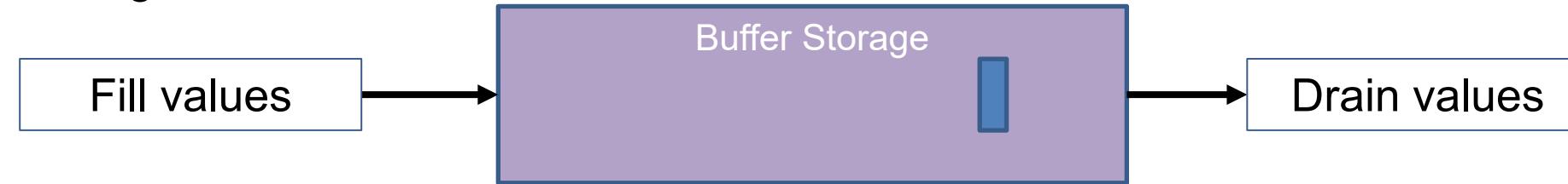
Fill and use:



Double buffer

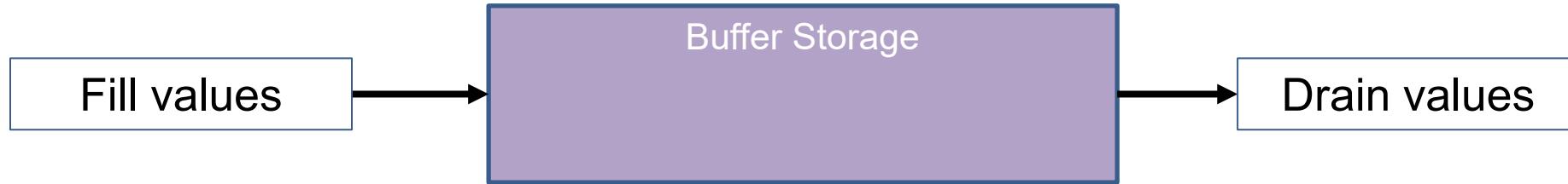


Rolling use

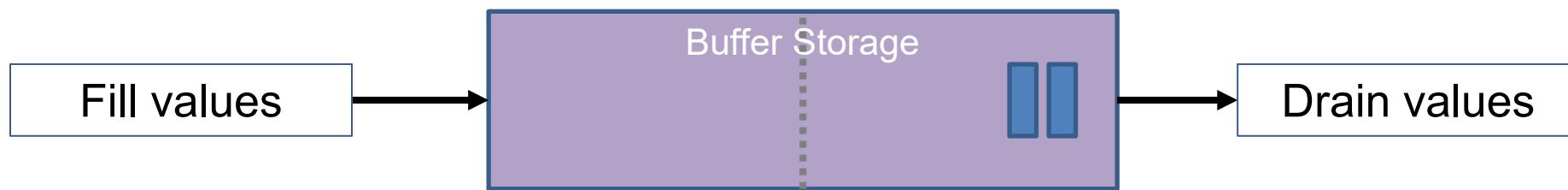


EDDO Strategies

Fill and use:



Double buffer



Rolling use

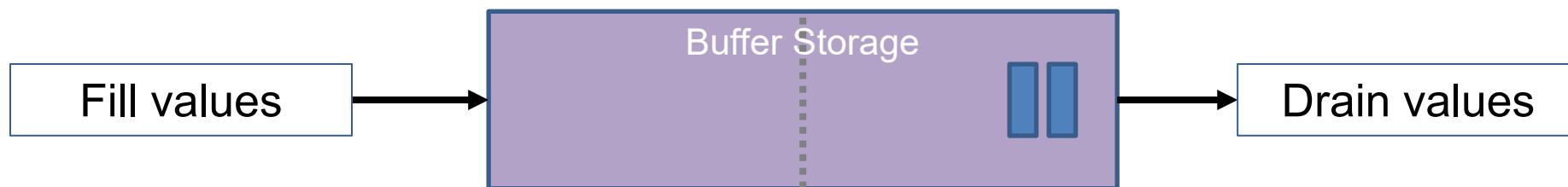


EDDO Strategies

Fill and use:



Double buffer

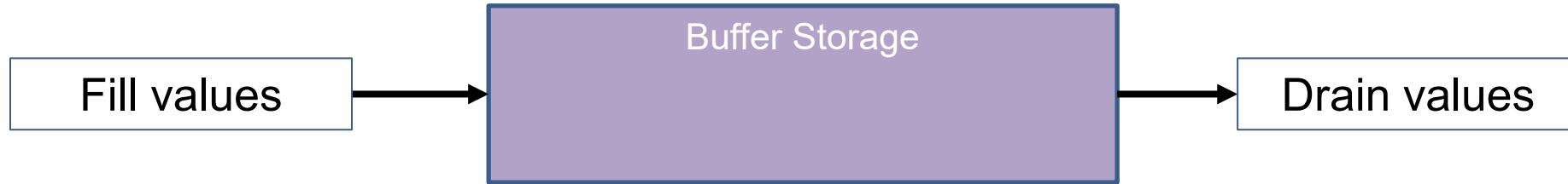


Rolling use

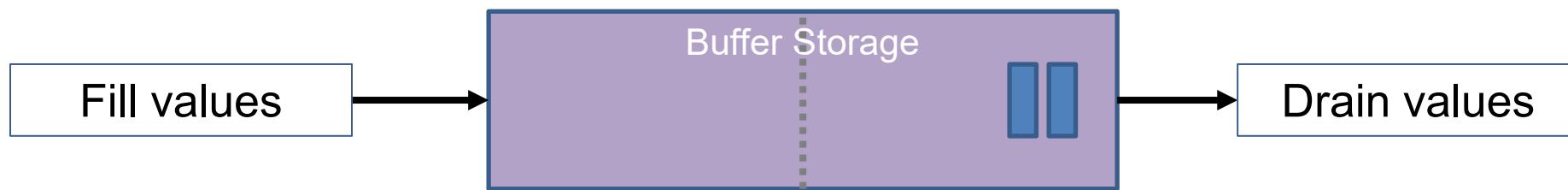


EDDO Strategies

Fill and use:



Double buffer



Rolling use

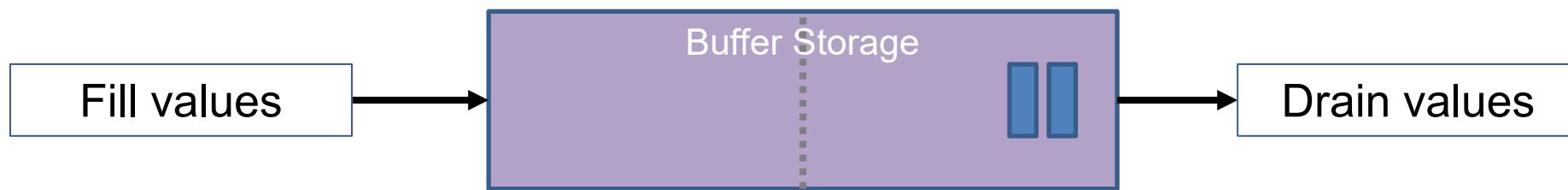


EDDO Strategies

Fill and use:



Double buffer



Rolling use

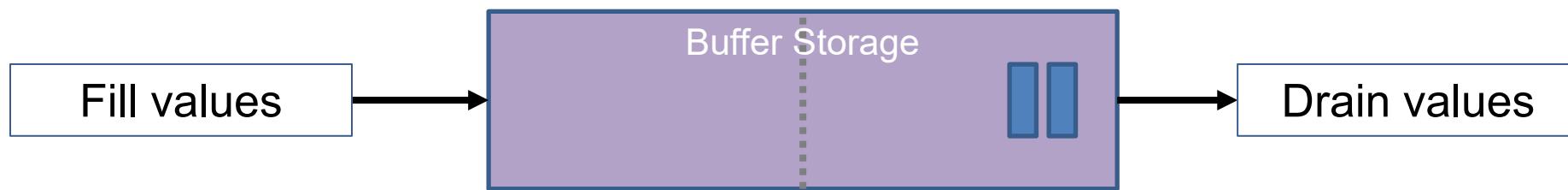


EDDO Strategies

Fill and use:



Double buffer



Rolling use

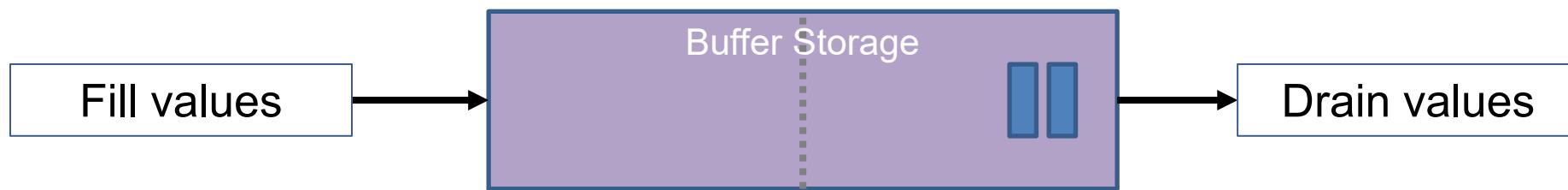


EDDO Strategies

Fill and use:



Double buffer



Rolling use

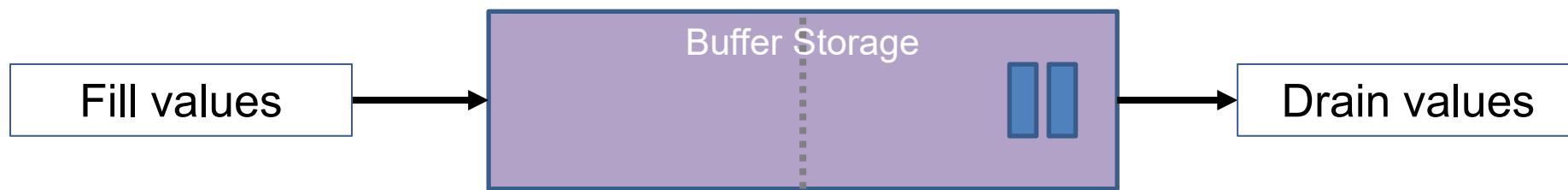


EDDO Strategies

Fill and use:



Double buffer

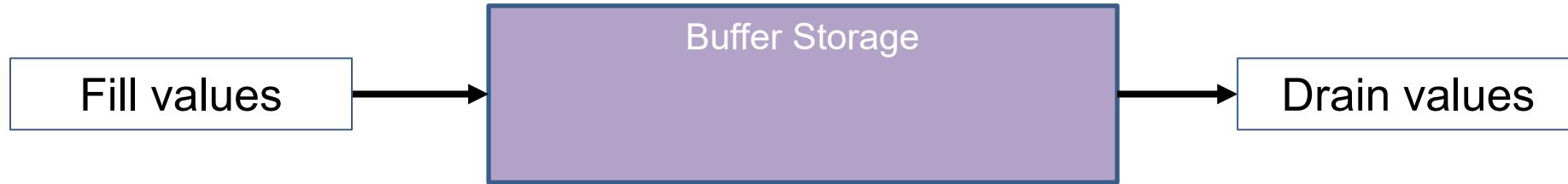


Rolling use

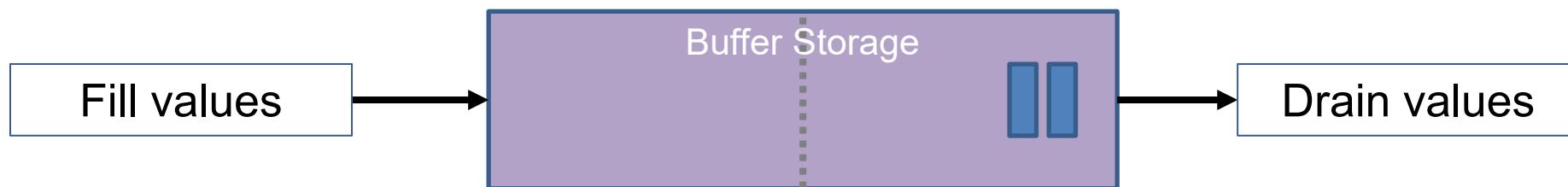


EDDO Strategies

Fill and use:



Double buffer



Rolling use

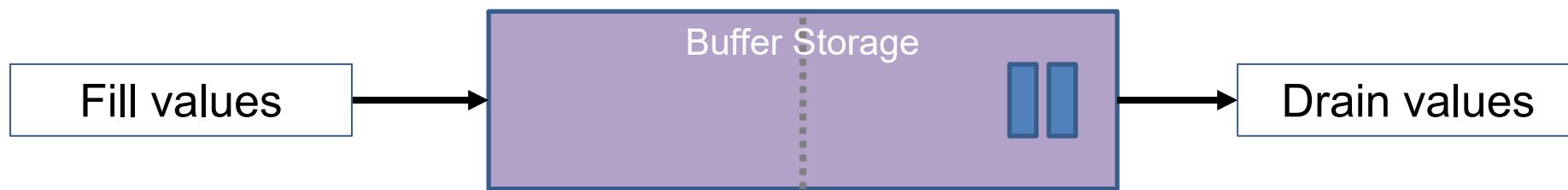


EDDO Strategies

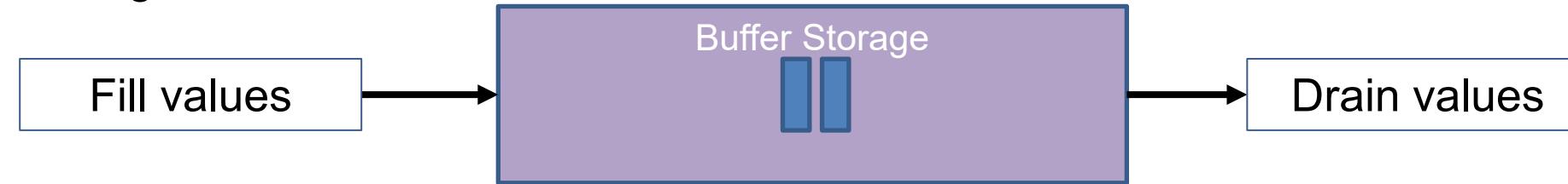
Fill and use:



Double buffer

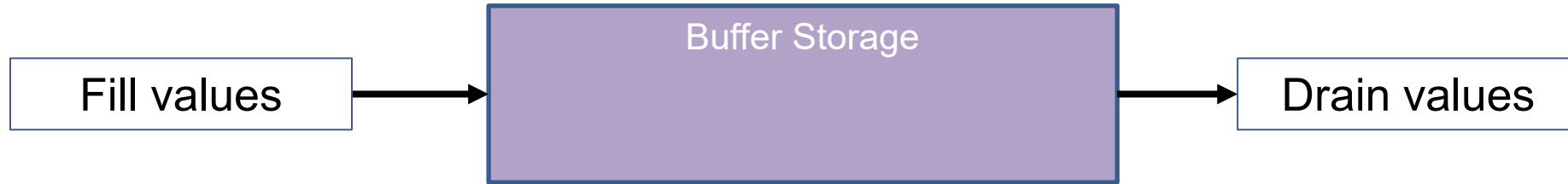


Rolling use

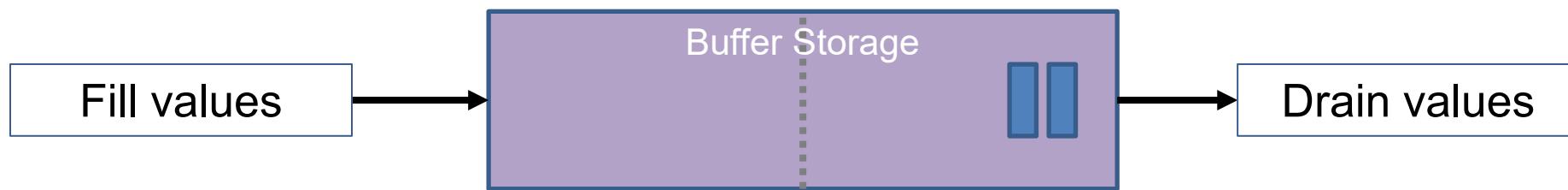


EDDO Strategies

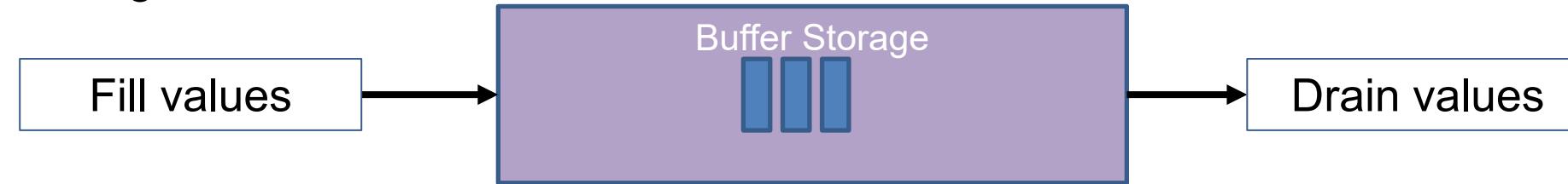
Fill and use:



Double buffer



Rolling use

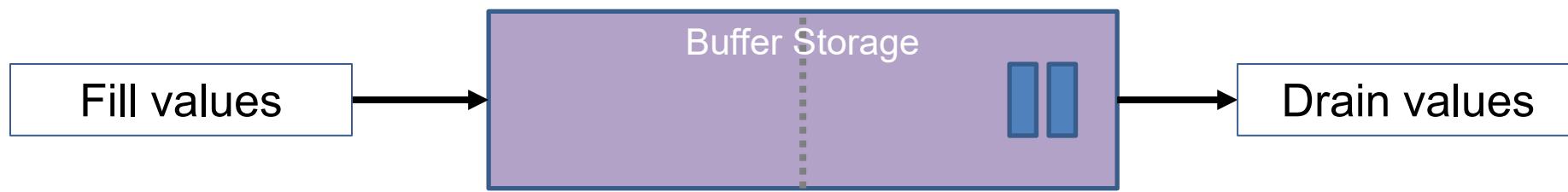


EDDO Strategies

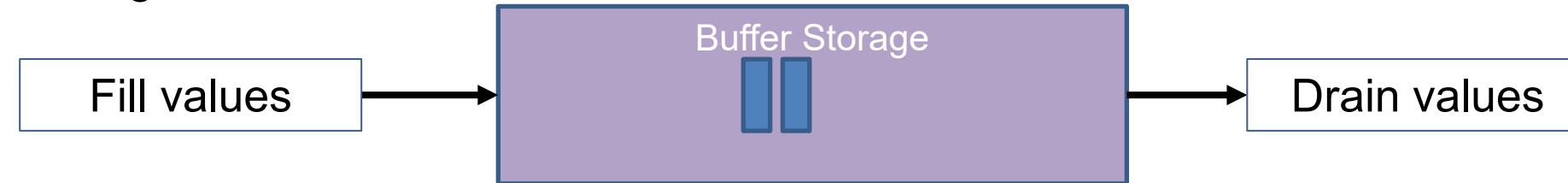
Fill and use:



Double buffer



Rolling use

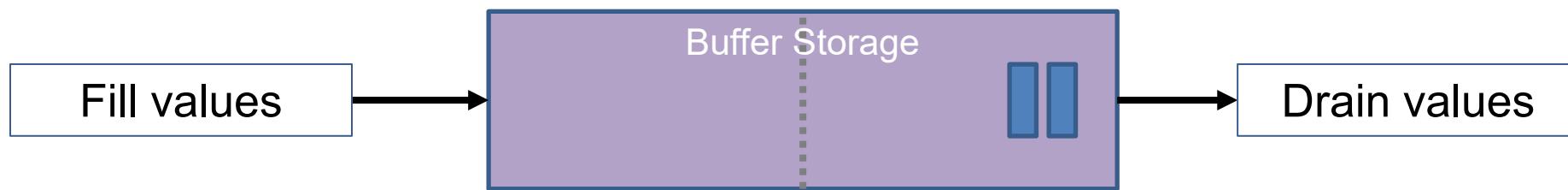


EDDO Strategies

Fill and use:



Double buffer



Rolling use

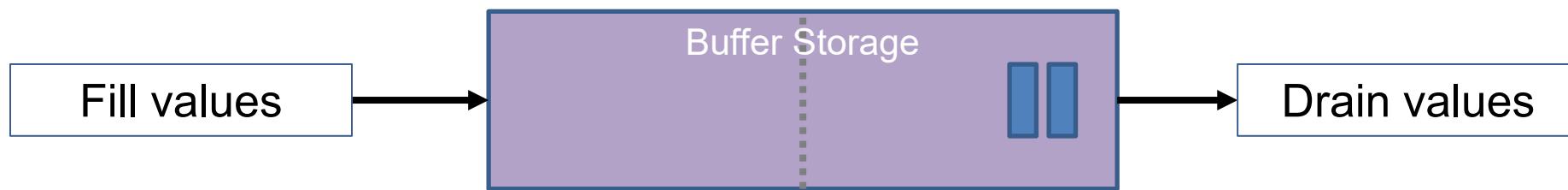


EDDO Strategies

Fill and use:



Double buffer

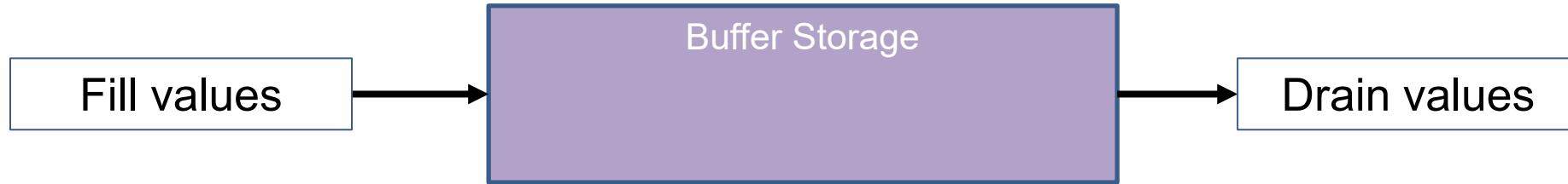


Rolling use

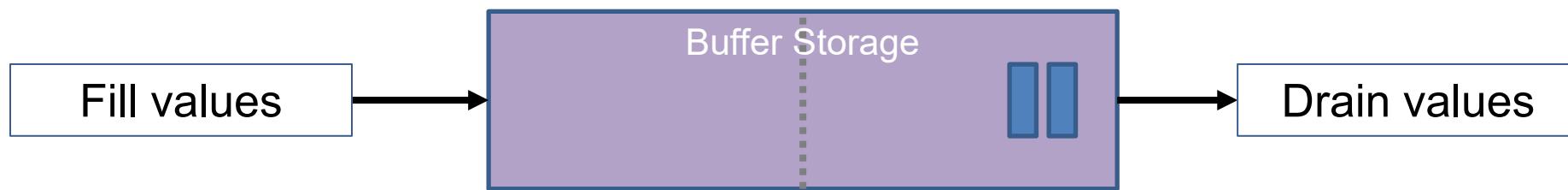


EDDO Strategies

Fill and use:



Double buffer

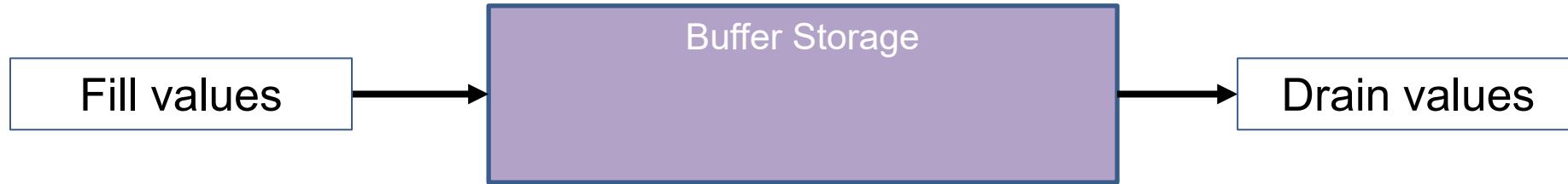


Rolling use

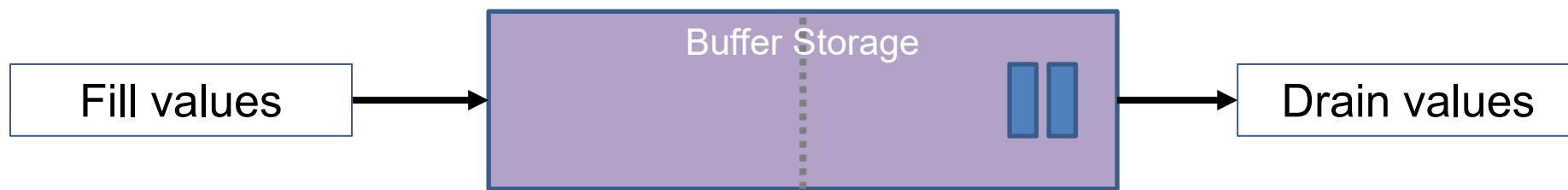


EDDO Strategies

Fill and use:



Double buffer



Rolling use

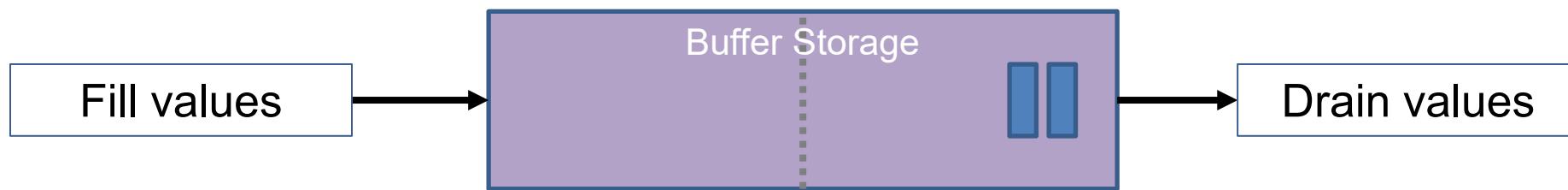


EDDO Strategies

Fill and use:



Double buffer

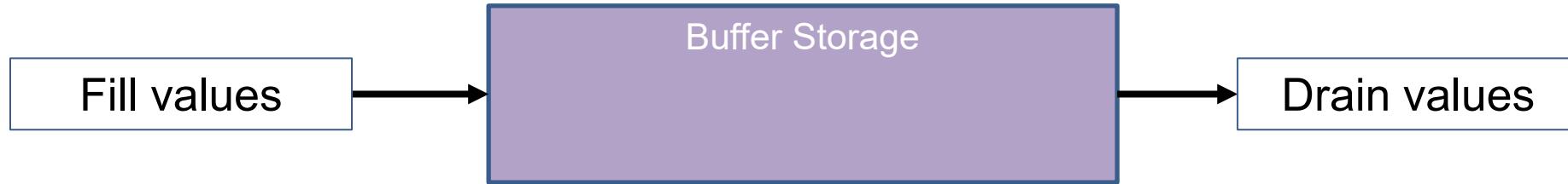


Rolling use

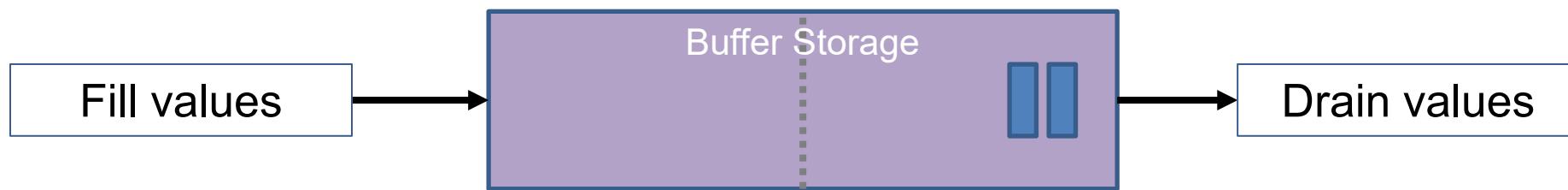


EDDO Strategies

Fill and use:



Double buffer

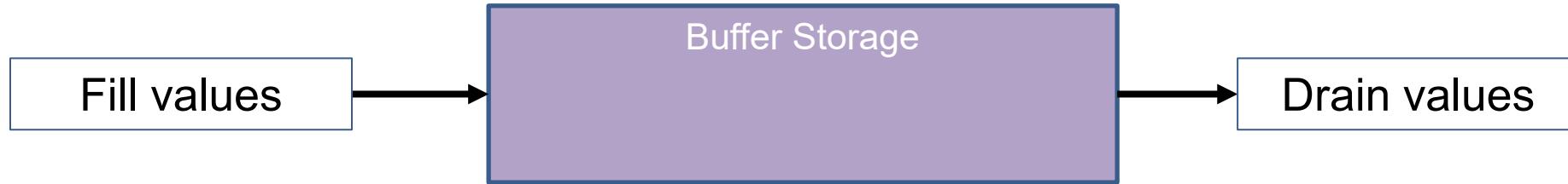


Rolling use

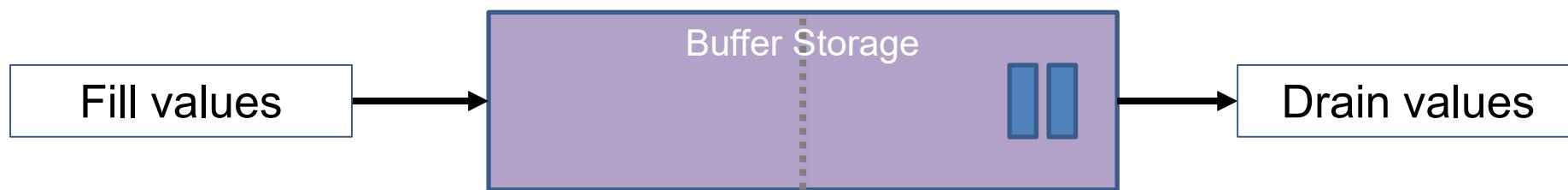


EDDO Strategies

Fill and use:



Double buffer



Rolling use

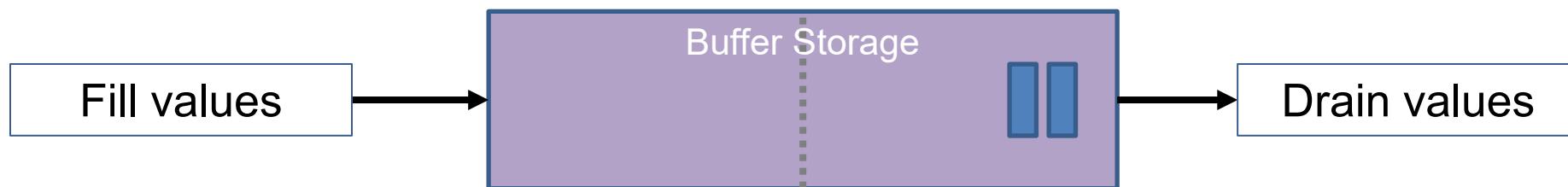


EDDO Strategies

Fill and use:



Double buffer

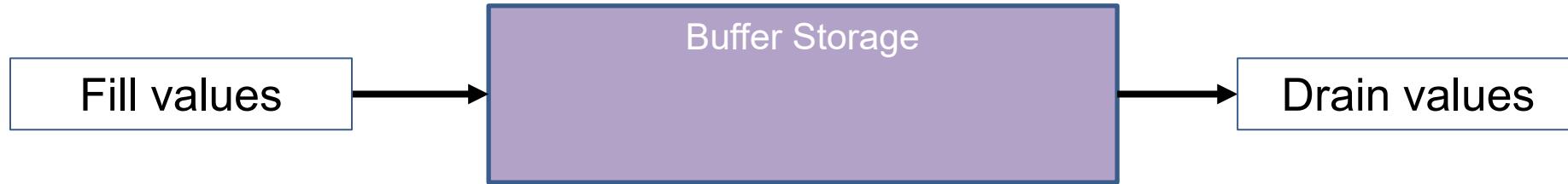


Rolling use

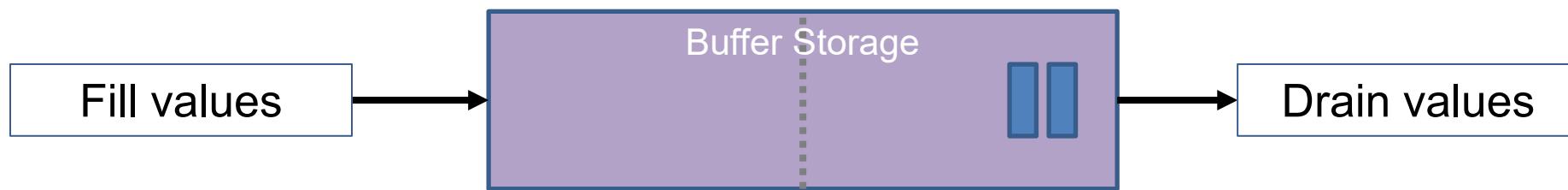


EDDO Strategies

Fill and use:



Double buffer

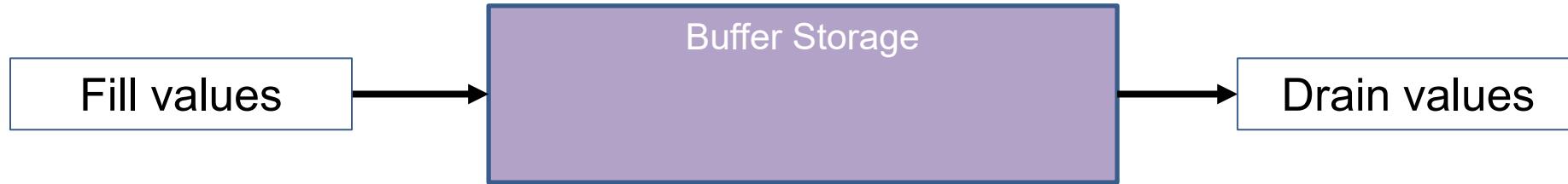


Rolling use

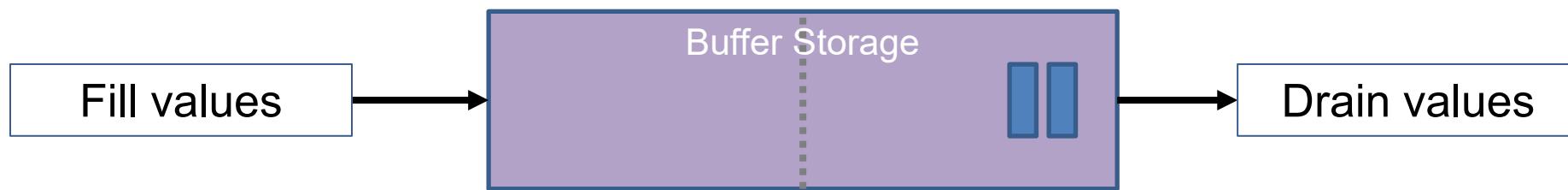


EDDO Strategies

Fill and use:



Double buffer

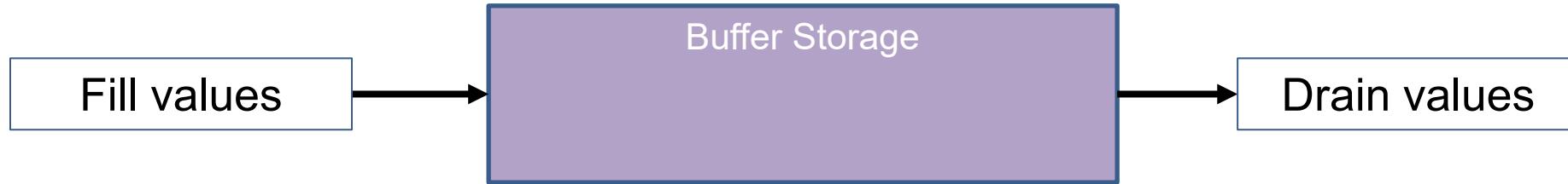


Rolling use

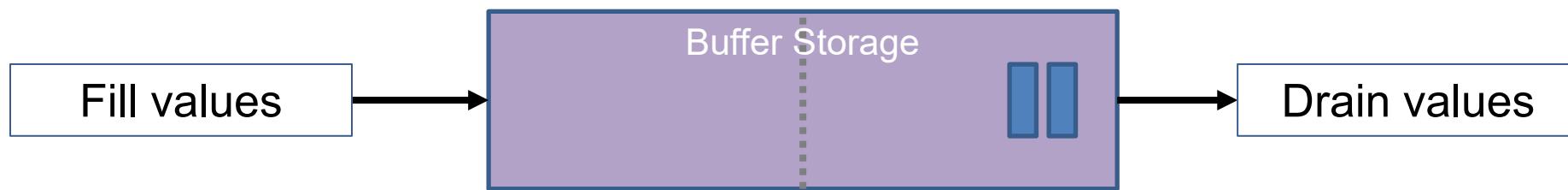


EDDO Strategies

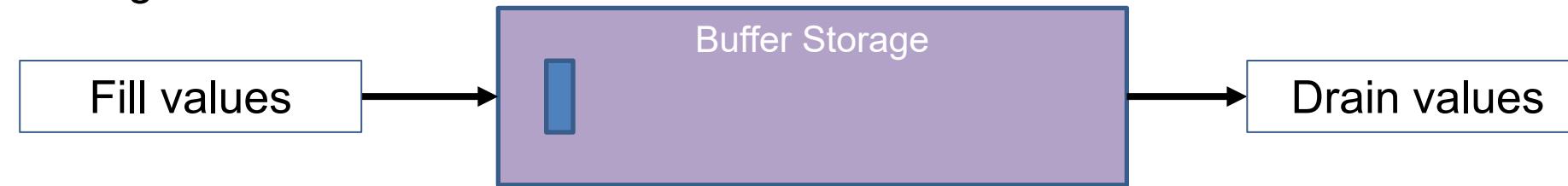
Fill and use:



Double buffer



Rolling use

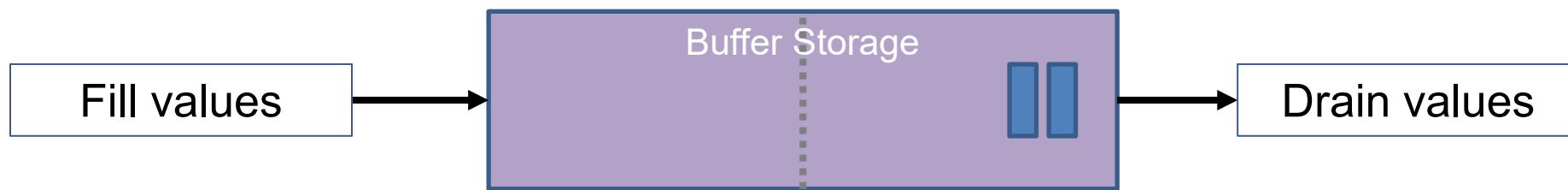


EDDO Strategies

Fill and use:



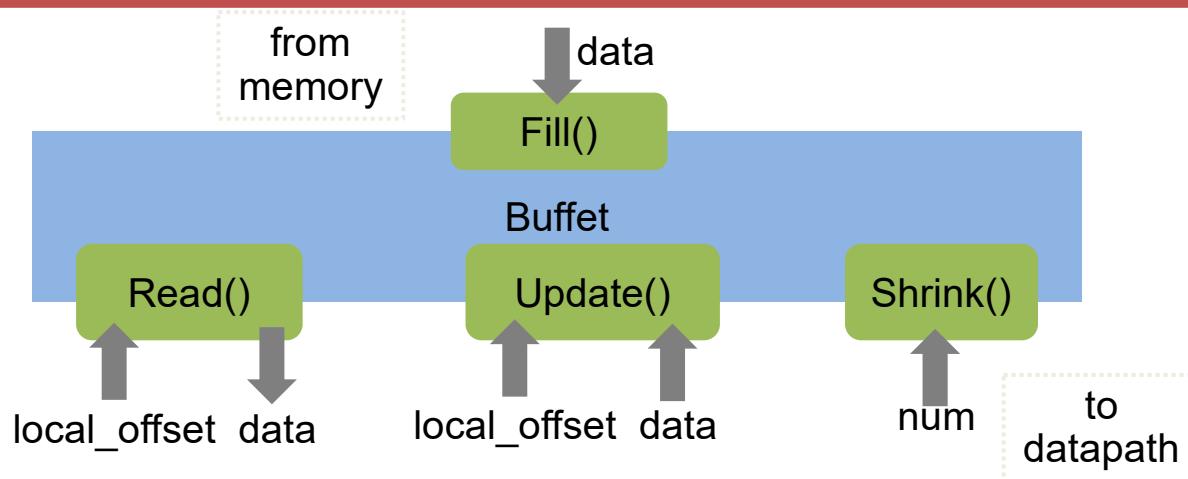
Double buffer



Rolling use



Buffets



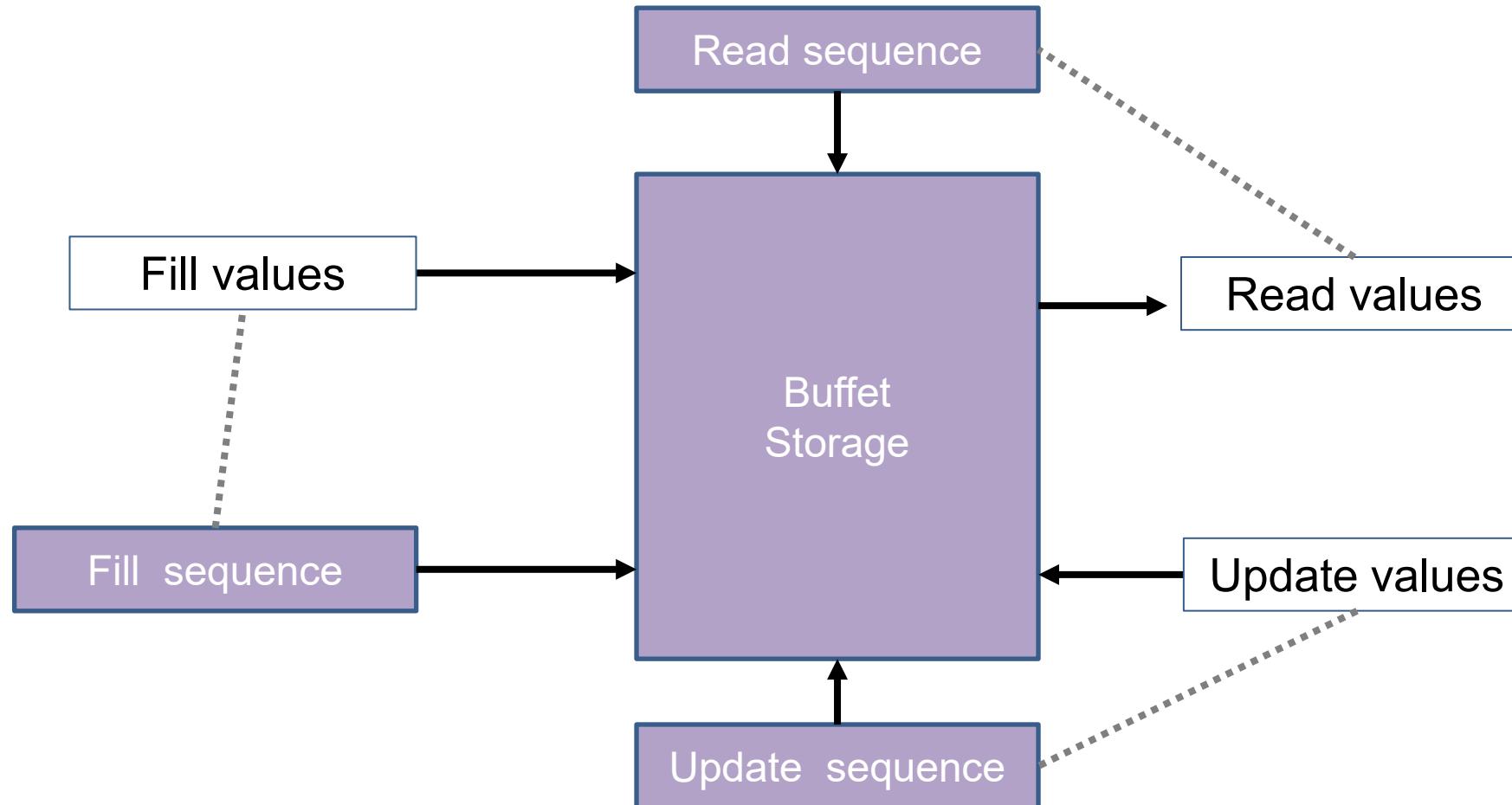
	Buffet
Hierarchically Composable	Yes
Fill/Access Sync.	Fine-grained
Operations	<code>Fill(data)</code> <code>Read(local_offset)</code> <code>Update(local_offset, data)</code> <code>Shrink(num)</code>
In-Place Updates	Yes
Hardware Complexity	RAM, head/tail sync logic, RAW hazard scoreboarding

- Compared to FIFO
 - Allows random access into live window
 - Allows updates of values in live window
- Compared to scratchpad:
 - Adds scoreboard for synchronization
 - Allows arbitrary degrees of buffering
- Compared to cache
 - Addresses local (therefore fewer bits) and no tag store
 - Push model versus pull model for smaller landing zone
 - Raises level of abstraction beyond single value transfers

Fill → (Read → Update?) → Read → Shrink*

Can be specialized, for example: “read-only” buffets that have fills but not updates

Buffet Usage Model



Buffet Behavioral Attributes

- Based on ‘fill’ address sequence, the buffet will pop values from ‘fill’ channel until buffer is full.
- Based on ‘read’ address sequence the buffet will try to **push** values down ‘read’ LI channel, but only if the value has been ‘filled’.
- ‘Read’ address sequence can also inform buffet that a value can be **dropped**, i.e., space freed. This is routed to the **shrink** control port.
- Based on ‘update’ address sequence the buffet will try to pop values from the ‘update’ channel
- Implementation may include multiple logical buffers inside a single physical buffer.

Sliding Window – 4

Tensor: I [W]

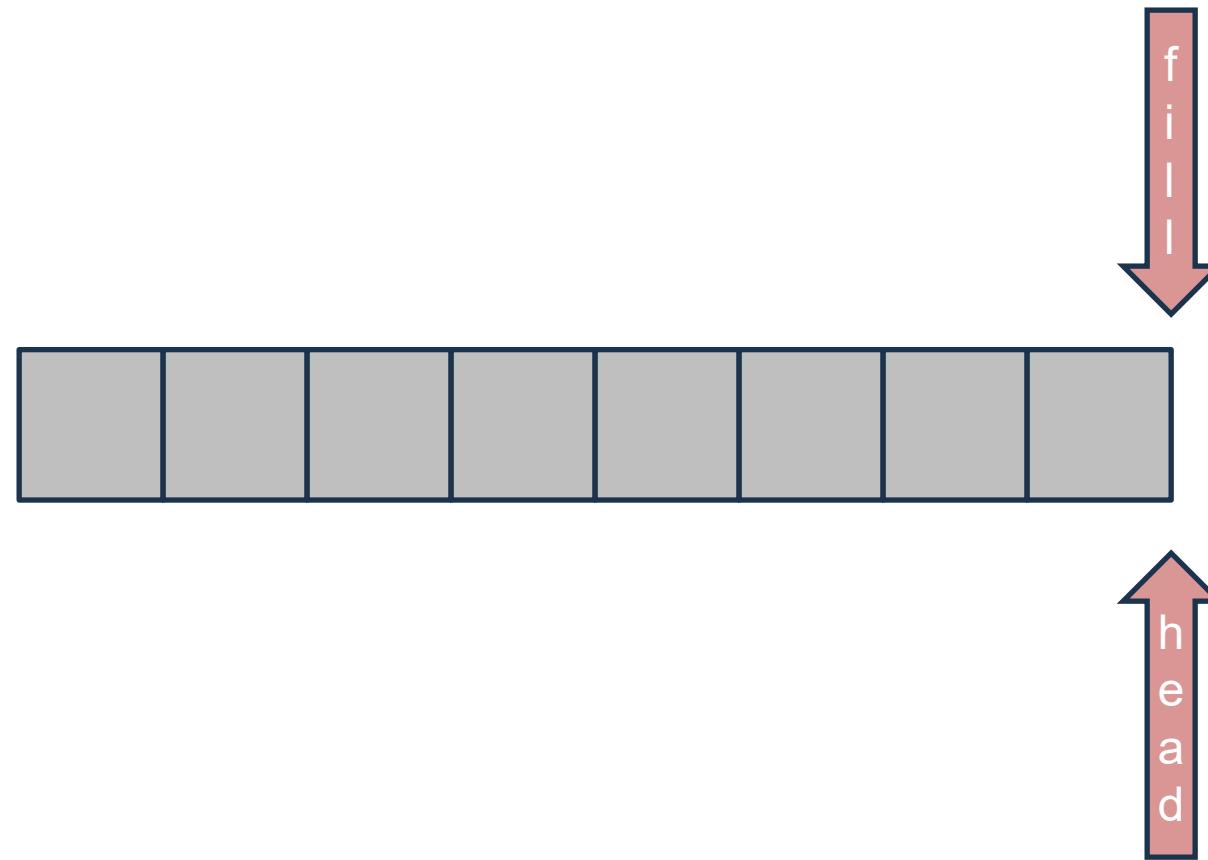
Rank: W

0 1 2 3 4 5 6 7 8 9 10 11

A	B	C	D	E	F	G	H	I	J	K	L
---	---	---	---	---	---	---	---	---	---	---	---

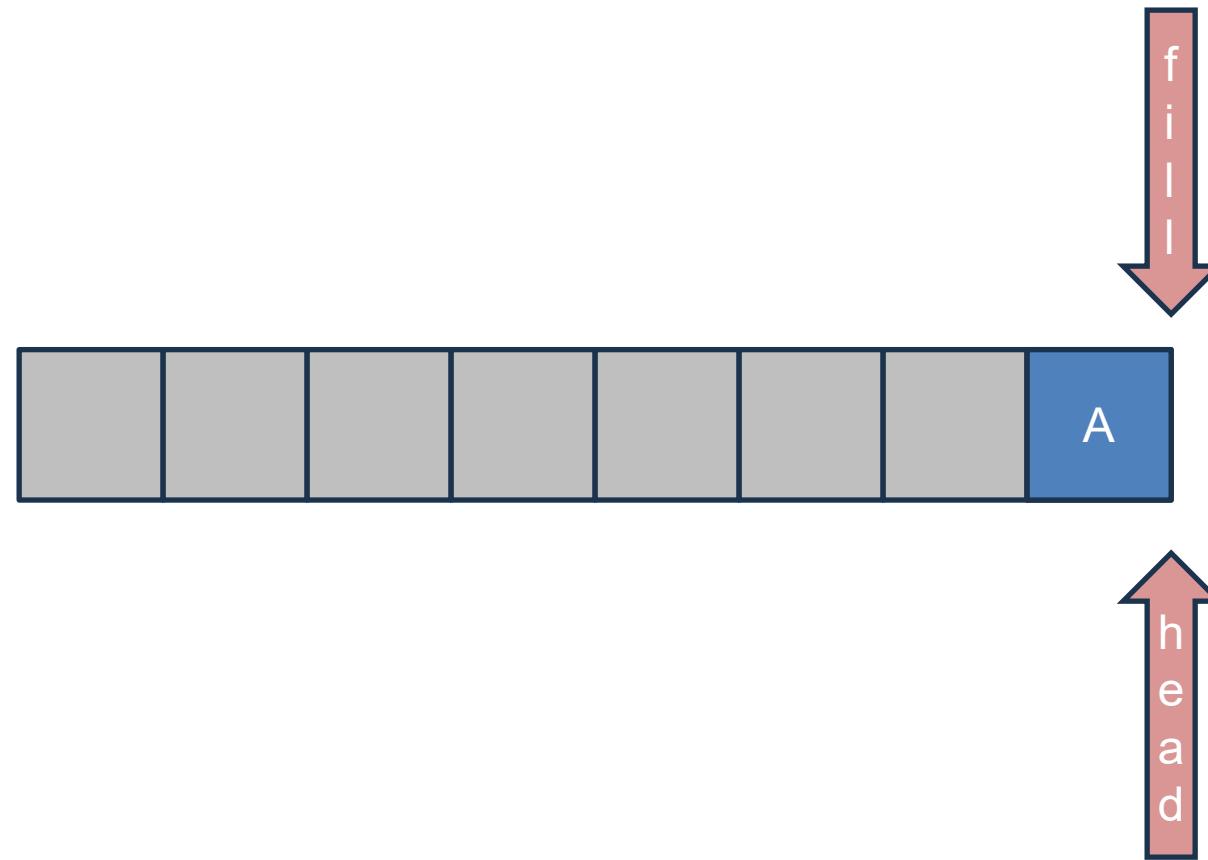
Buffet Control Example

Four-wide sliding window



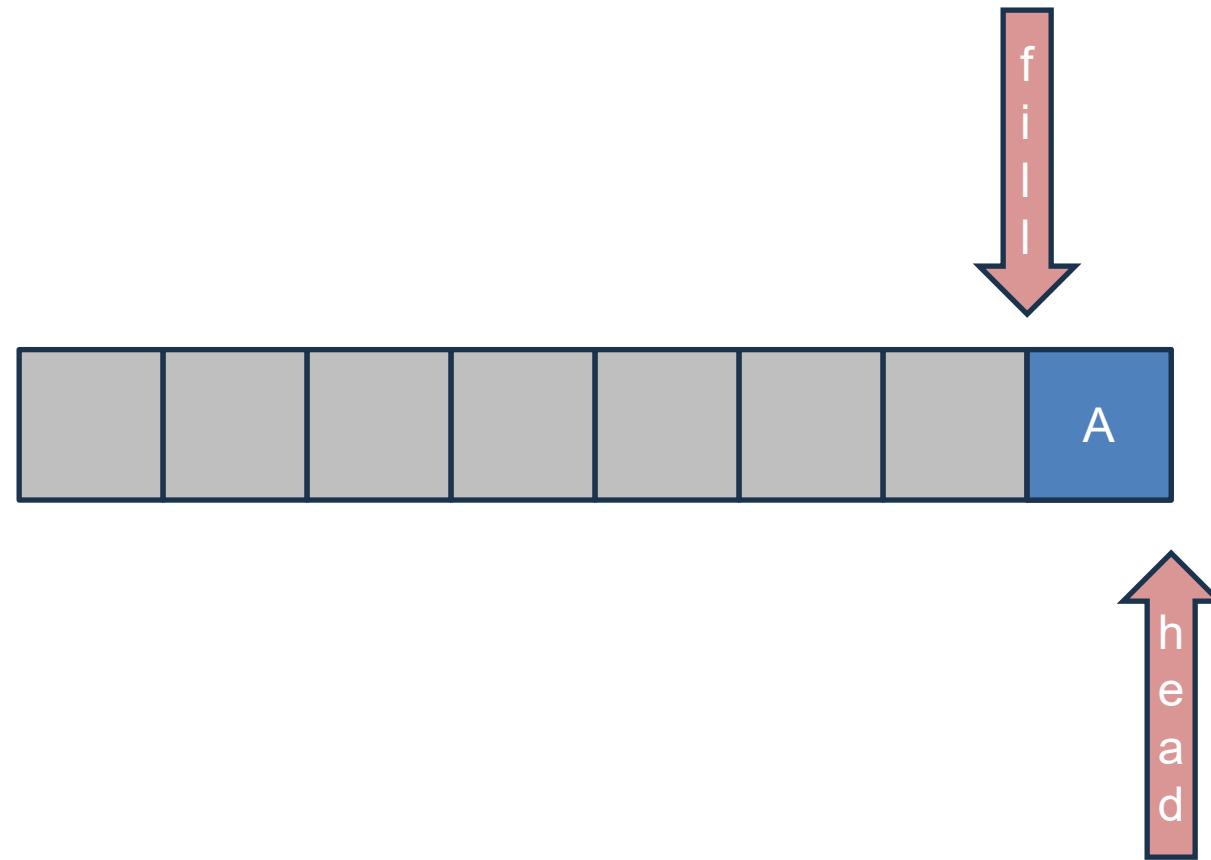
Buffet Control Example

Four-wide sliding window



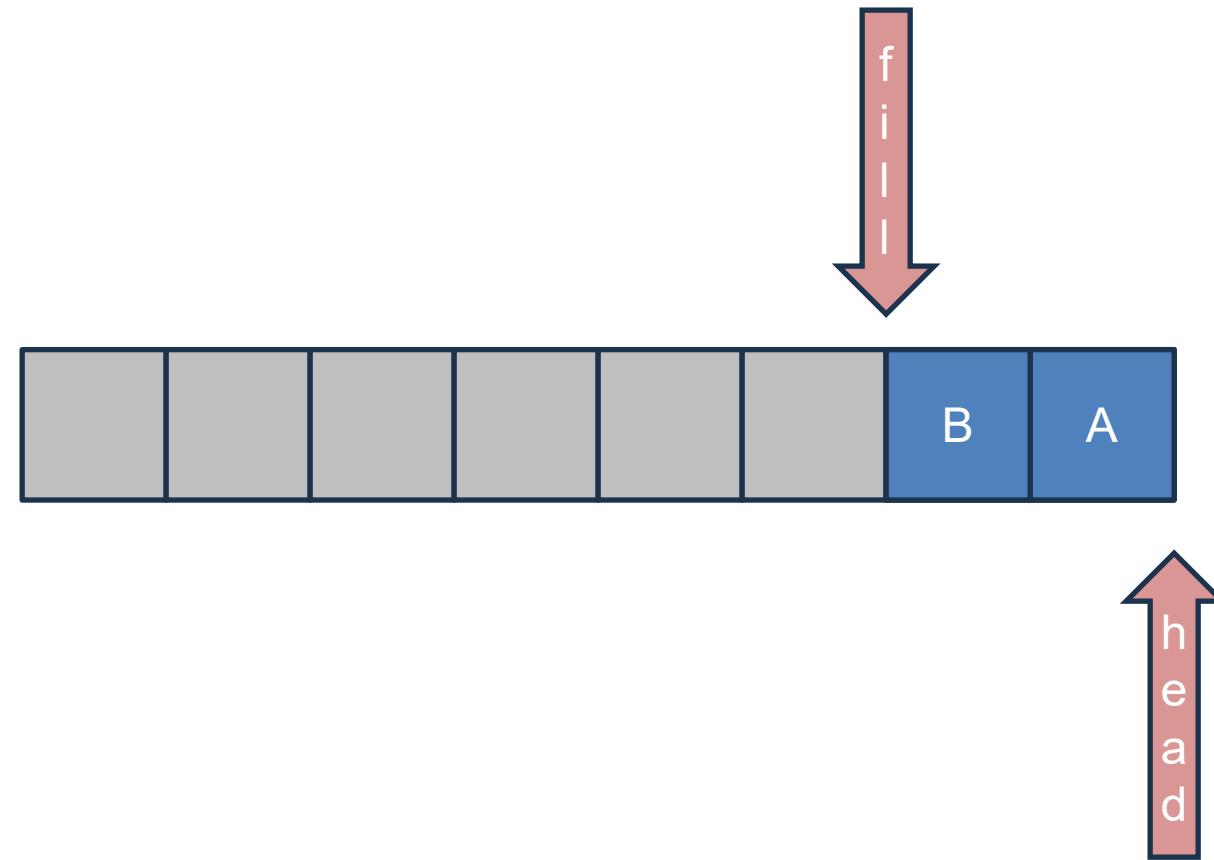
Buffet Control Example

Four-wide sliding window



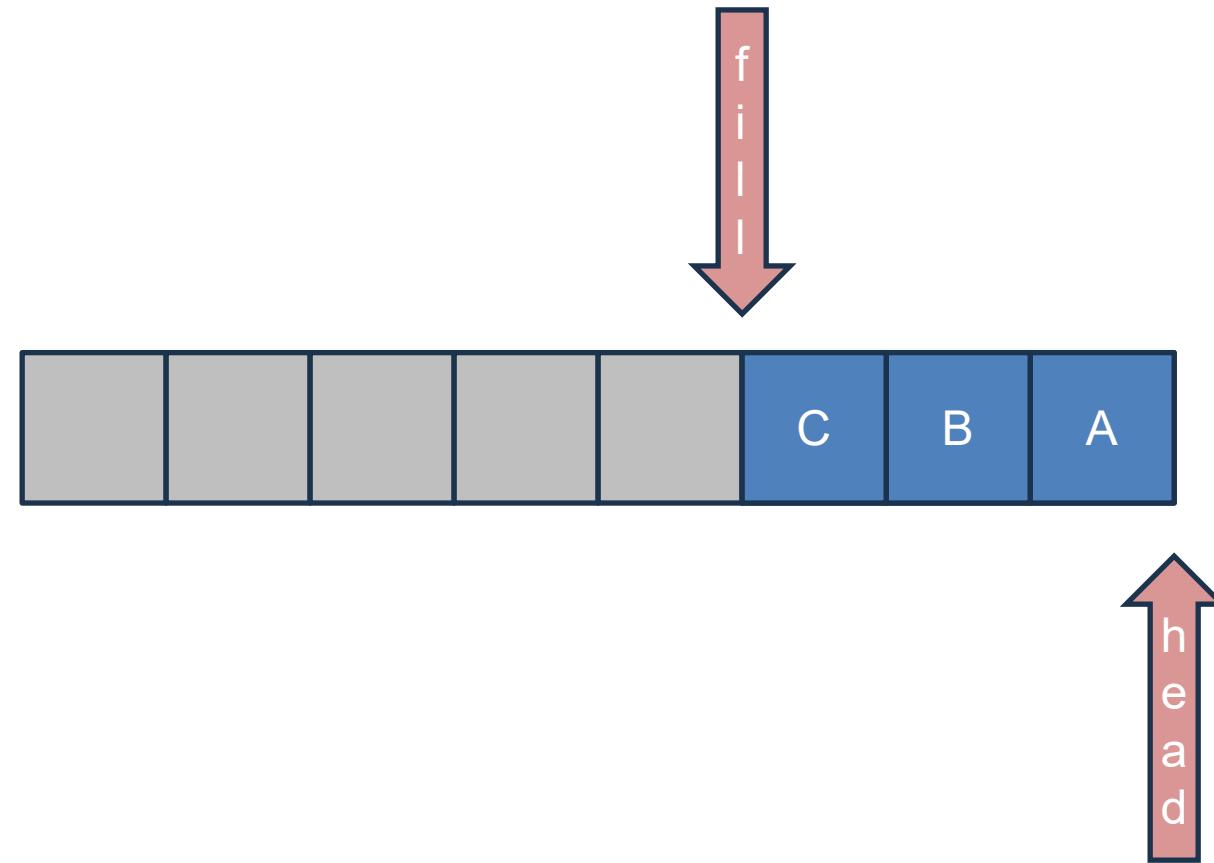
Buffet Control Example

Four-wide sliding window



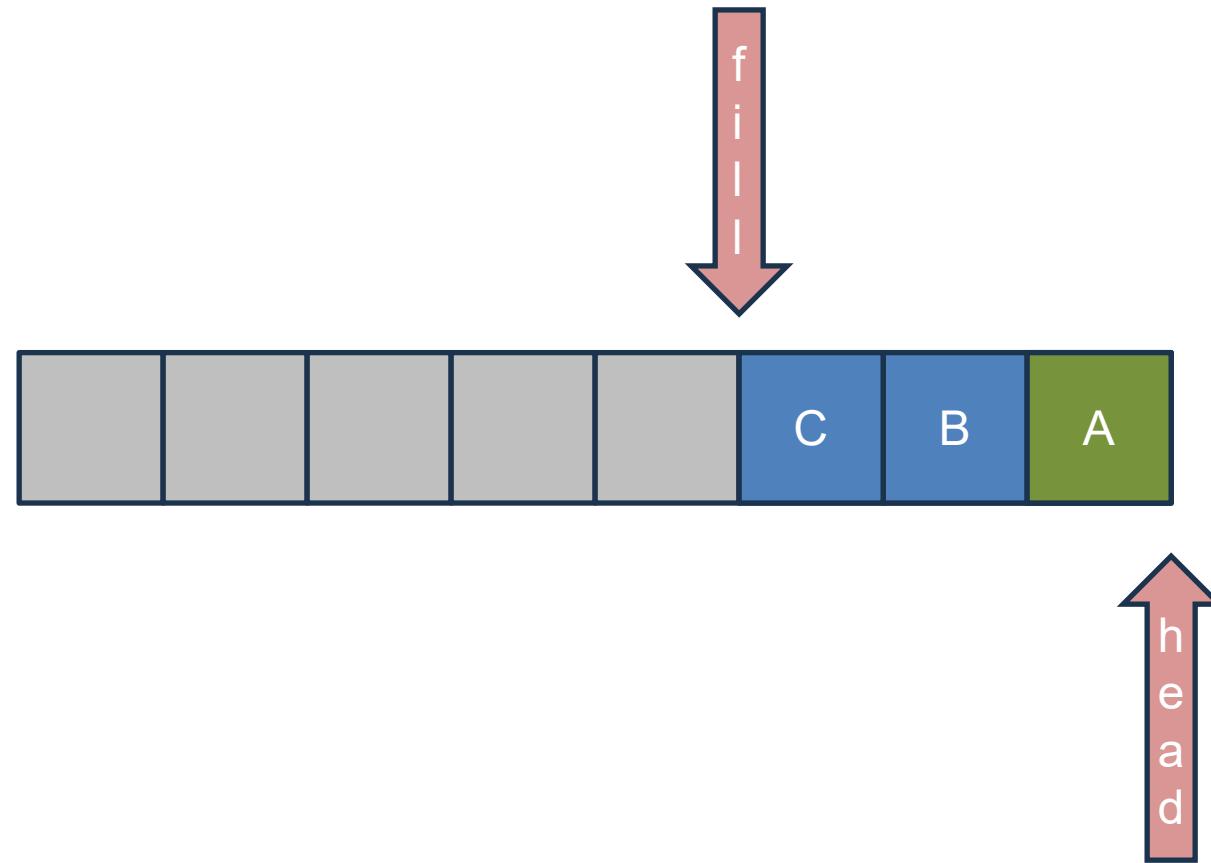
Buffet Control Example

Four-wide sliding window



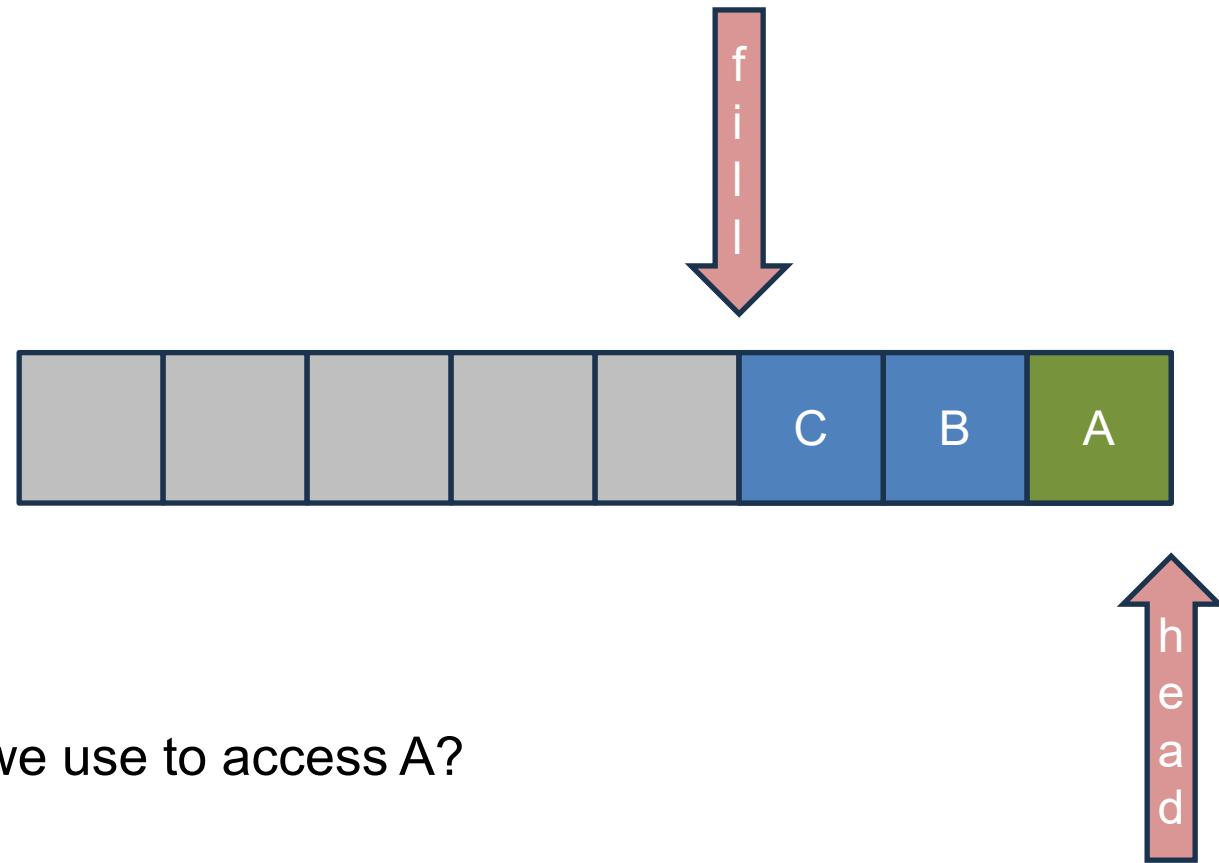
Buffet Control Example

Four-wide sliding window



Buffet Control Example

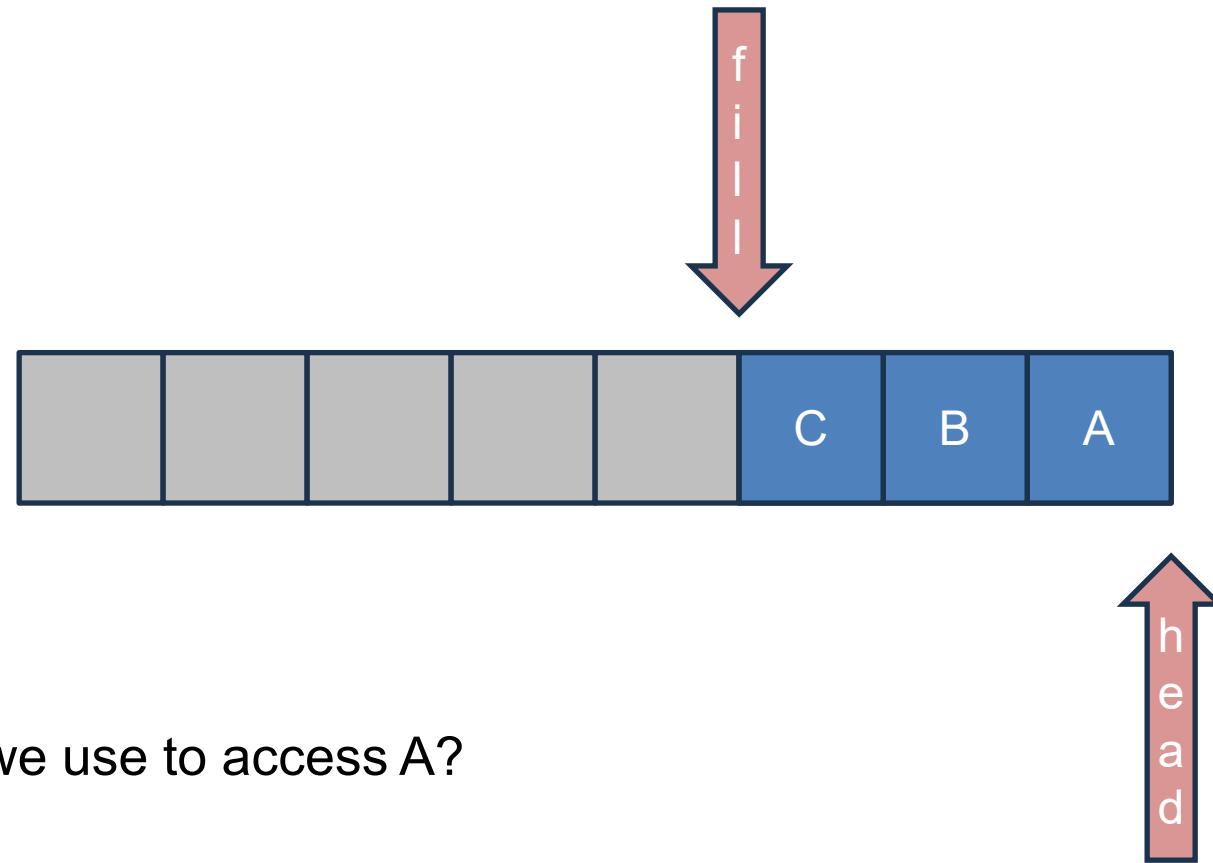
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

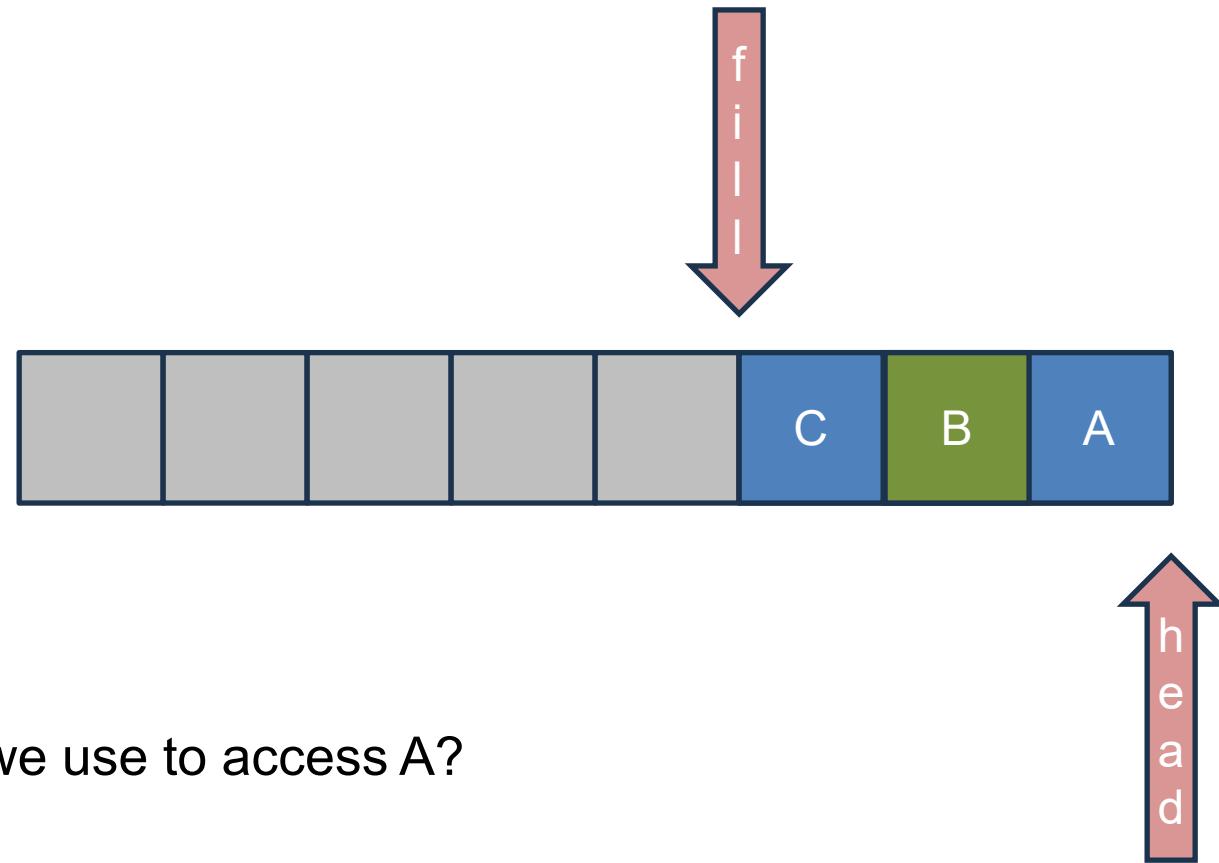
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

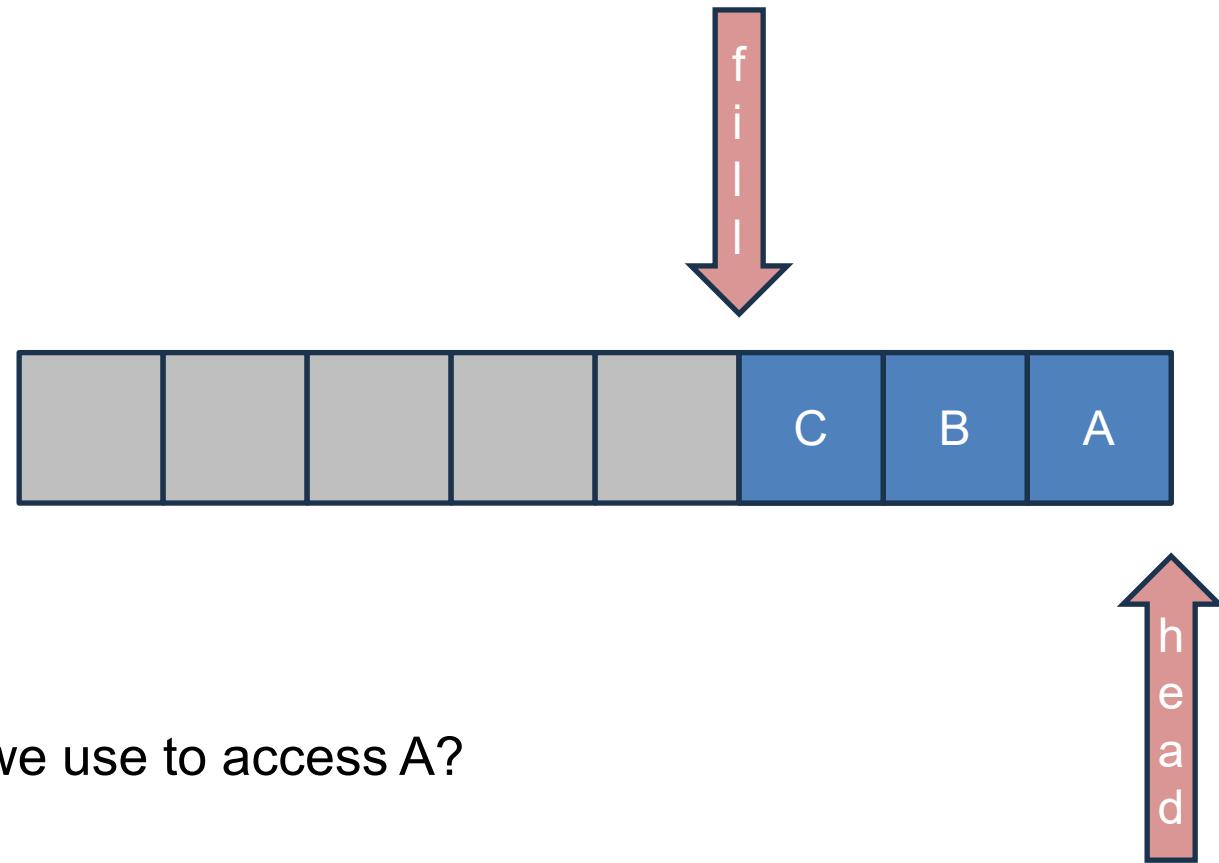
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

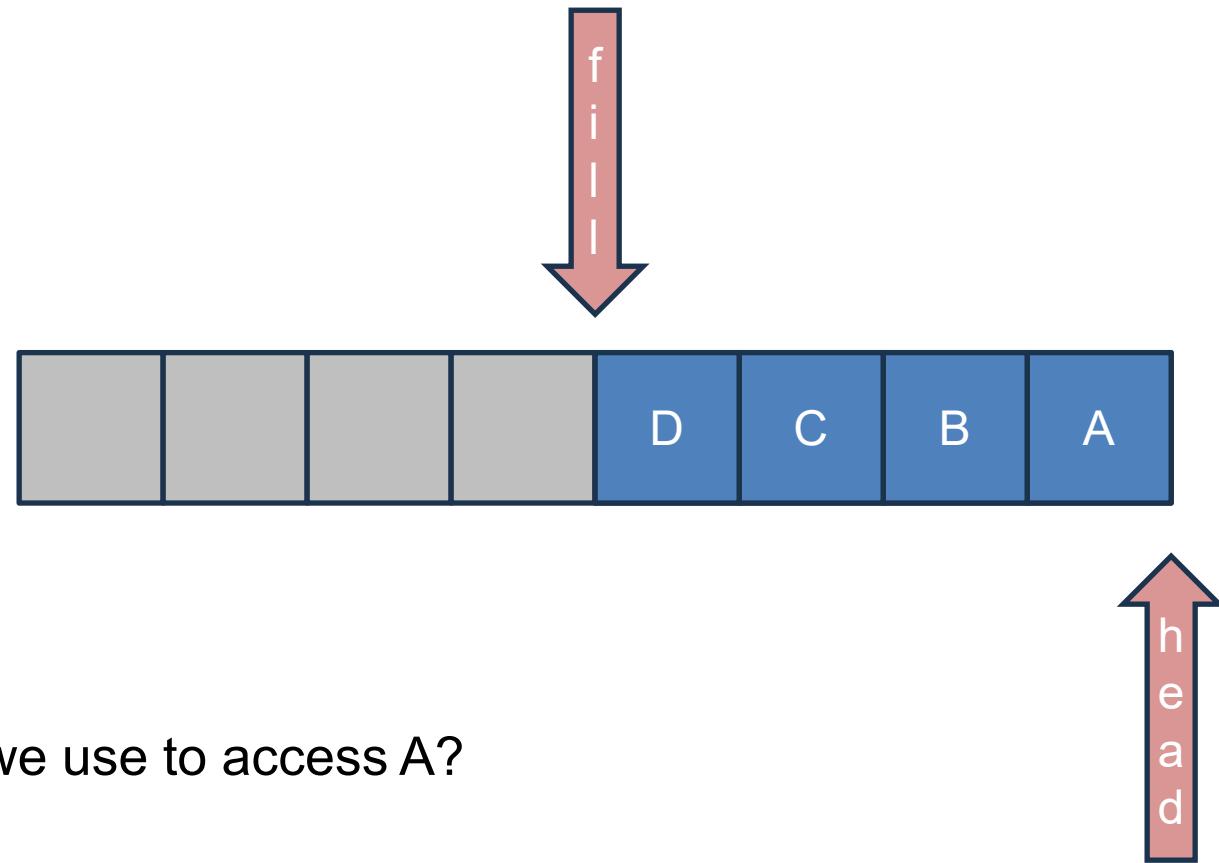
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

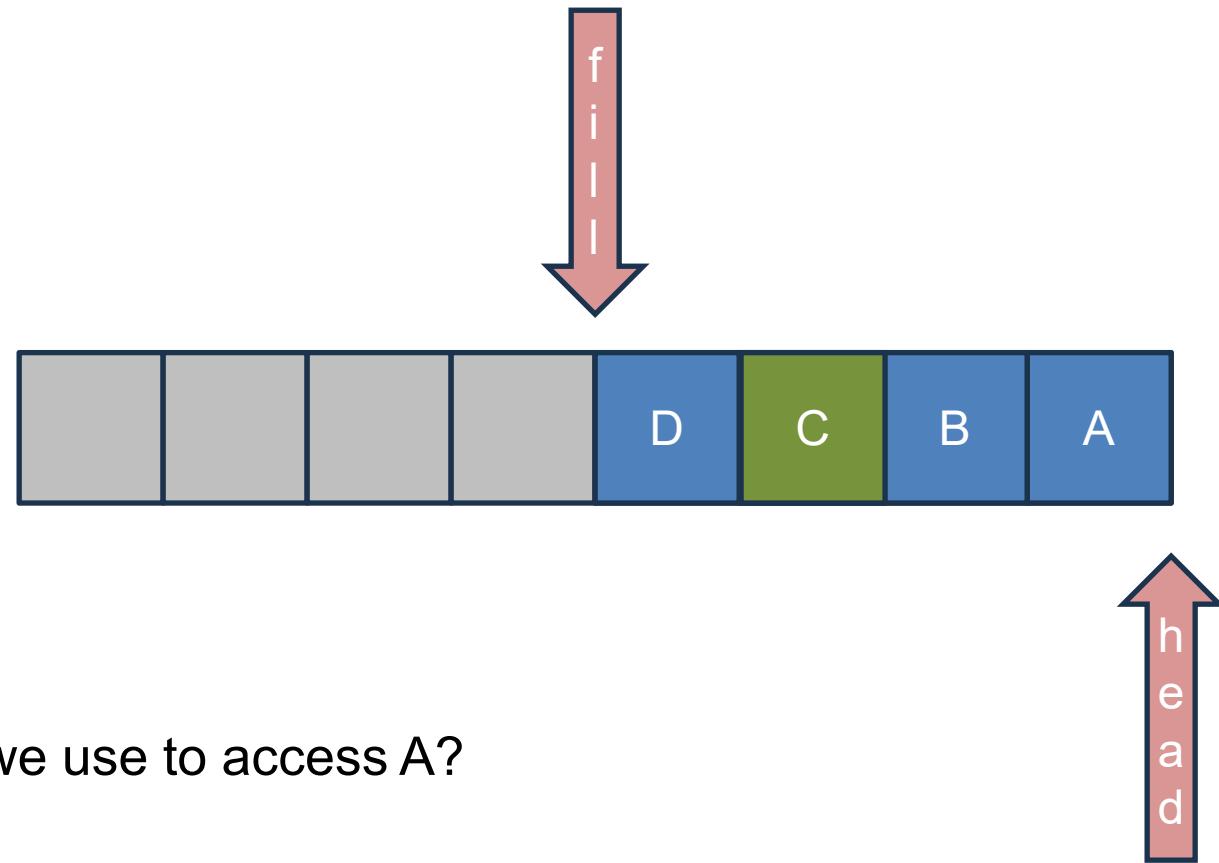
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

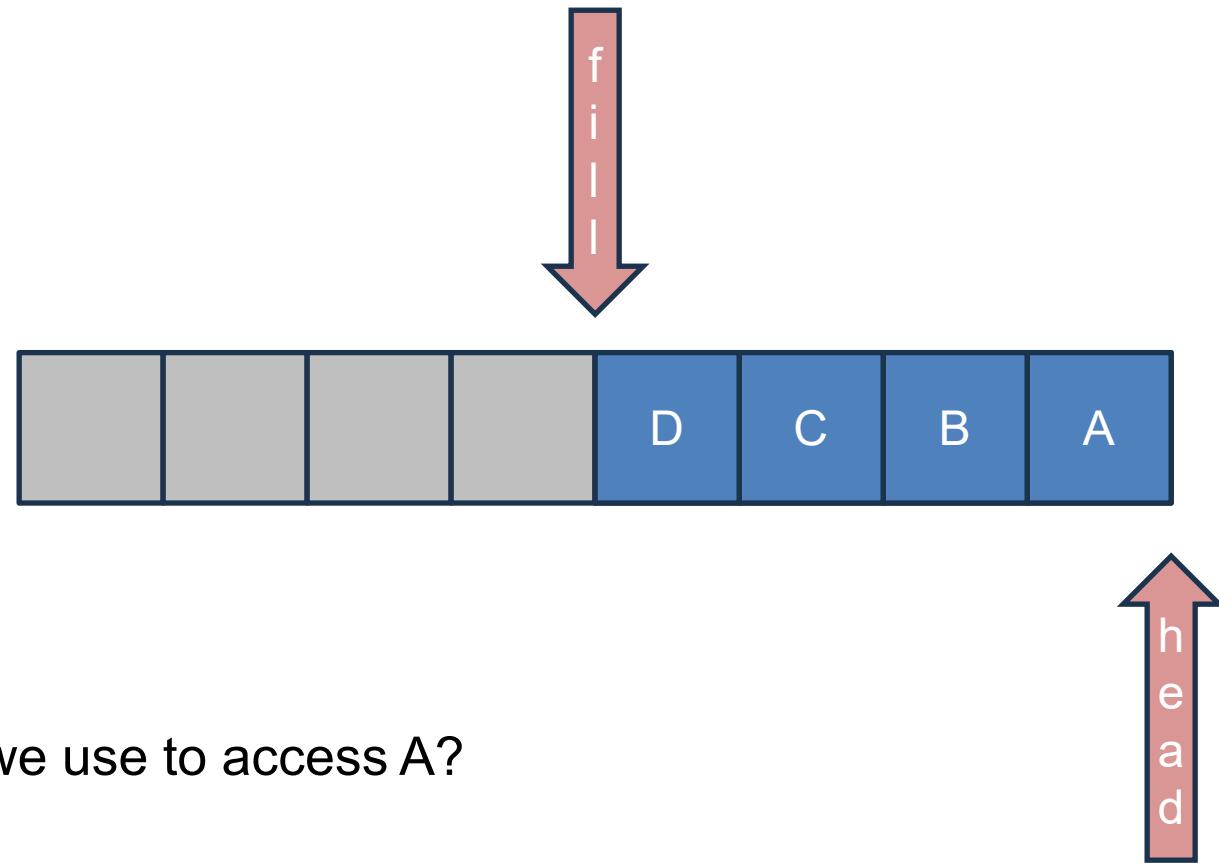
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

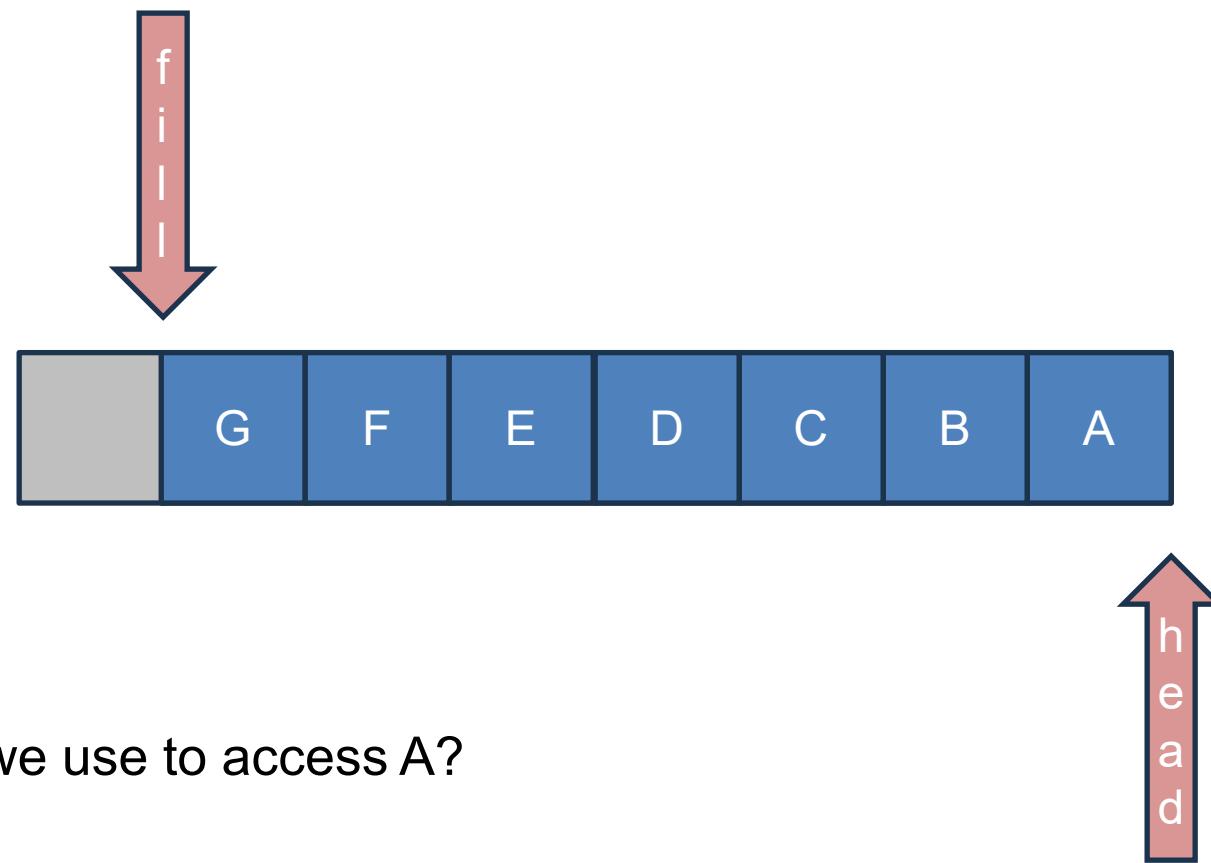
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

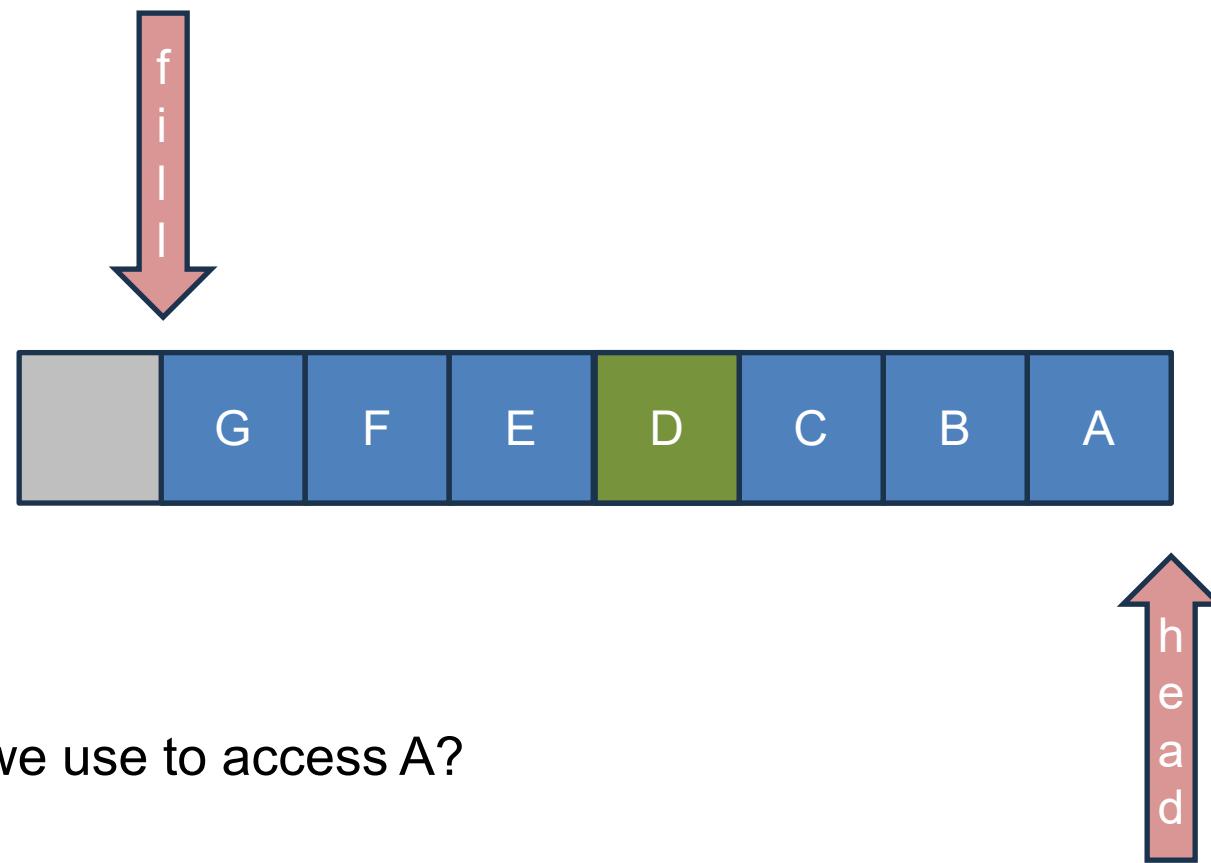
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

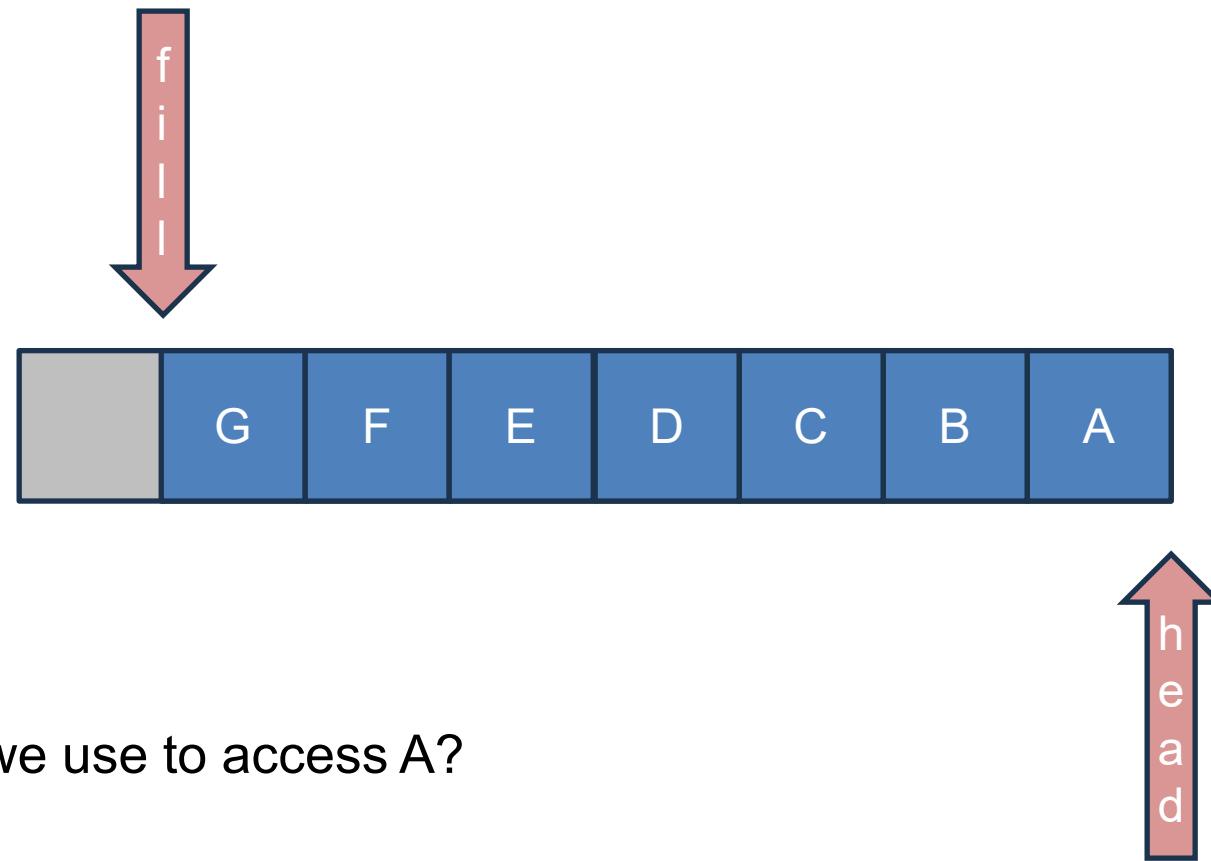
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

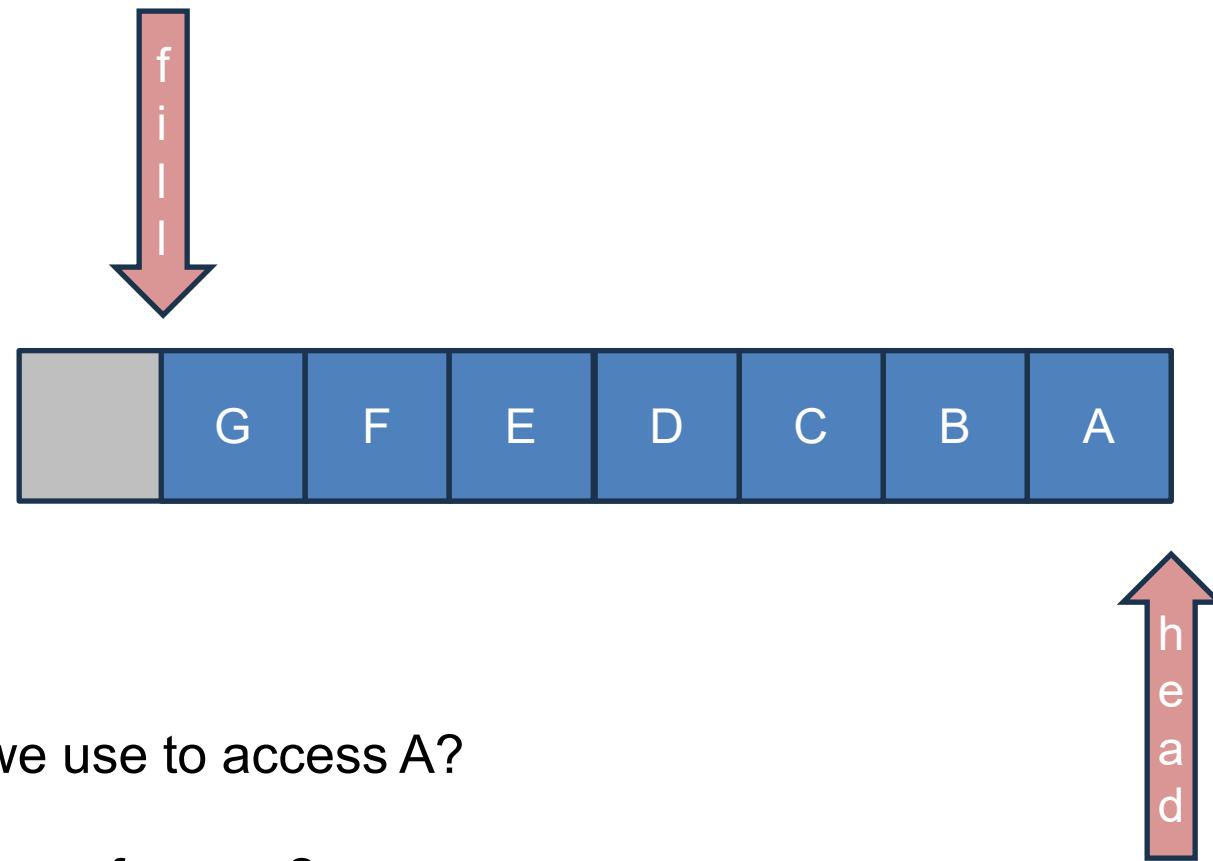
Four-wide sliding window



What offset do we use to access A?

Buffet Control Example

Four-wide sliding window

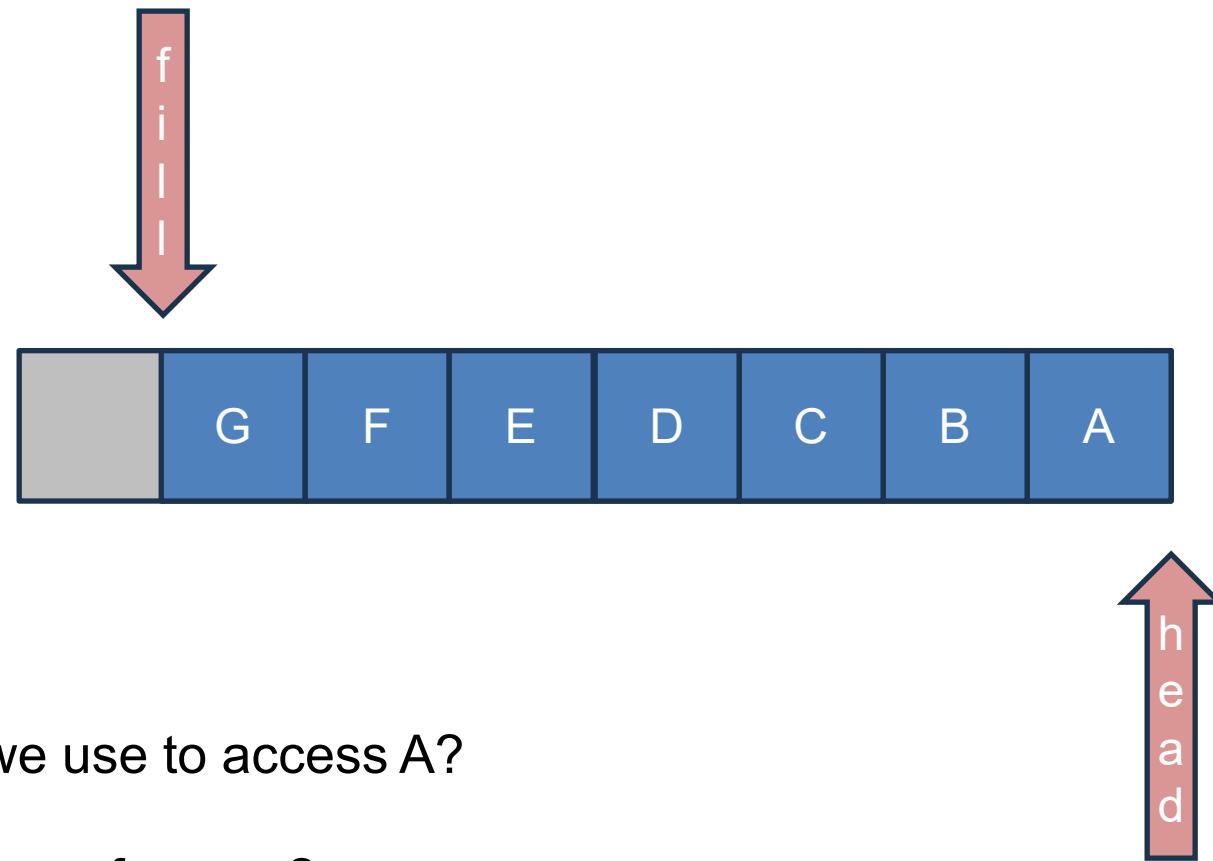


What offset do we use to access A?

Now where do we reference?

Buffet Control Example

Four-wide sliding window



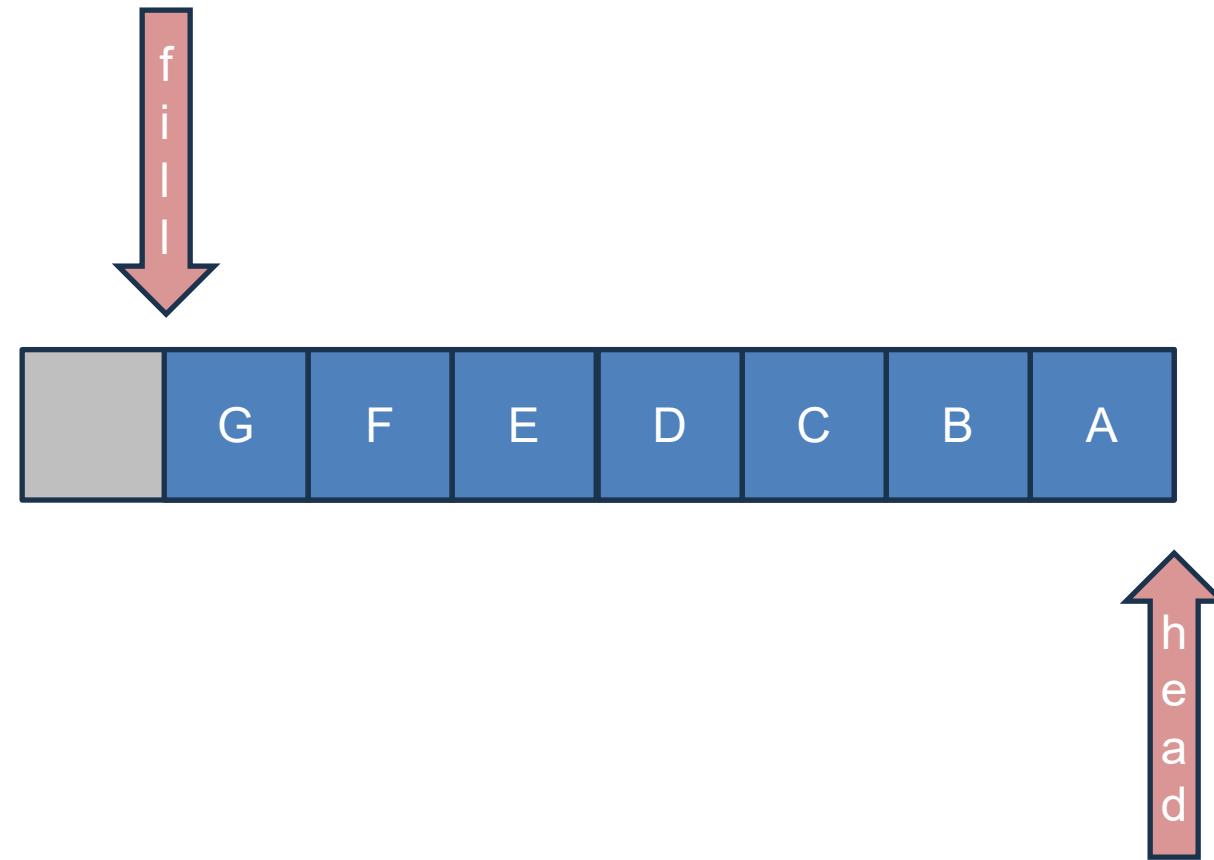
What offset do we use to access A?

Now where do we reference?

But do we need A anymore?

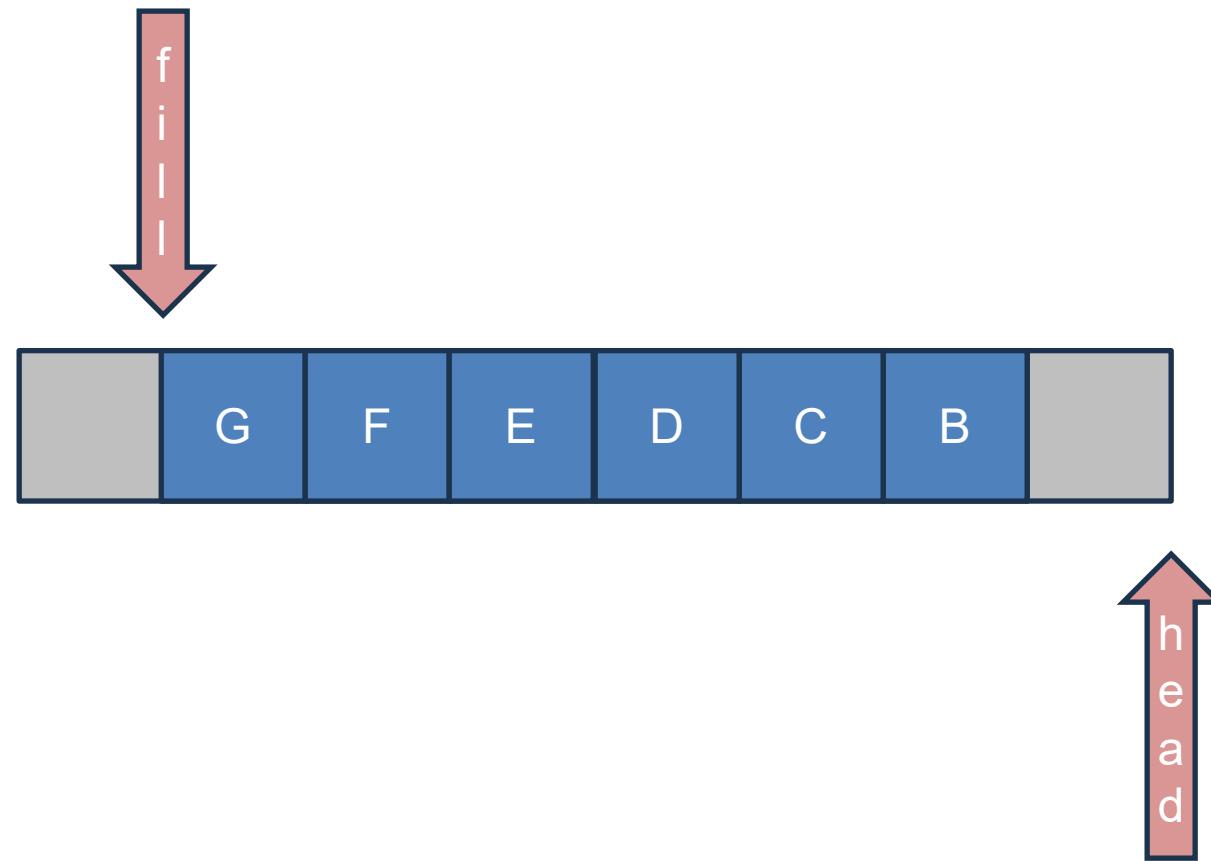
Buffet Control Example

Four-wide sliding window



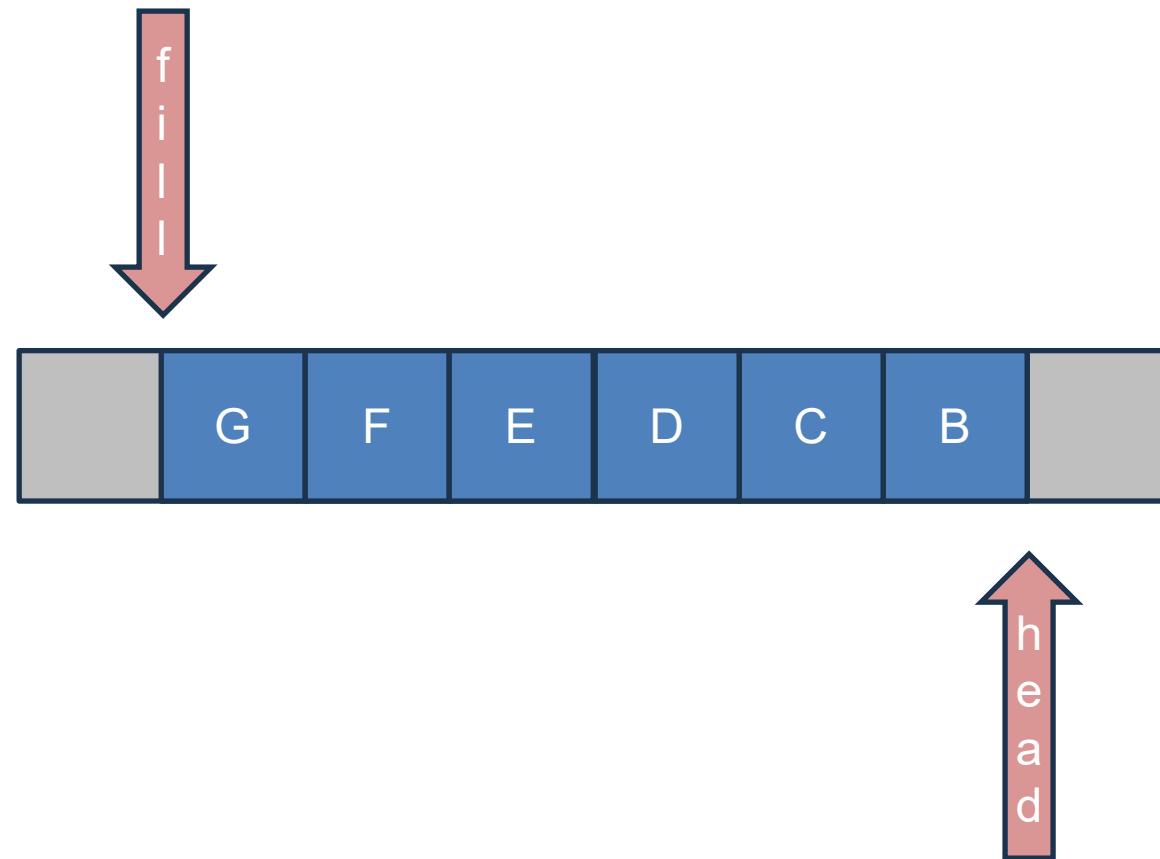
Buffet Control Example

Four-wide sliding window



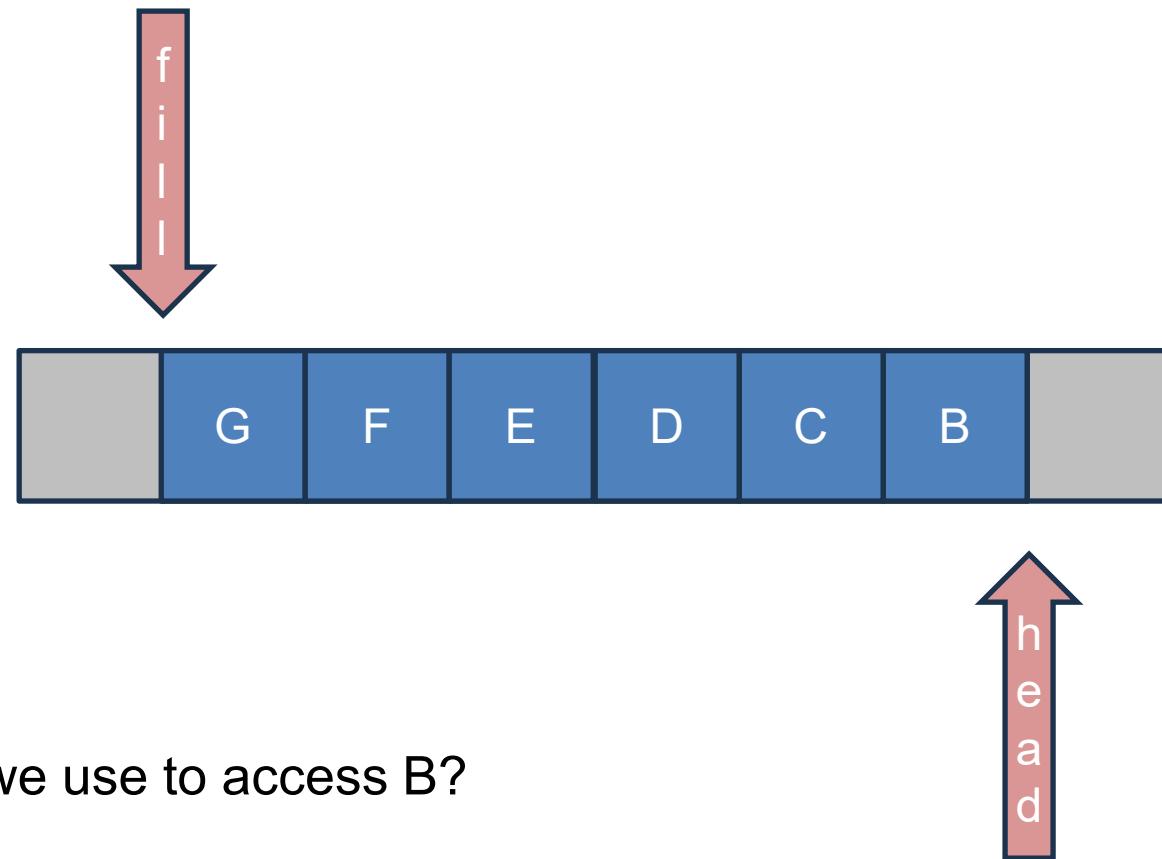
Buffet Control Example

Four-wide sliding window



Buffet Control Example

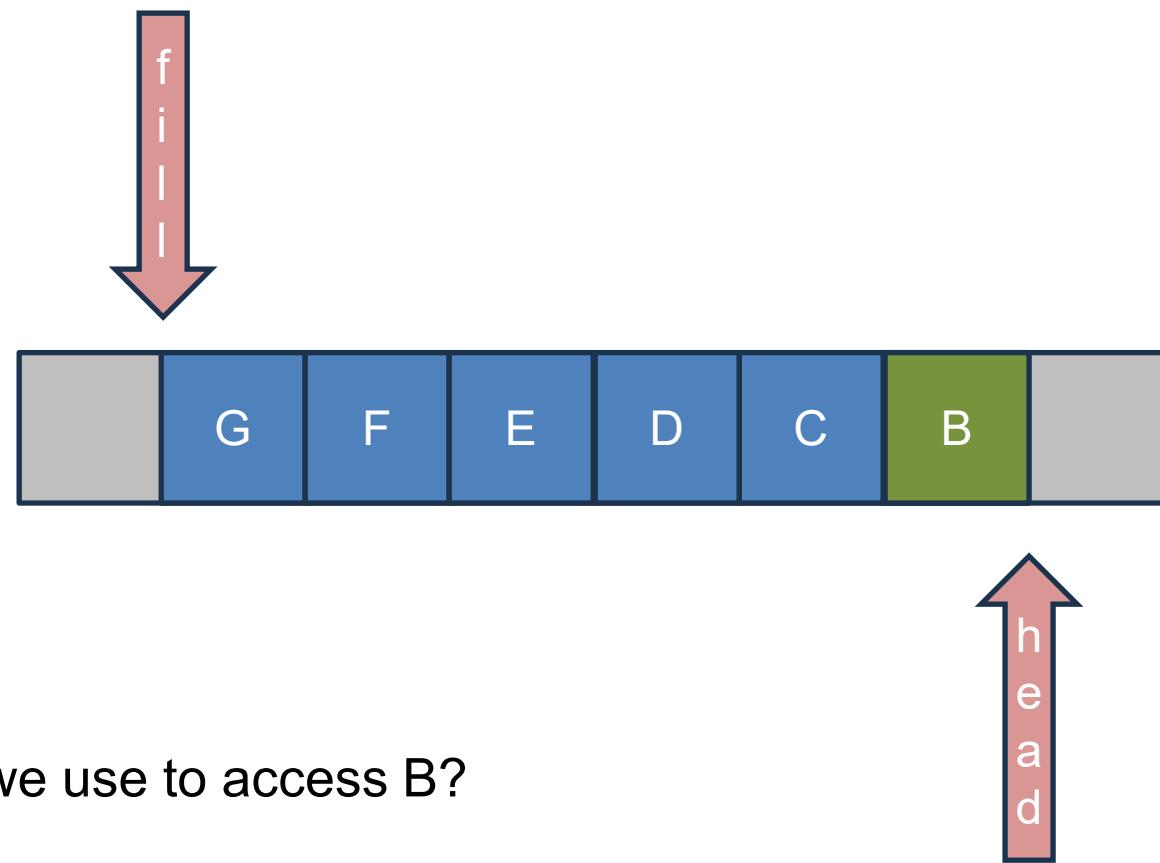
Four-wide sliding window



What offset do we use to access B?

Buffet Control Example

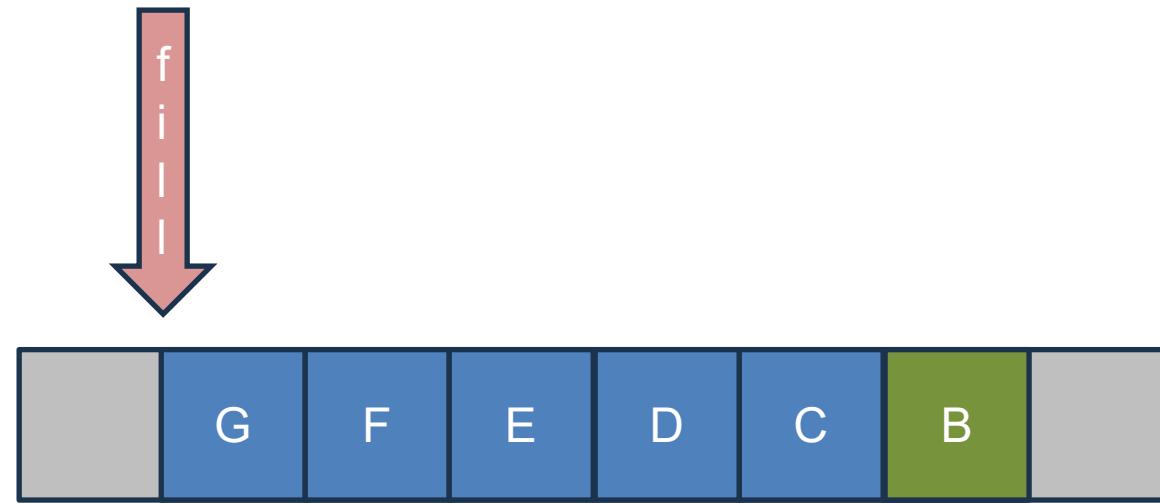
Four-wide sliding window



What offset do we use to access B?

Buffet Control Example

Four-wide sliding window

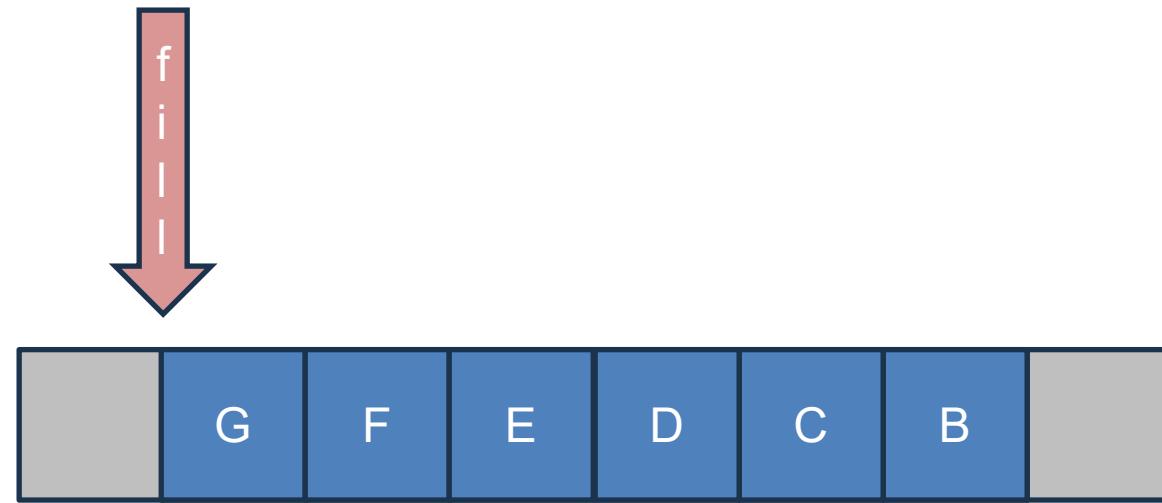


What offset do we use to access B?

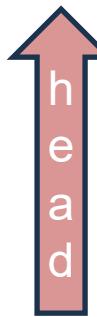
What element is the end of this window?

Buffet Control Example

Four-wide sliding window



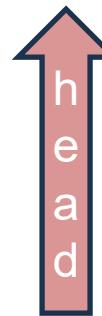
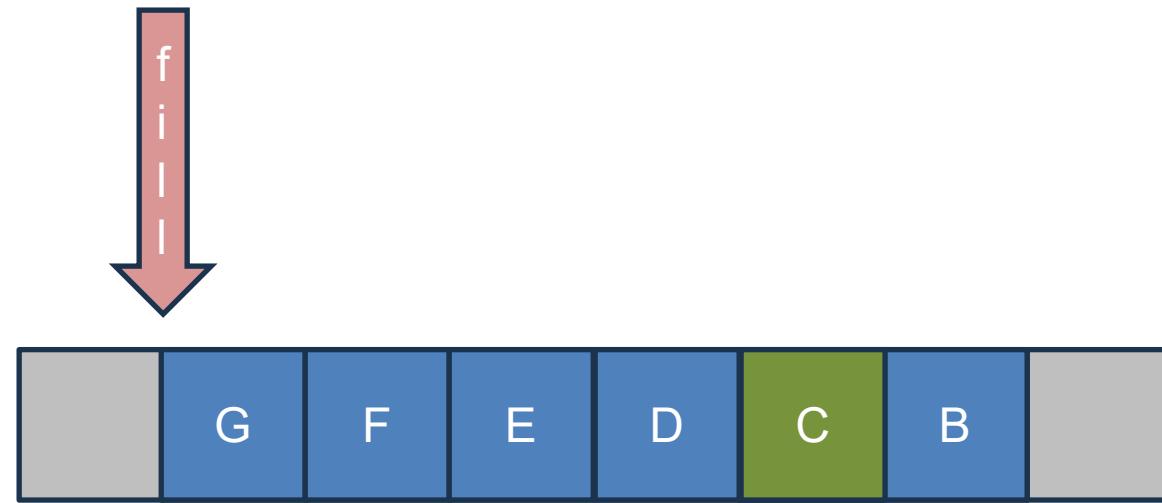
What offset do we use to access B?



What element is the end of this window?

Buffet Control Example

Four-wide sliding window

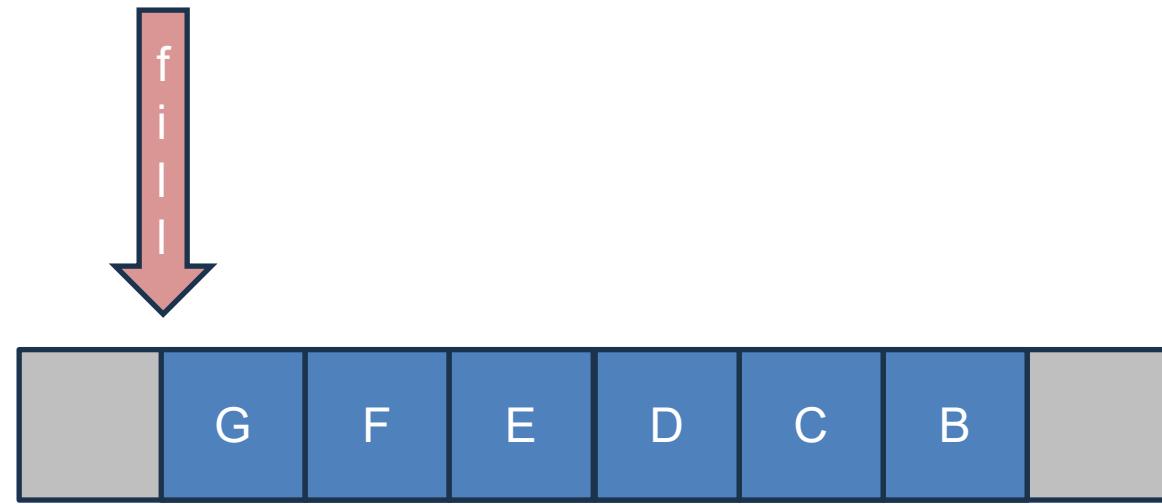


What offset do we use to access B?

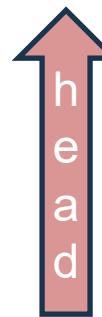
What element is the end of this window?

Buffet Control Example

Four-wide sliding window



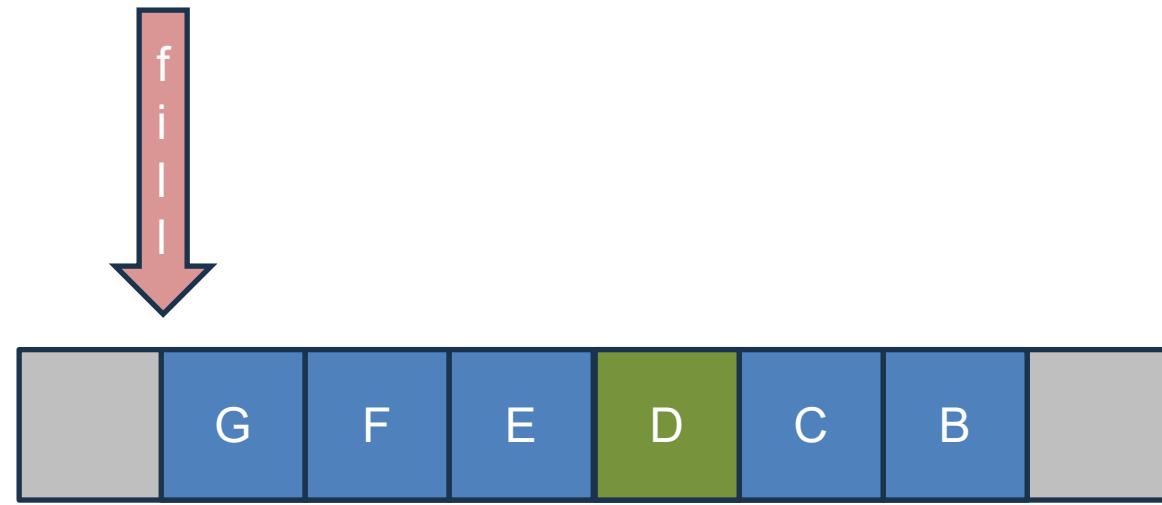
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Buffet Control Example

Four-wide sliding window

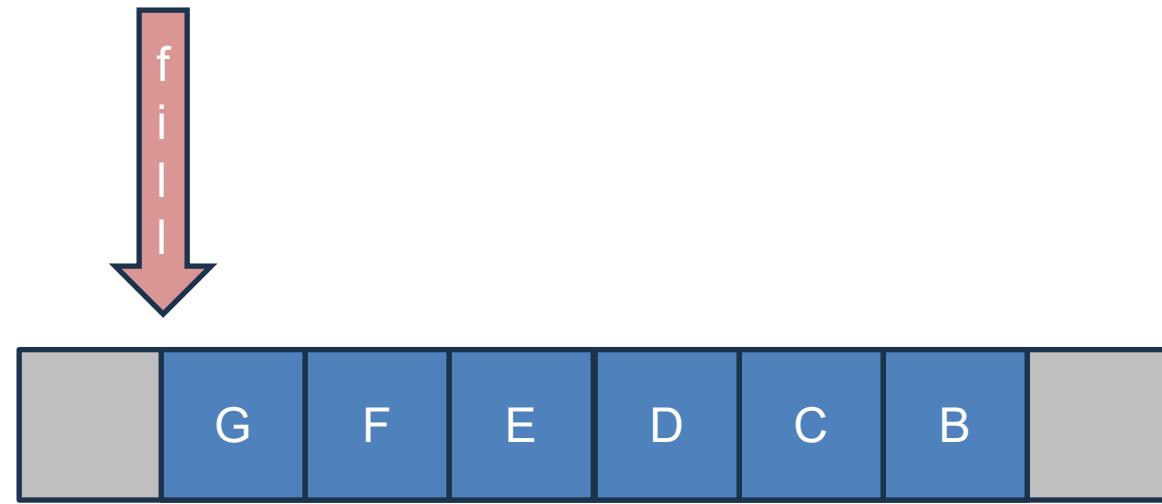


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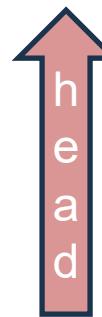
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Buffet Control Example

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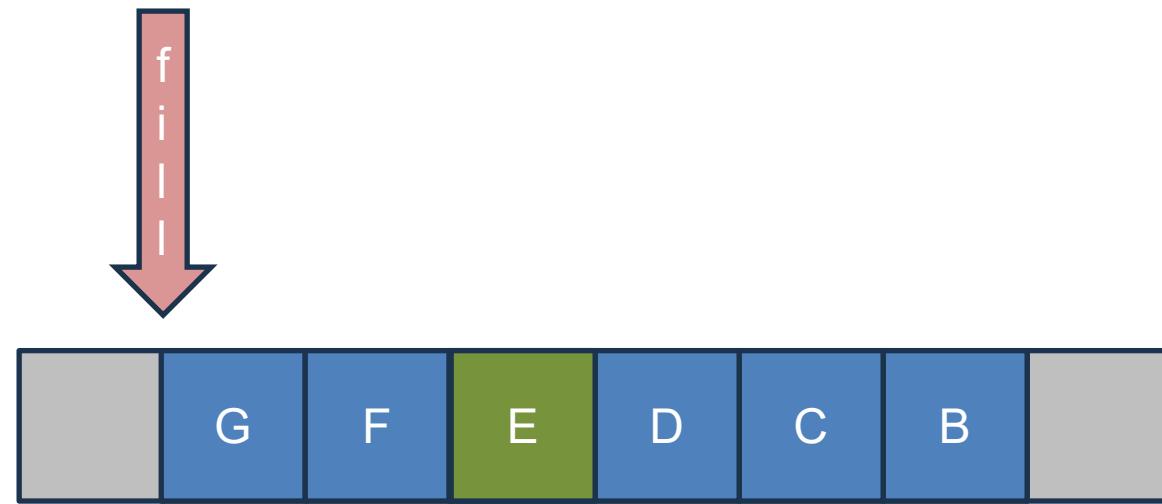
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Buffet Control Example

Four-wide sliding window

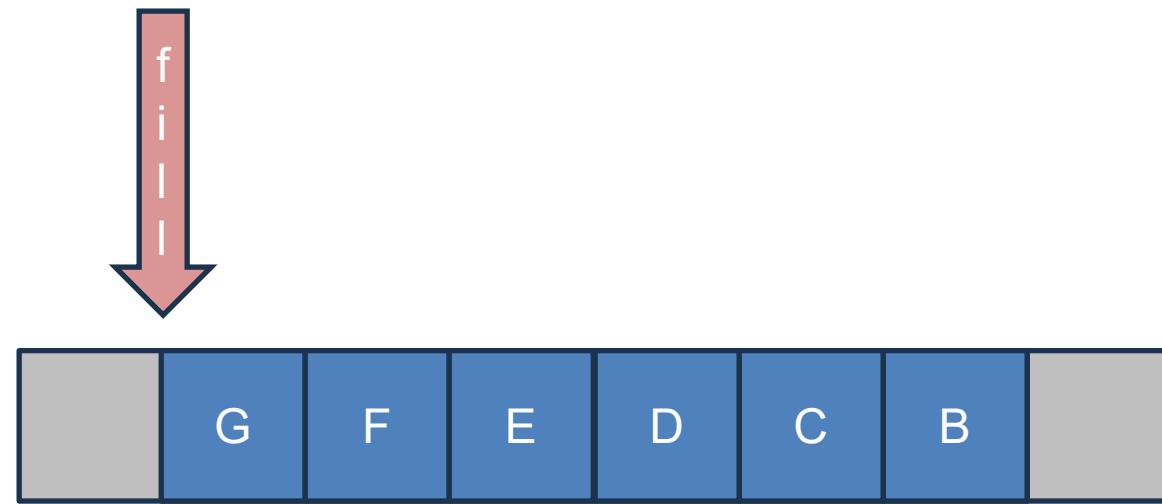


What offset do we use to access B?

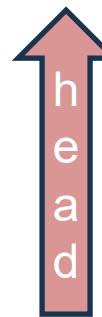
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Buffet Control Example

Four-wide sliding window



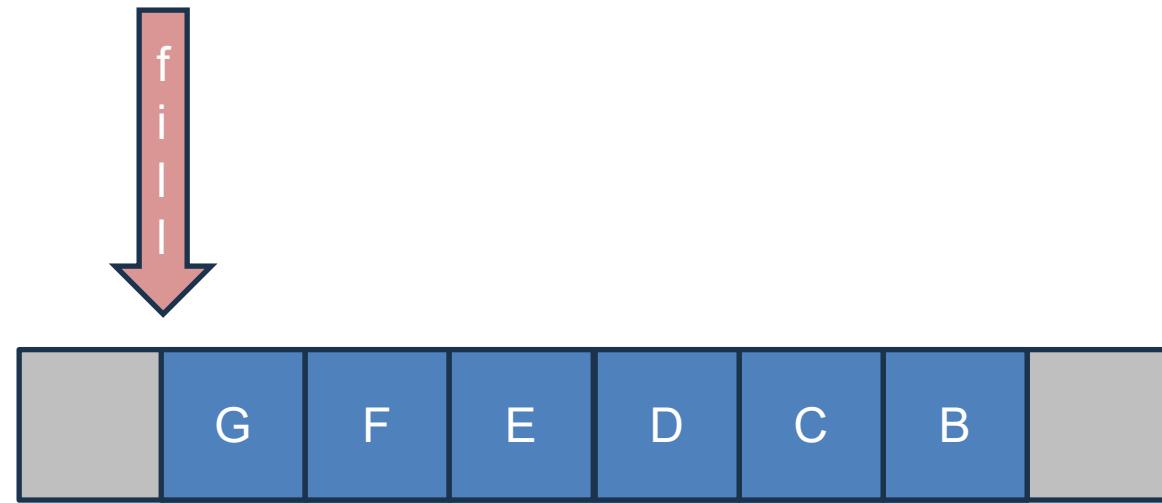
What offset do we use to access B?



What element is the end of this window?

Buffet Control Example

Four-wide sliding window

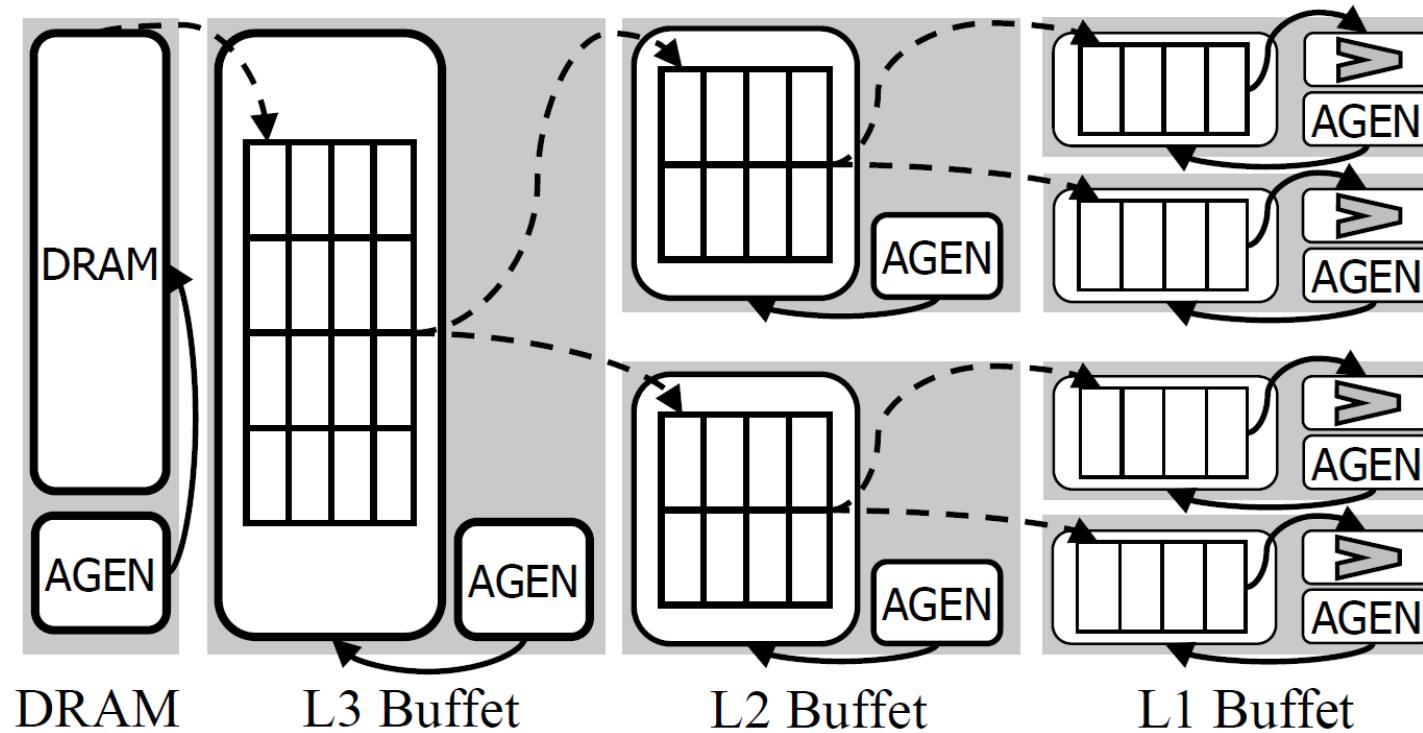


What offset do we use to access B?

What element is the end of this window?

What do we do now?

Buffets: Composable Idiom for E.D.D.O.



Transfers between levels only depend on a credit flow from the adjacent level.

This Lecture

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- Continue understanding representation of a convolution using loop nests, including mapping

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- Continue understanding representation of a convolution using loop nests, including mapping
- See how costs of a mapping can be determined from the loop nest representation.
- See how loop nest can guide configuration of an accelerator.
- Consider a loop nest representation for a full CNN layer and how to search for an optimal mapping
- Reading: Efficient Processing of Deep Neural Networks – Chap 5/6

Mapping Output Stationary to Hardware

1-D Convolution – Output Stationary

$$\begin{array}{ccc} \text{Weights} & & \text{Inputs} \\ \boxed{\textcolor{green}{S}} & * & \boxed{\textcolor{blue}{W}} \\ S & & W \end{array} = \boxed{\textcolor{red}{Q}} = W \cdot \text{ceil}(S/2)^\dagger$$

1-D Convolution – Output Stationary



$$O_q = I_{q+s} \times F_s$$

1-D Convolution – Output Stationary



$$O_q = I_{q+s} \times F_s$$

Traversal order (fastest to slowest): S, Q

1-D Convolution – Output Stationary



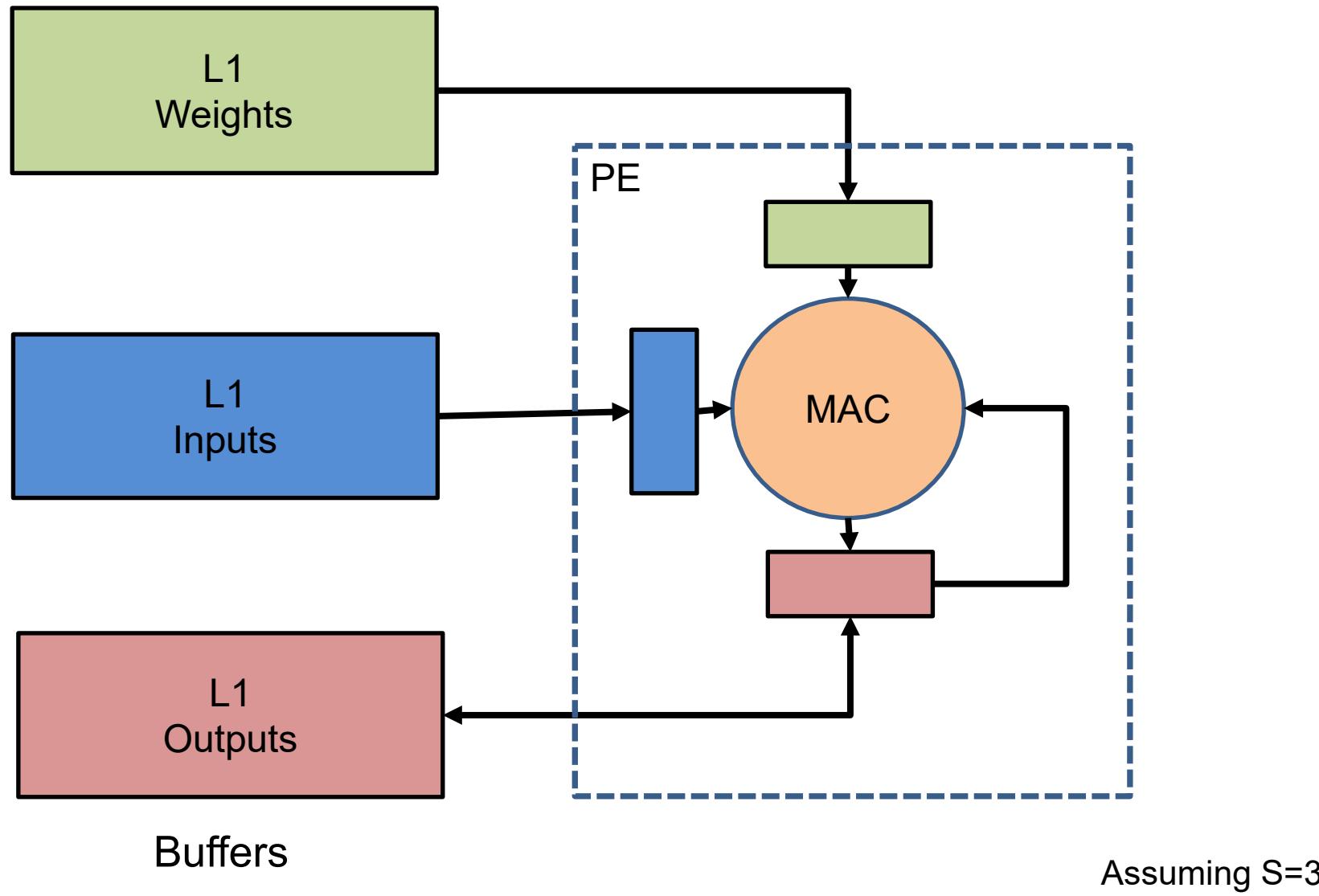
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Traversal order (fastest to slowest): S, Q

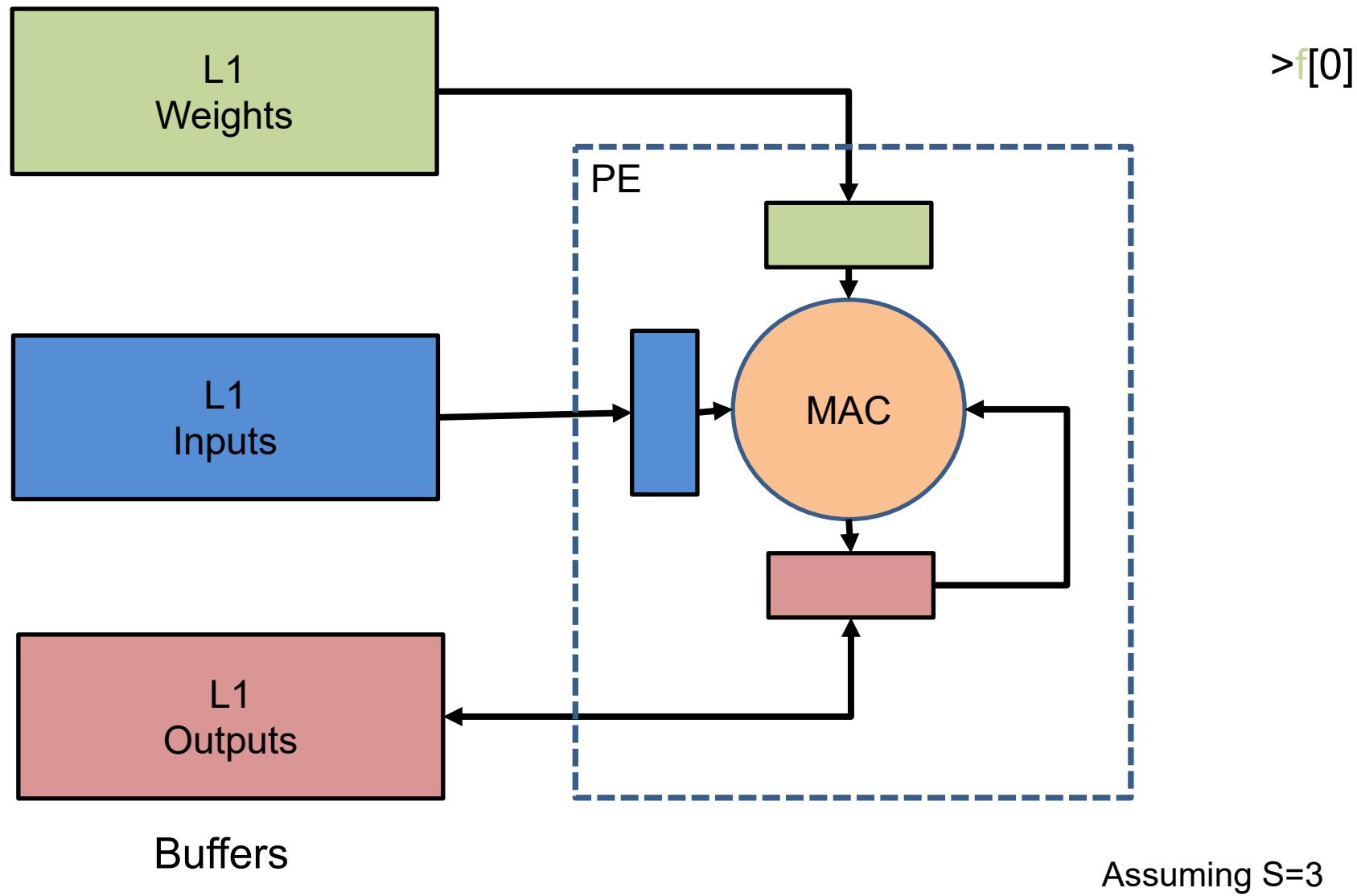
```
int i[W];      # Input activations
int f[S];      # Filter weights
int o[Q];      # Output activations

for q in [0, Q):
    for s in (0, S):
        o[q] += i[q+s]*f[s]
```

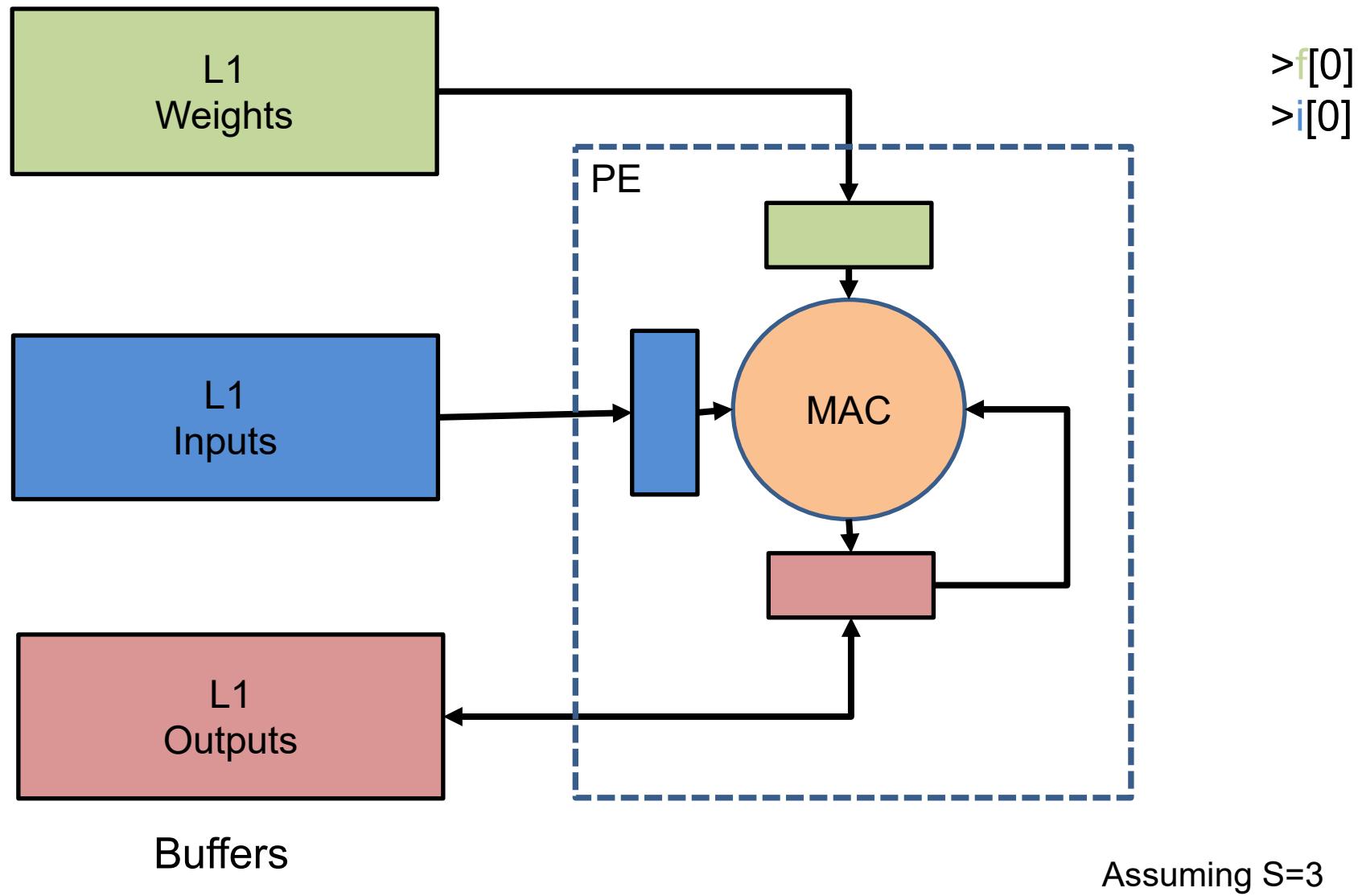
Single PE Output Stationary Flow



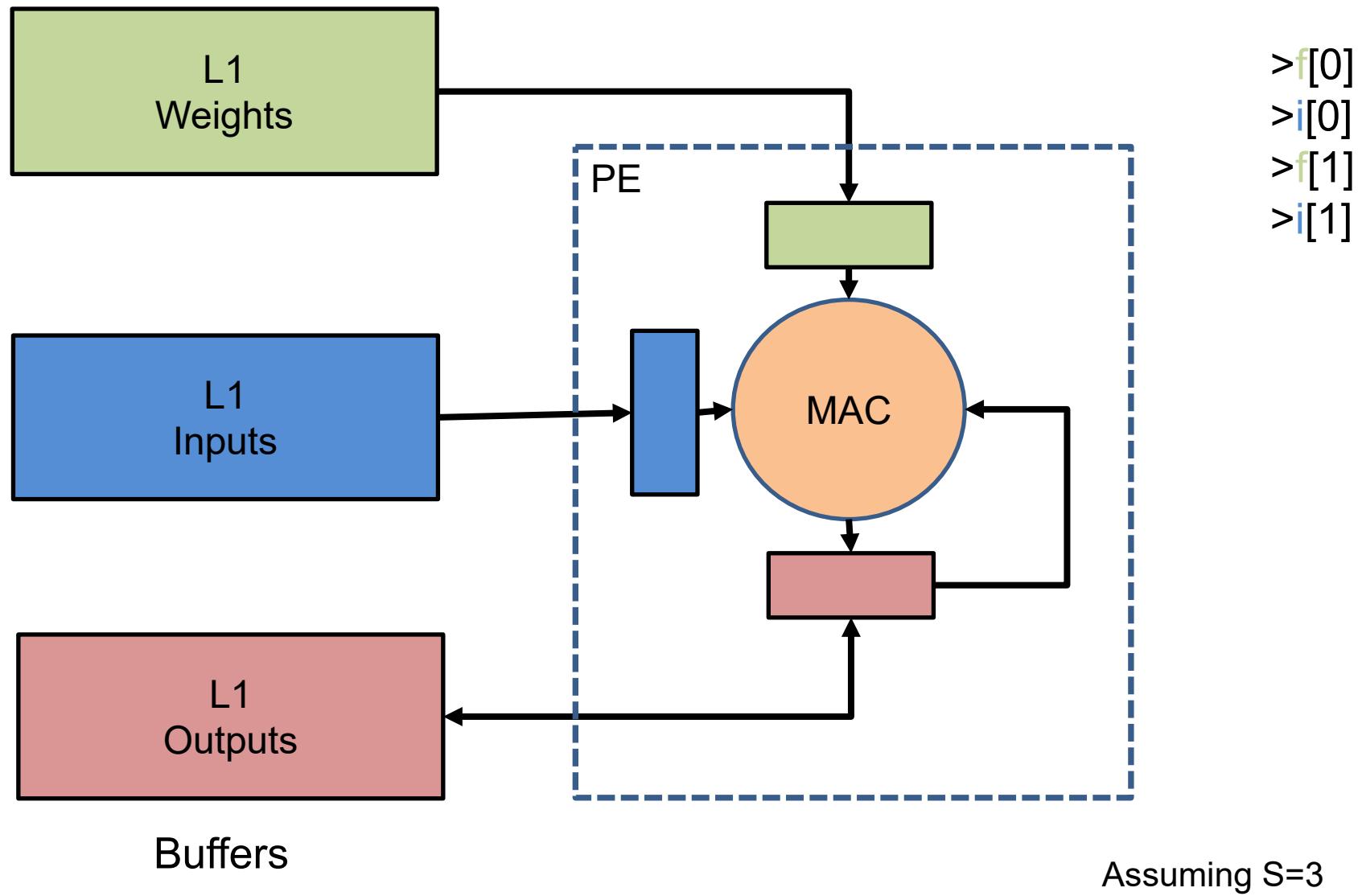
Single PE Output Stationary Flow



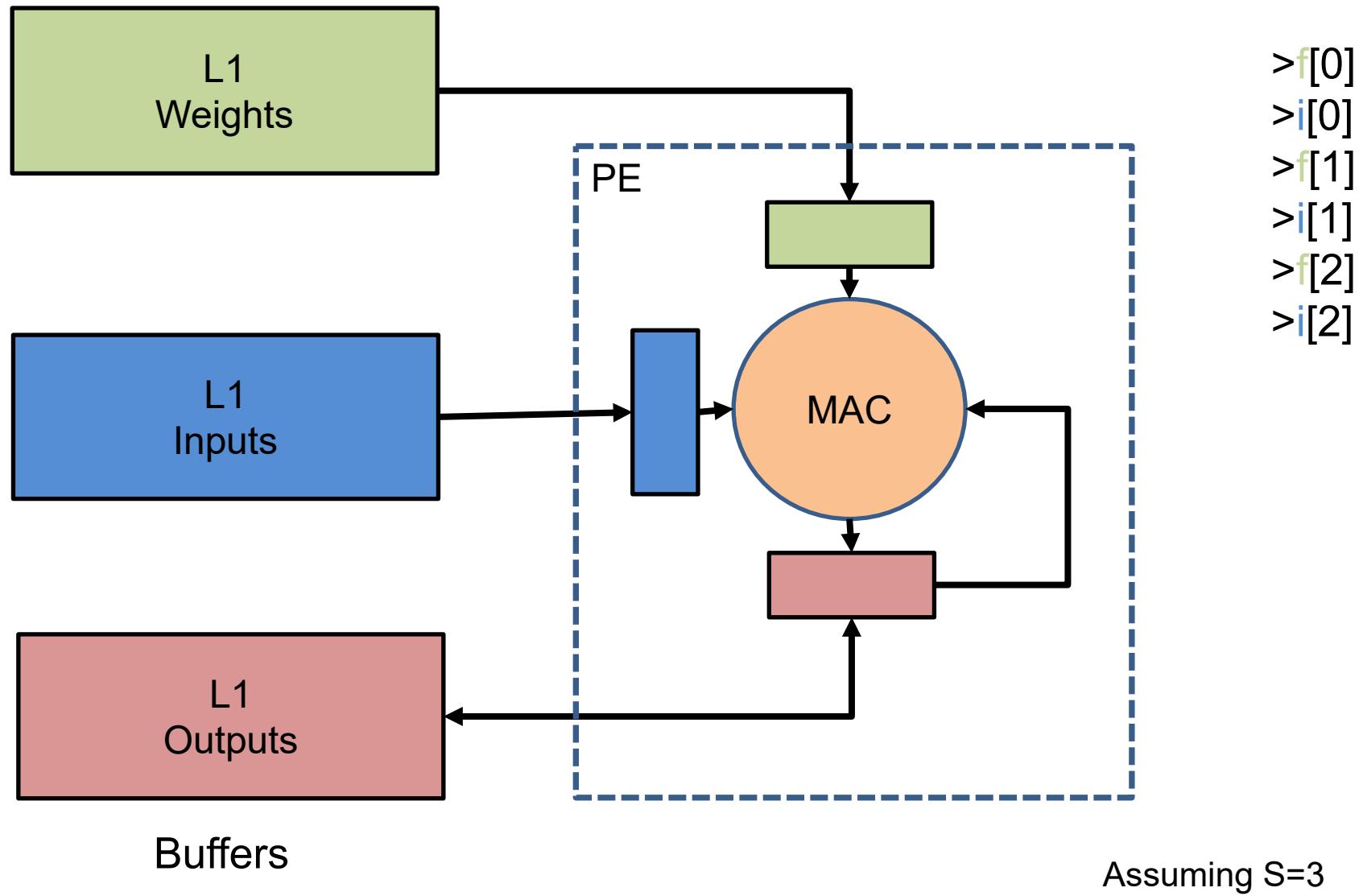
Single PE Output Stationary Flow



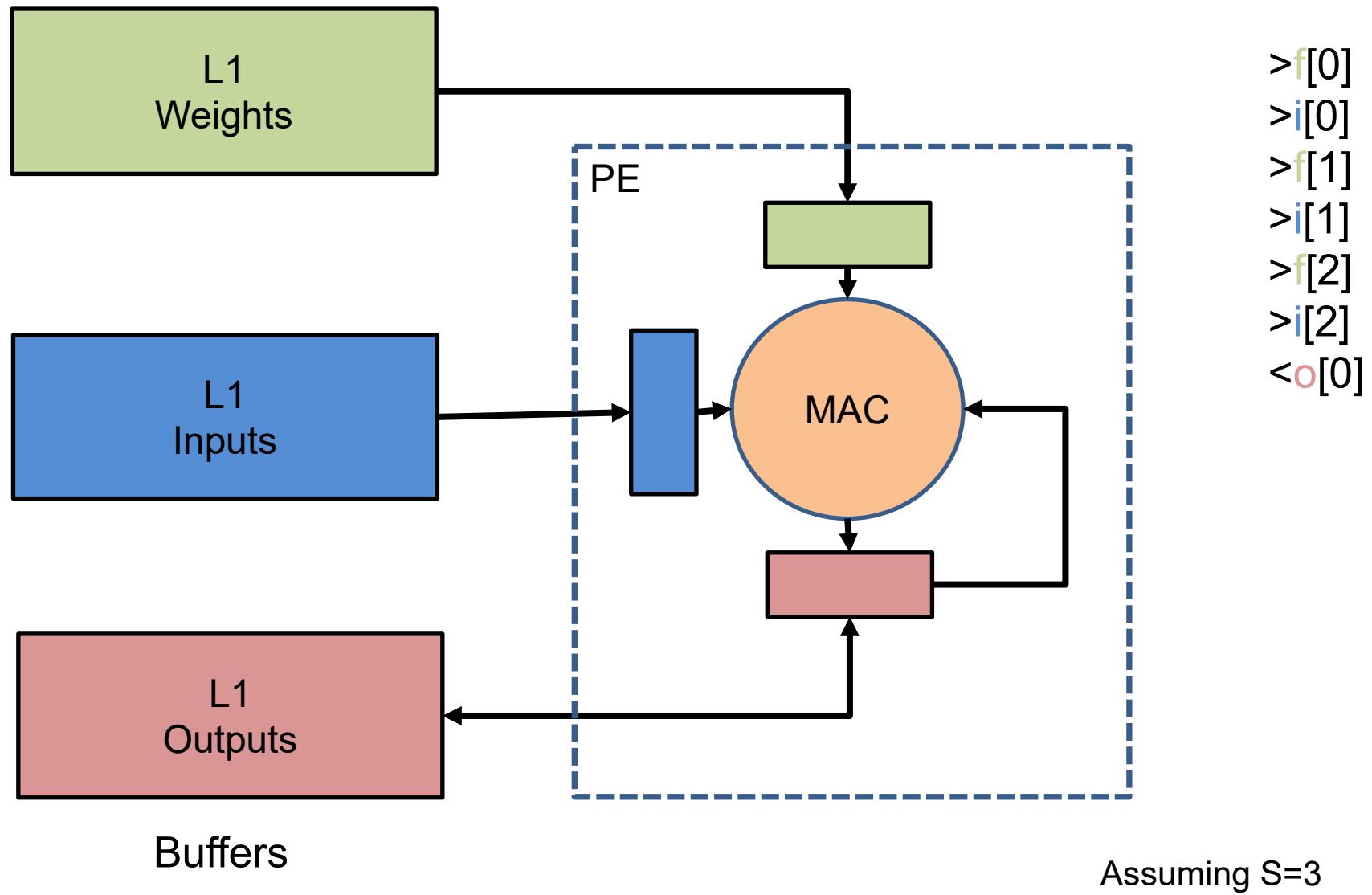
Single PE Output Stationary Flow



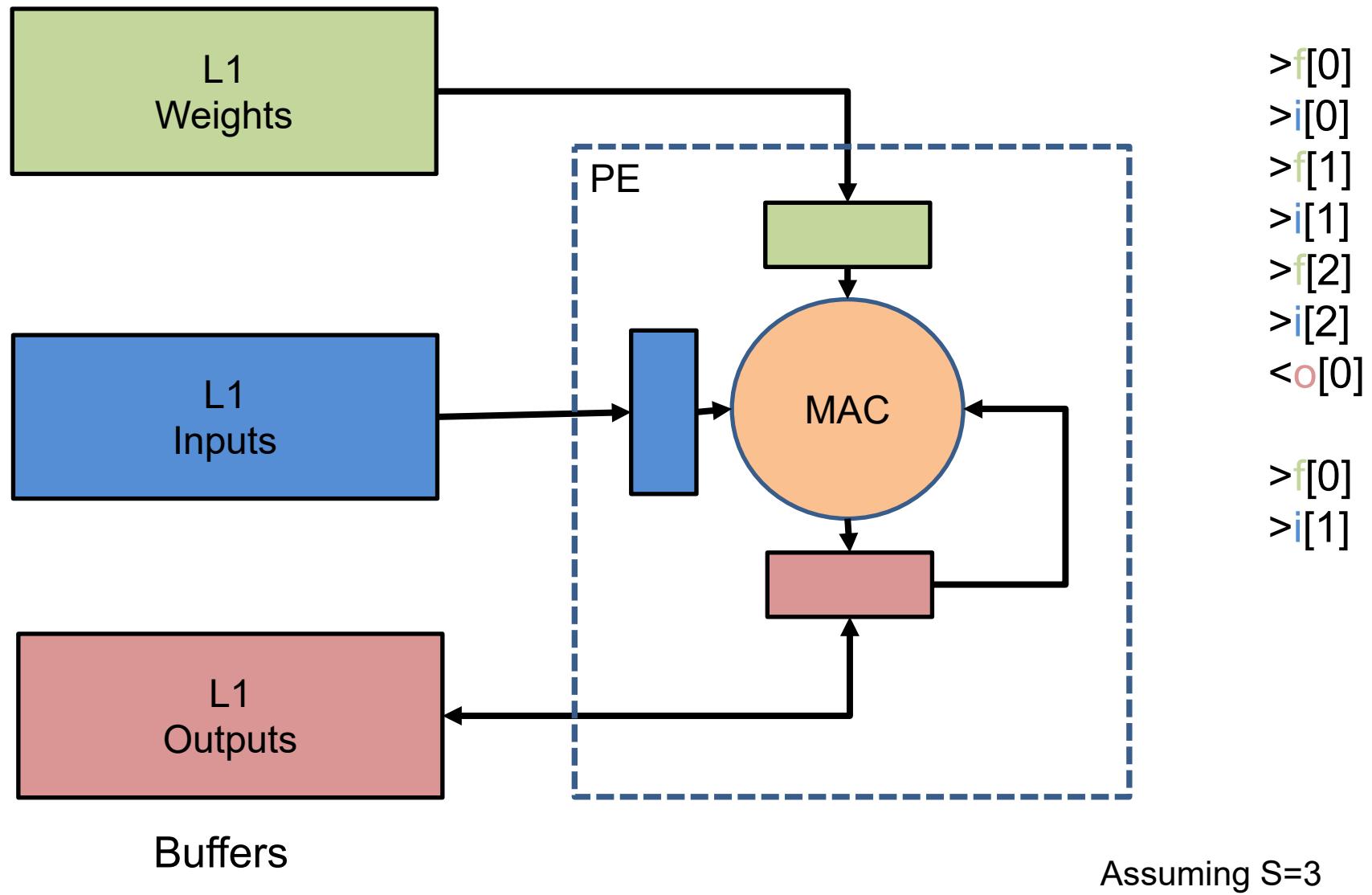
Single PE Output Stationary Flow



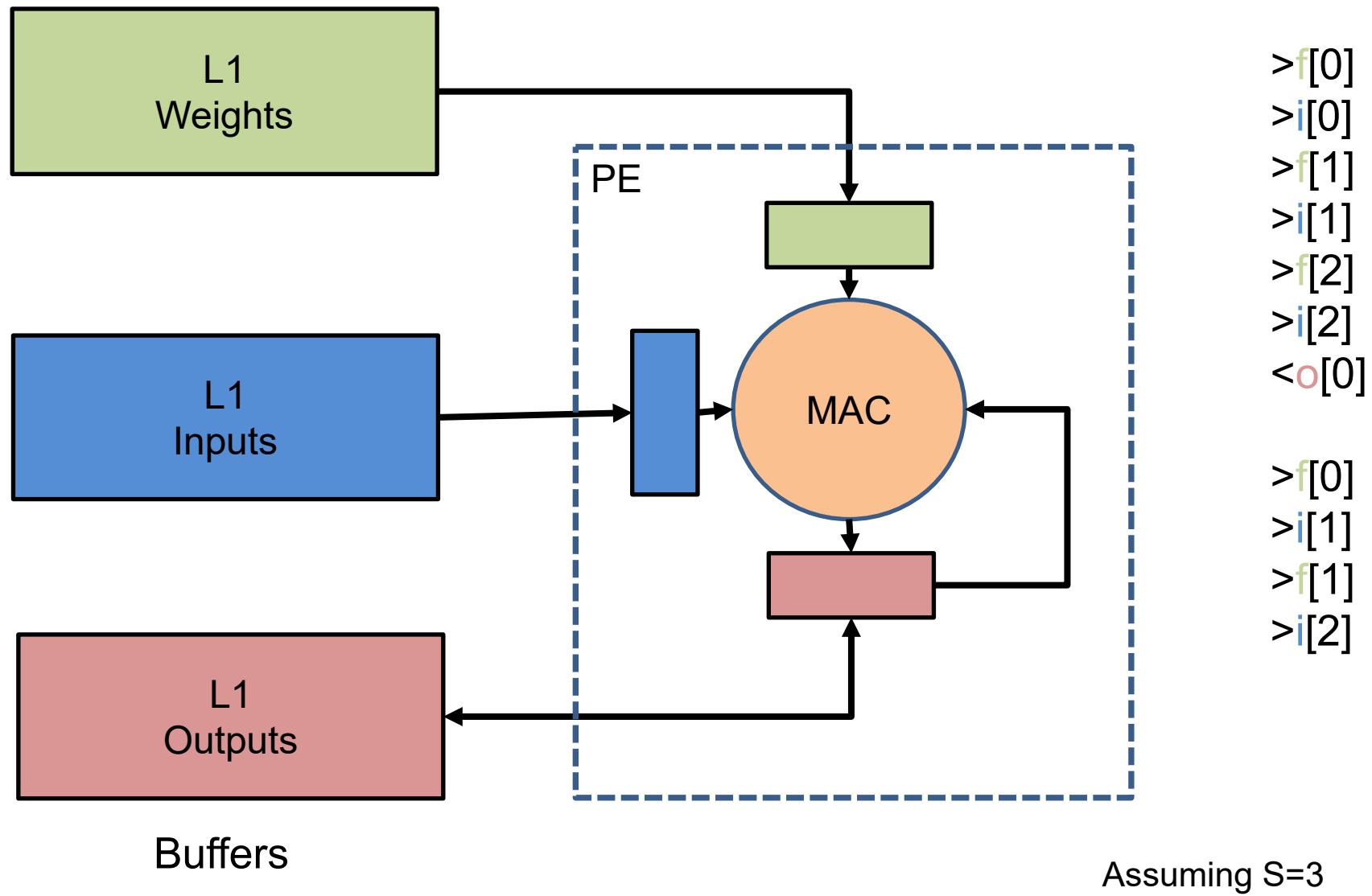
Single PE Output Stationary Flow



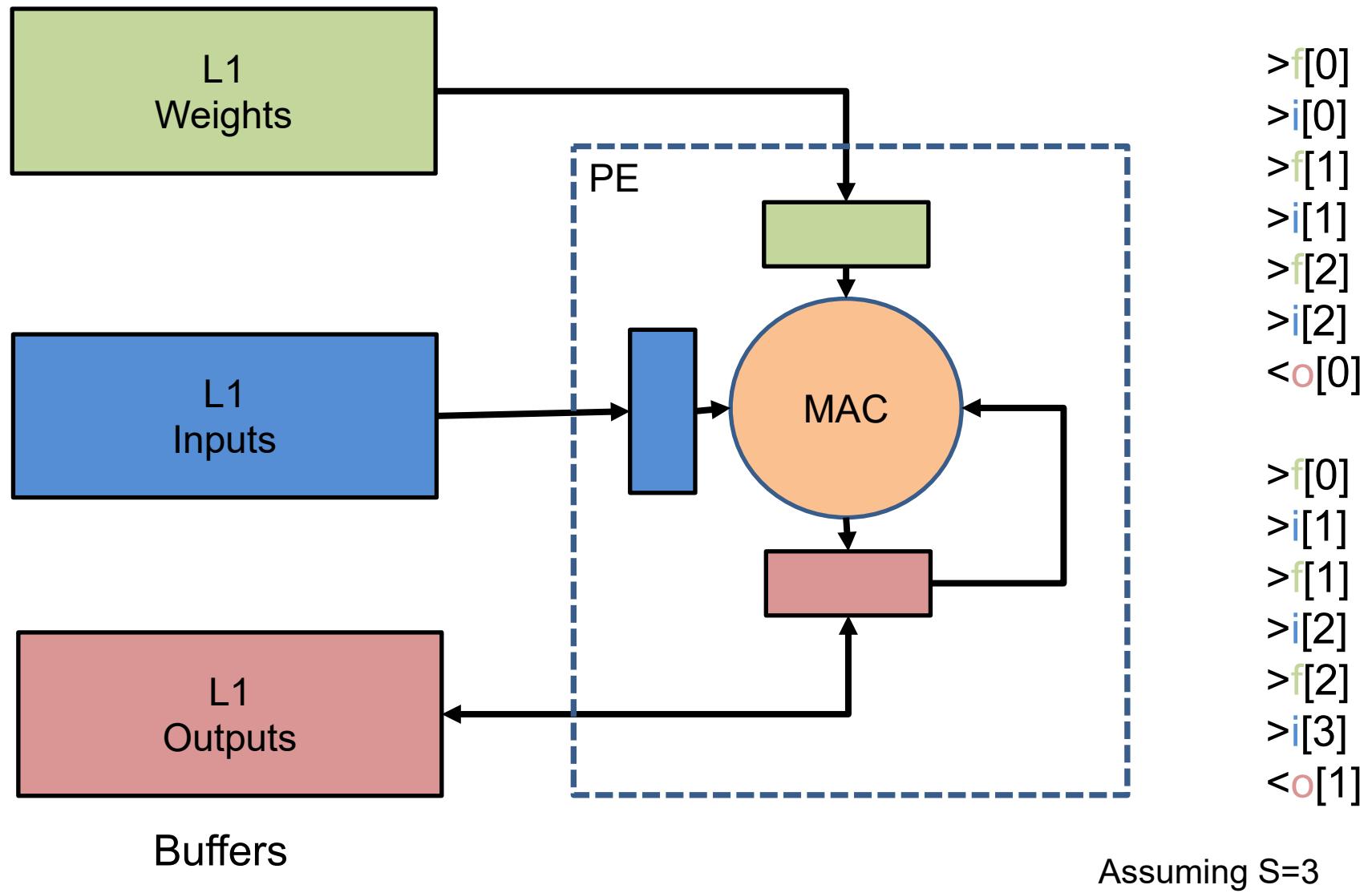
Single PE Output Stationary Flow



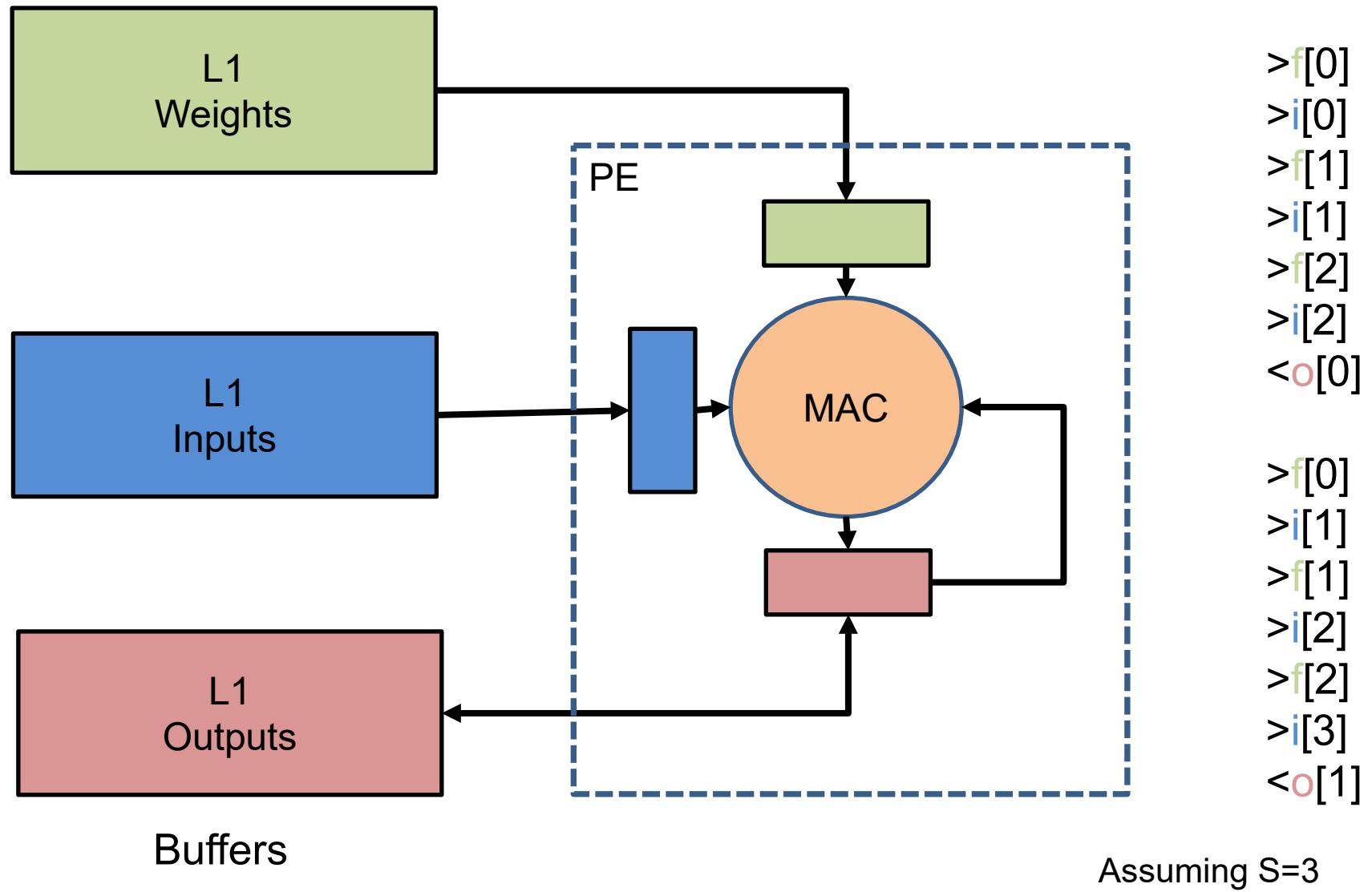
Single PE Output Stationary Flow



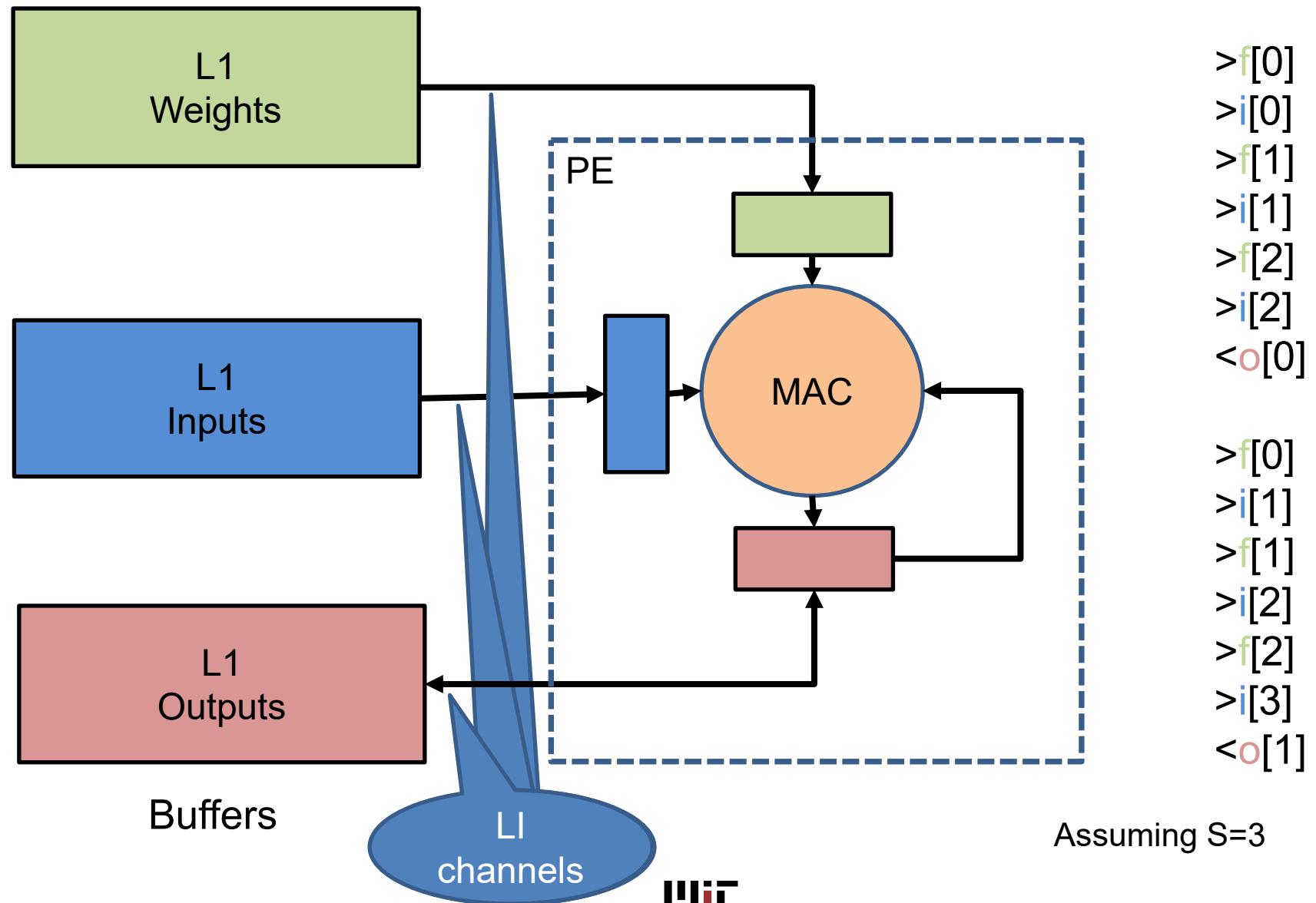
Single PE Output Stationary Flow



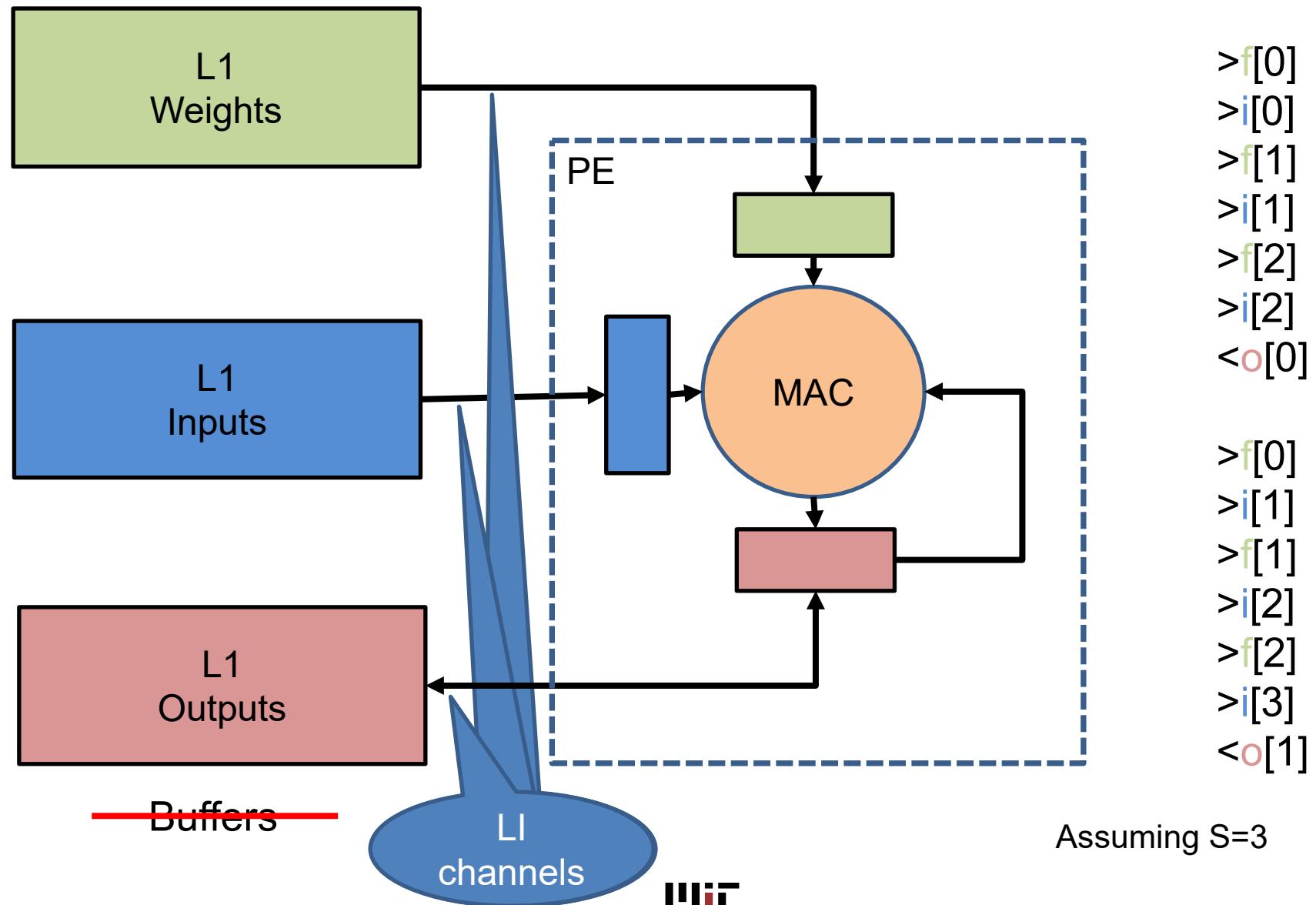
Single PE Output Stationary Flow



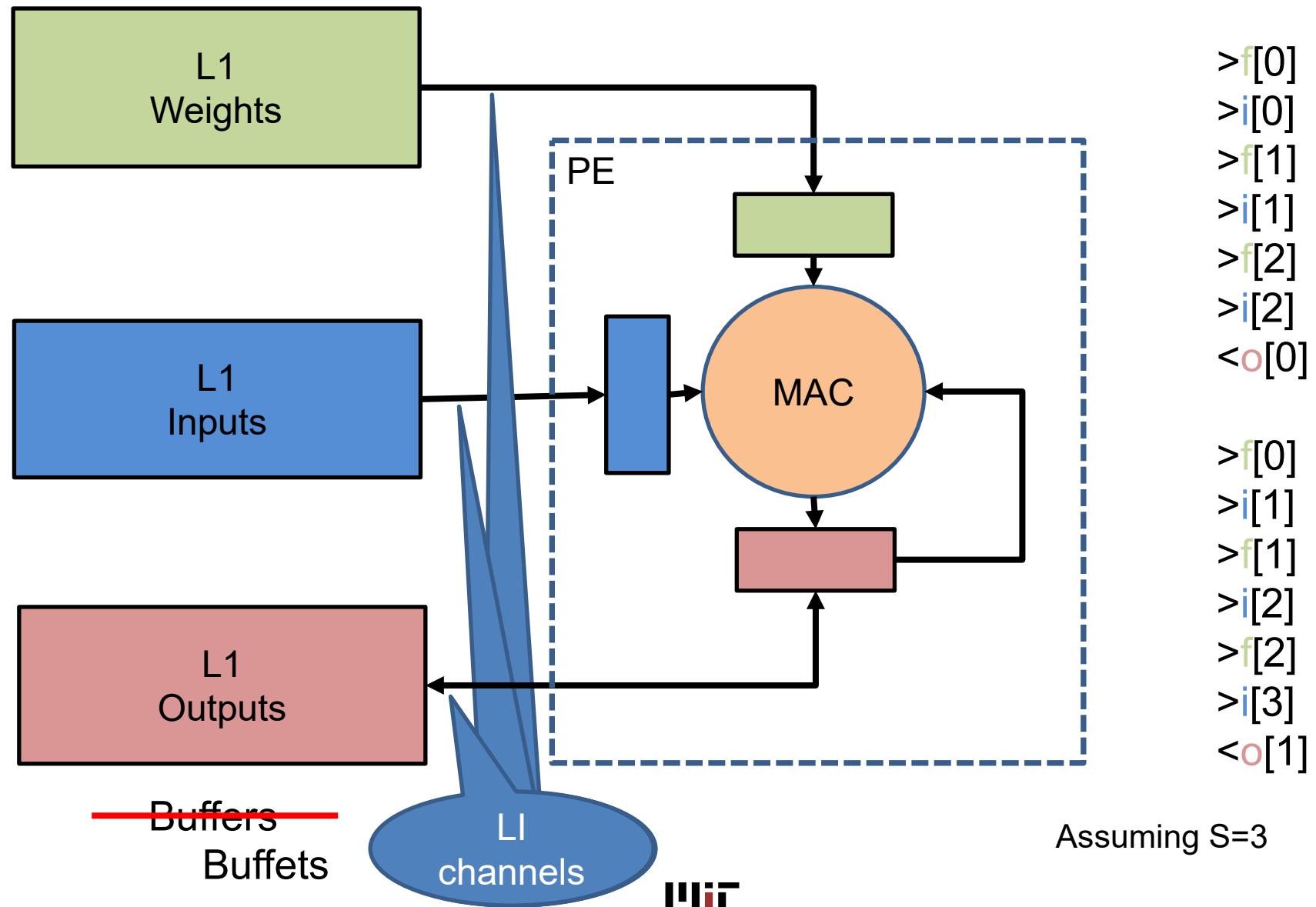
Single PE Output Stationary Flow



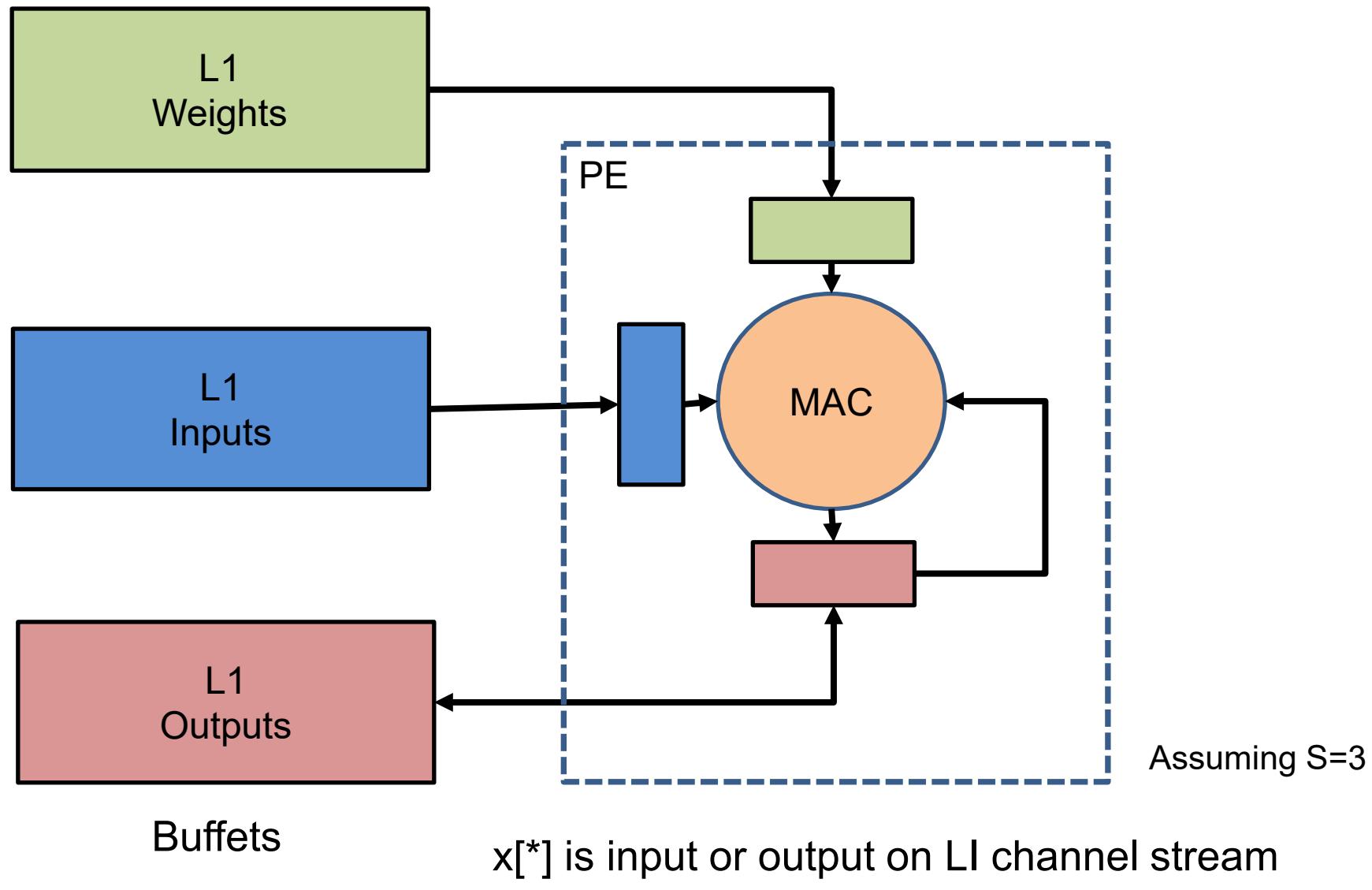
Single PE Output Stationary Flow



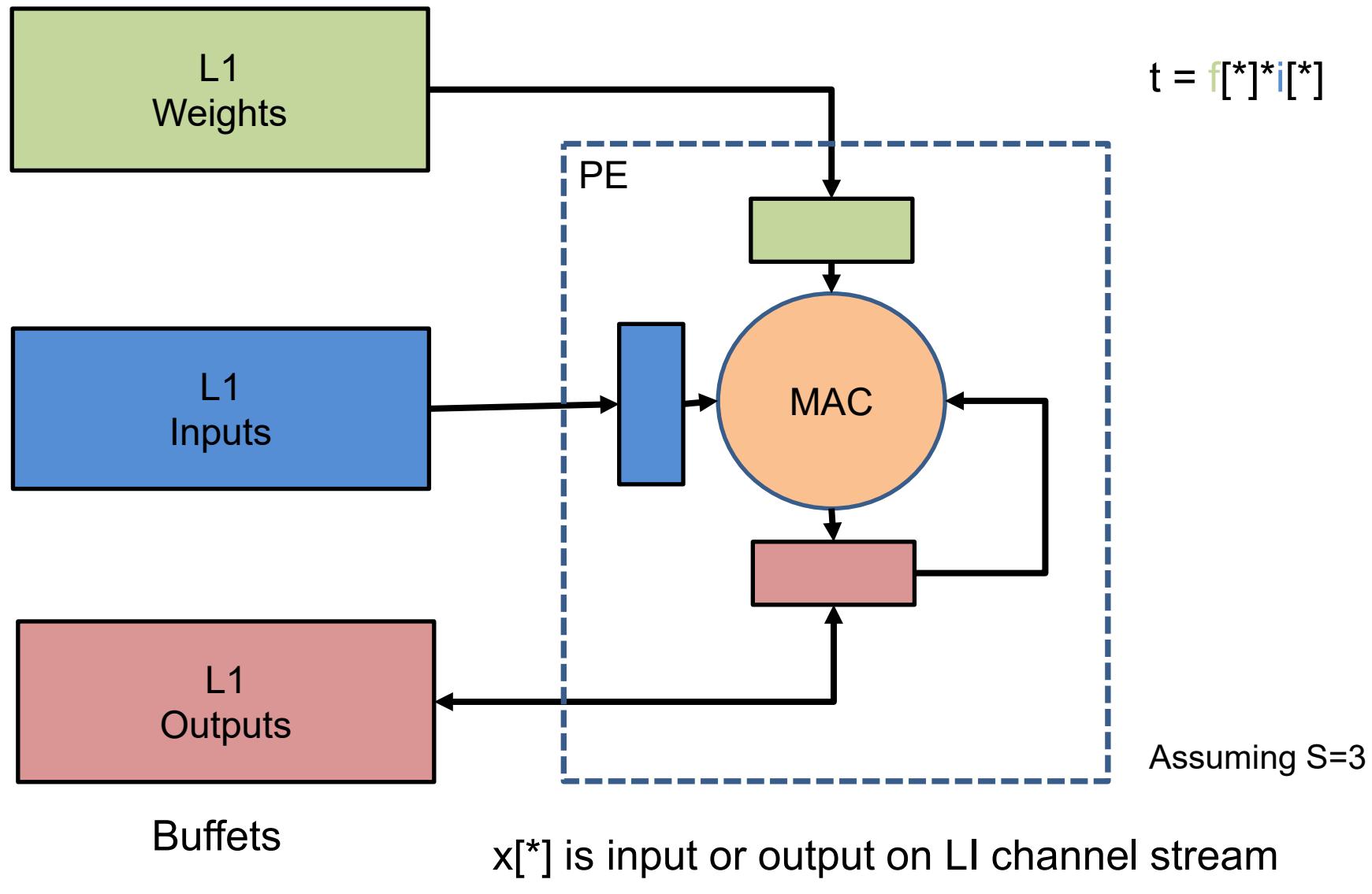
Single PE Output Stationary Flow



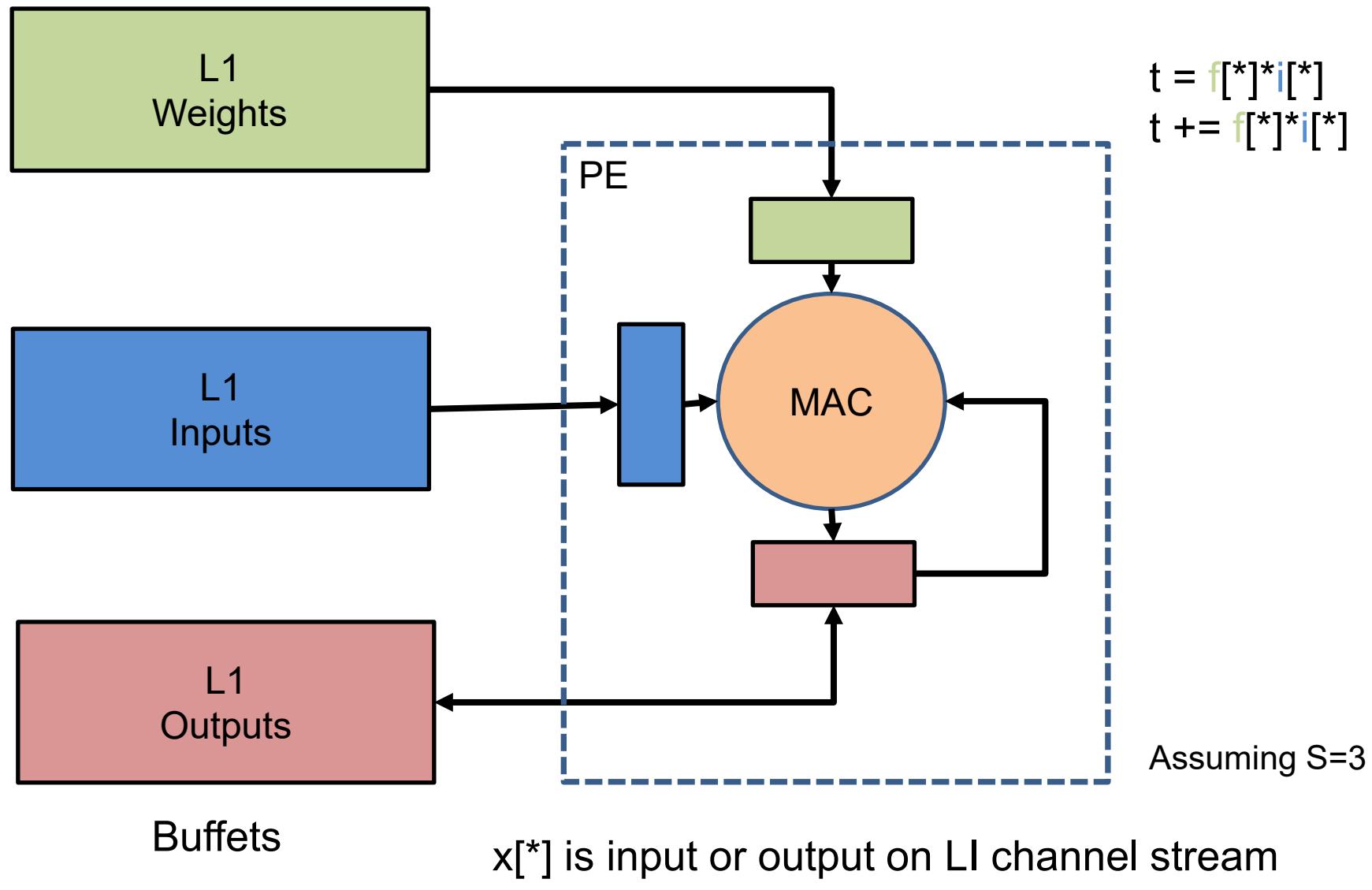
Single PE Output Stationary Flow



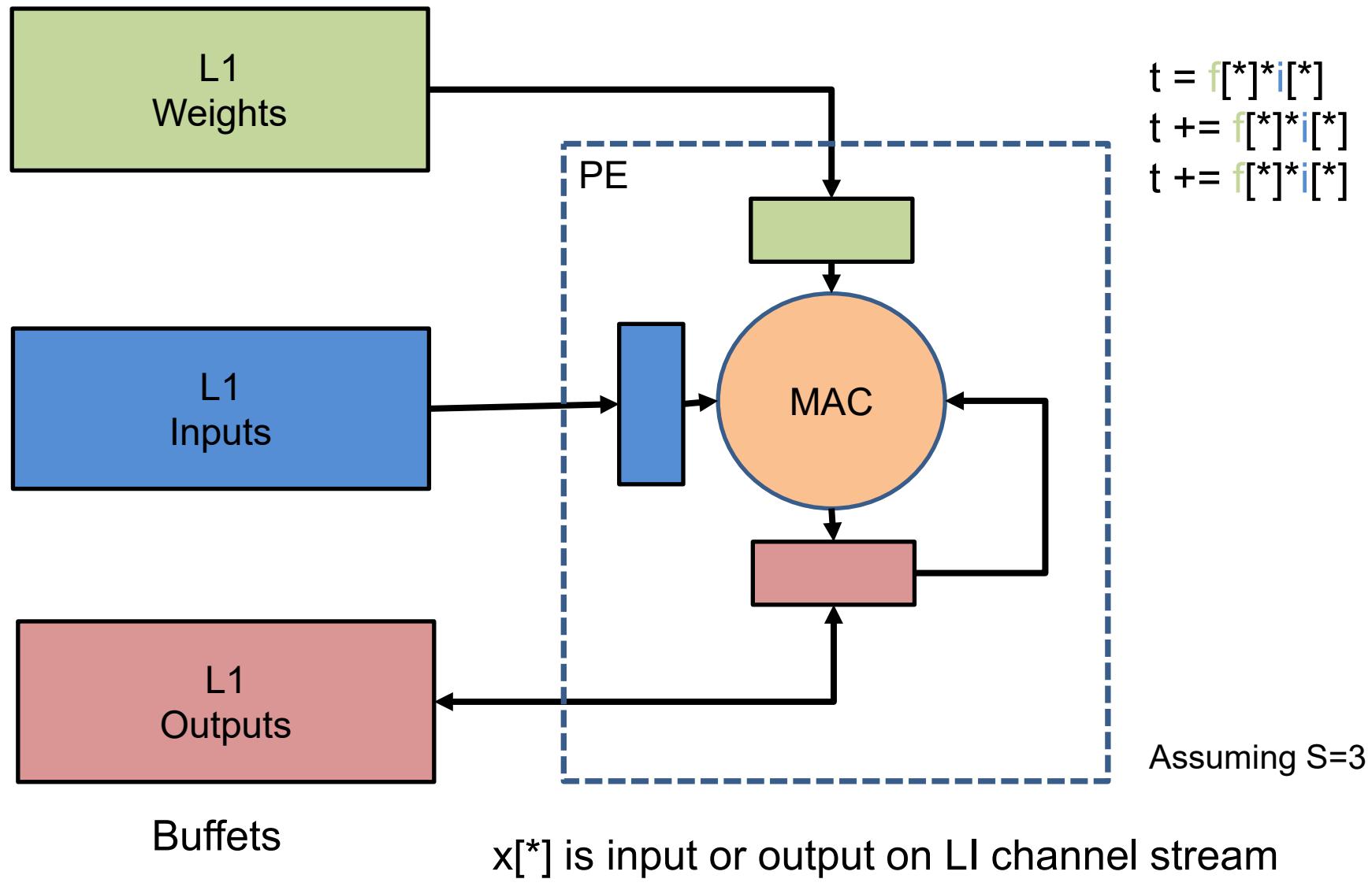
Single PE Output Stationary Flow



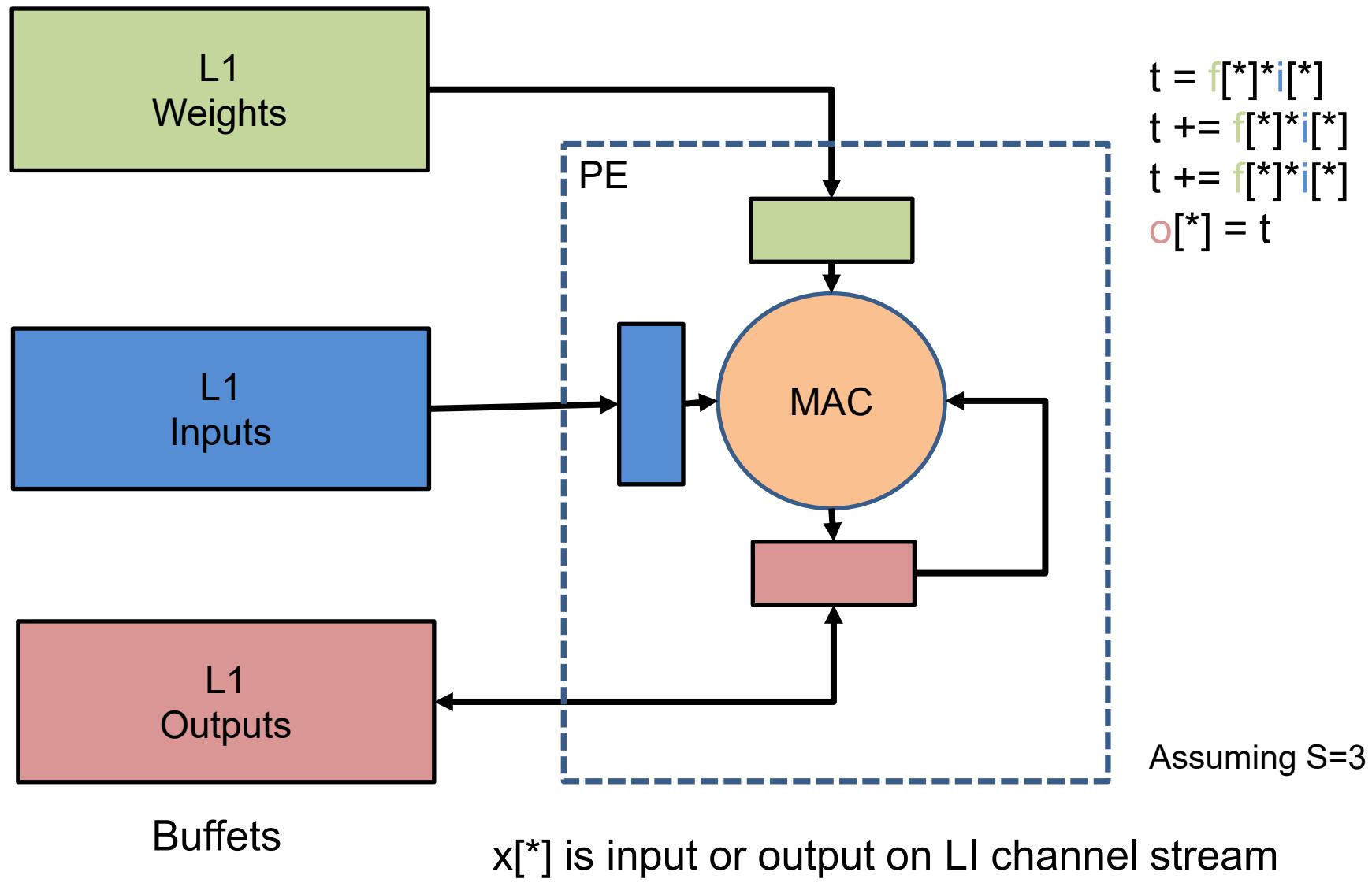
Single PE Output Stationary Flow



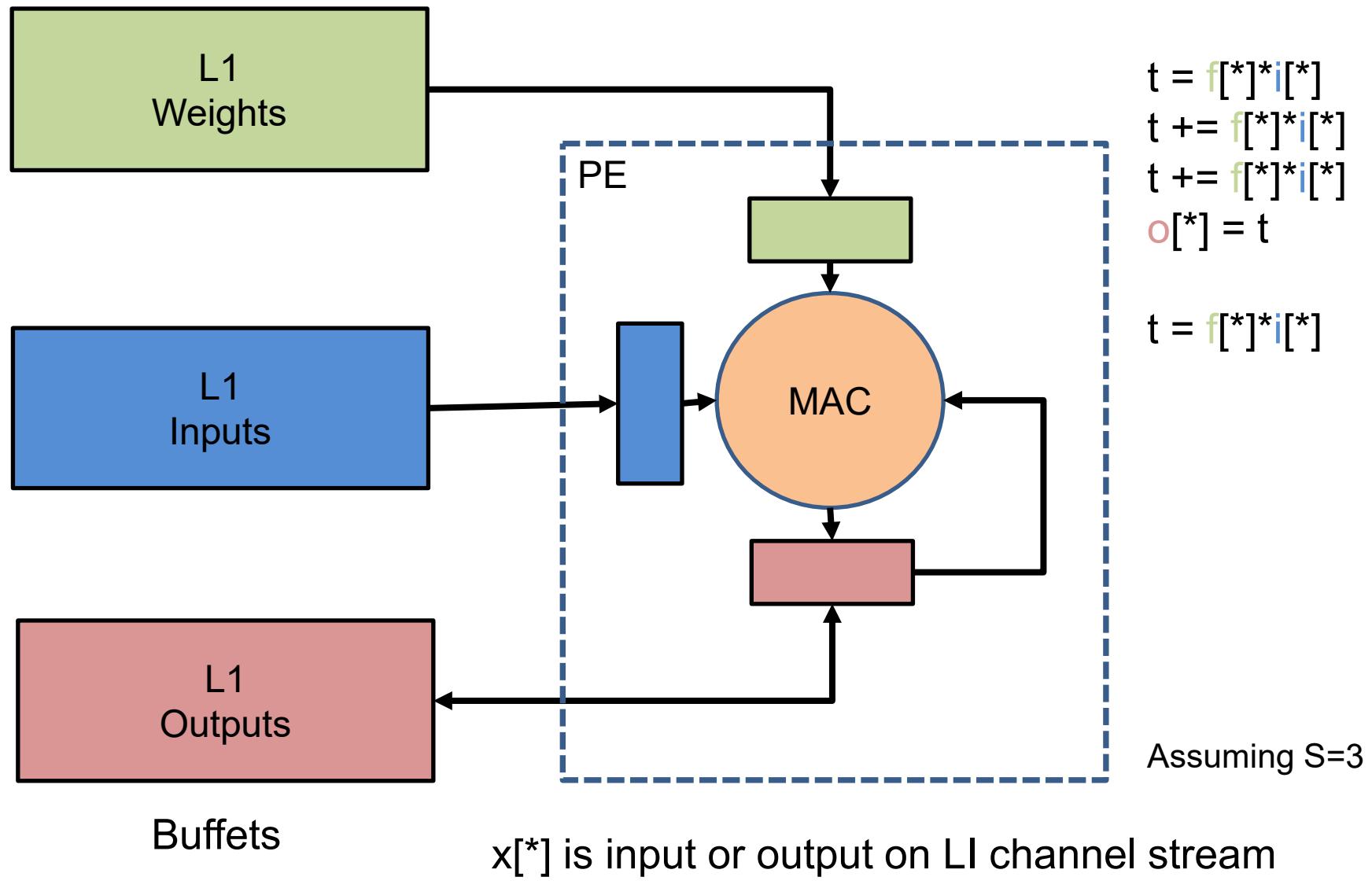
Single PE Output Stationary Flow



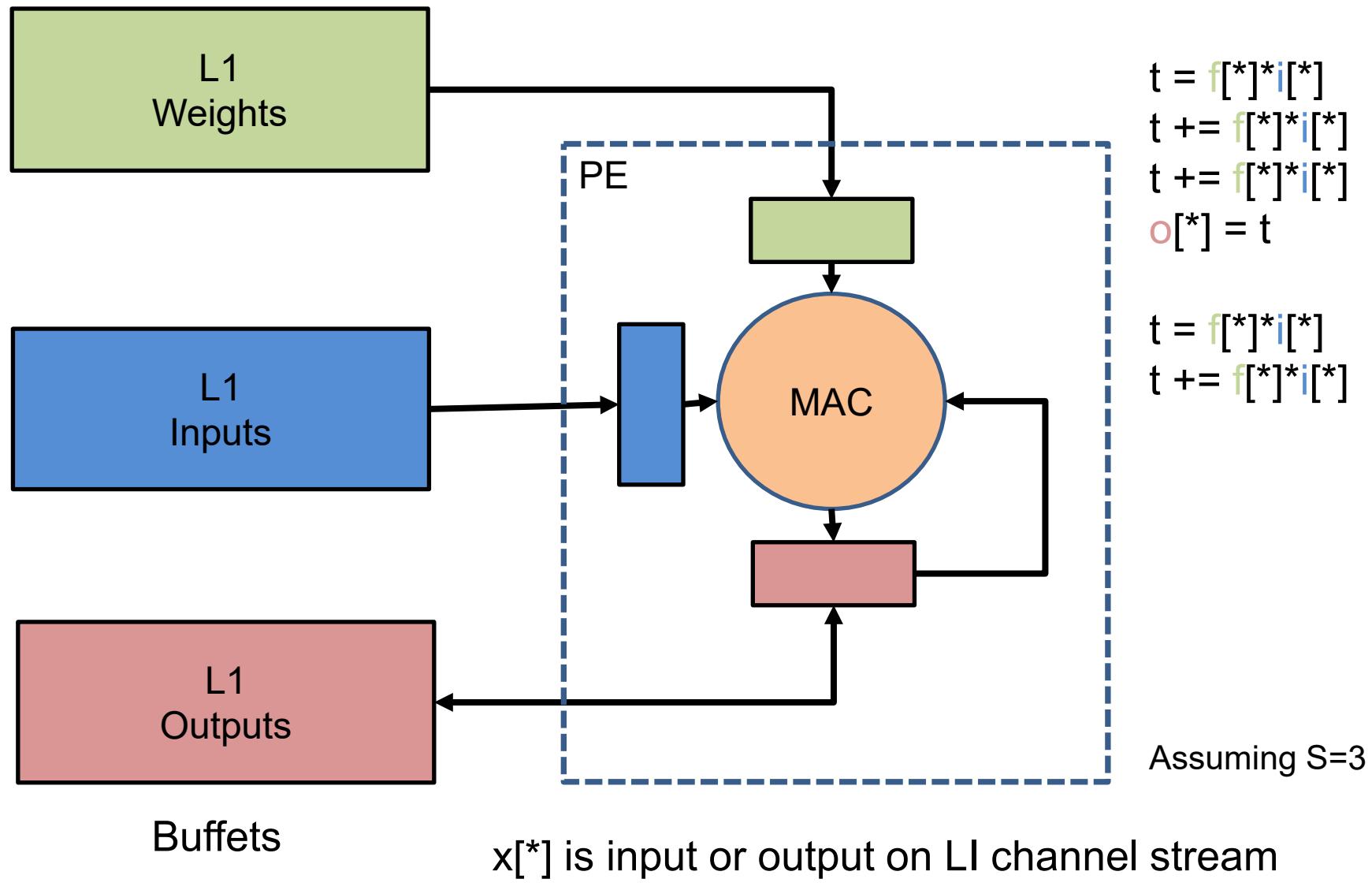
Single PE Output Stationary Flow



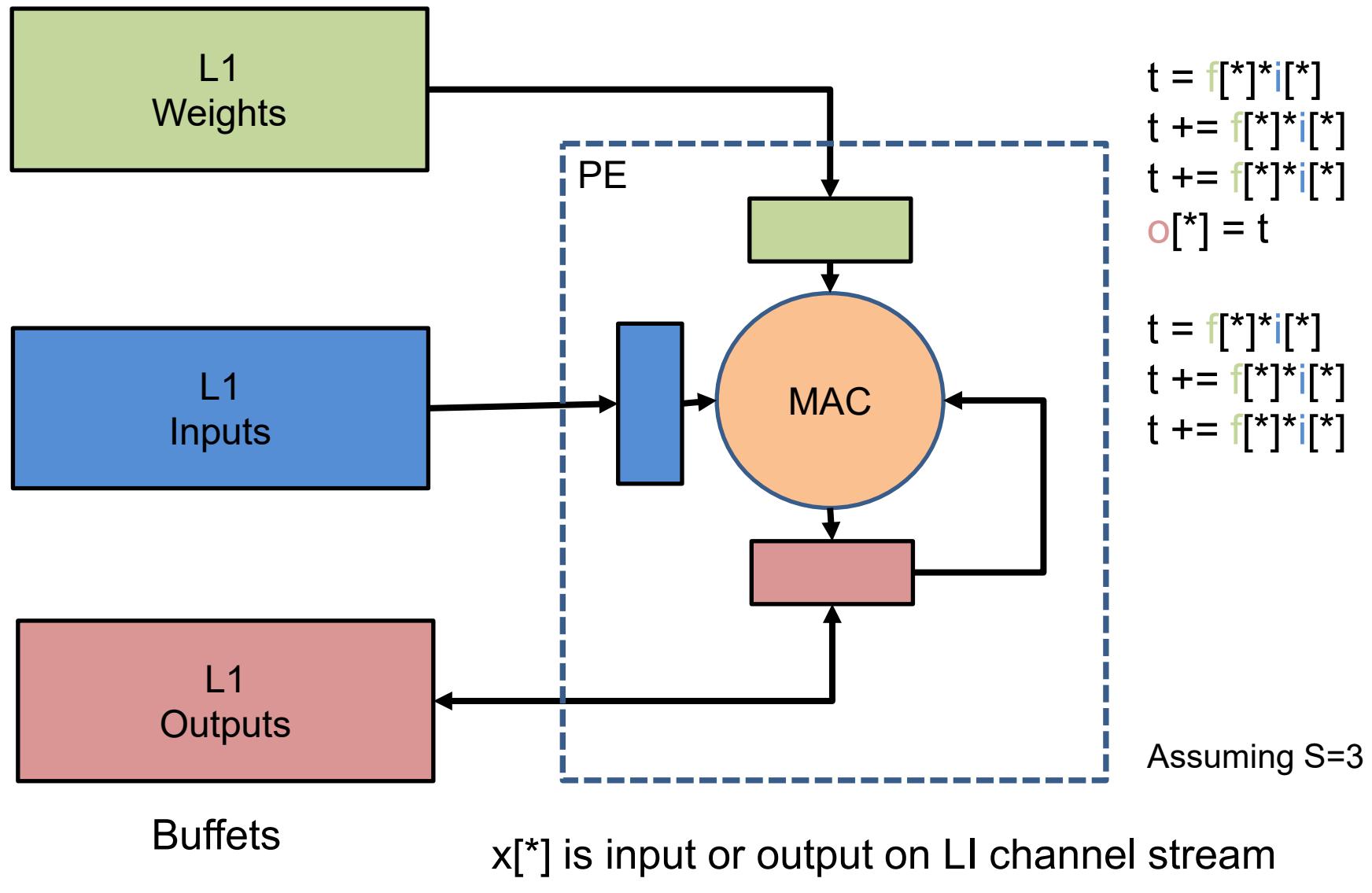
Single PE Output Stationary Flow



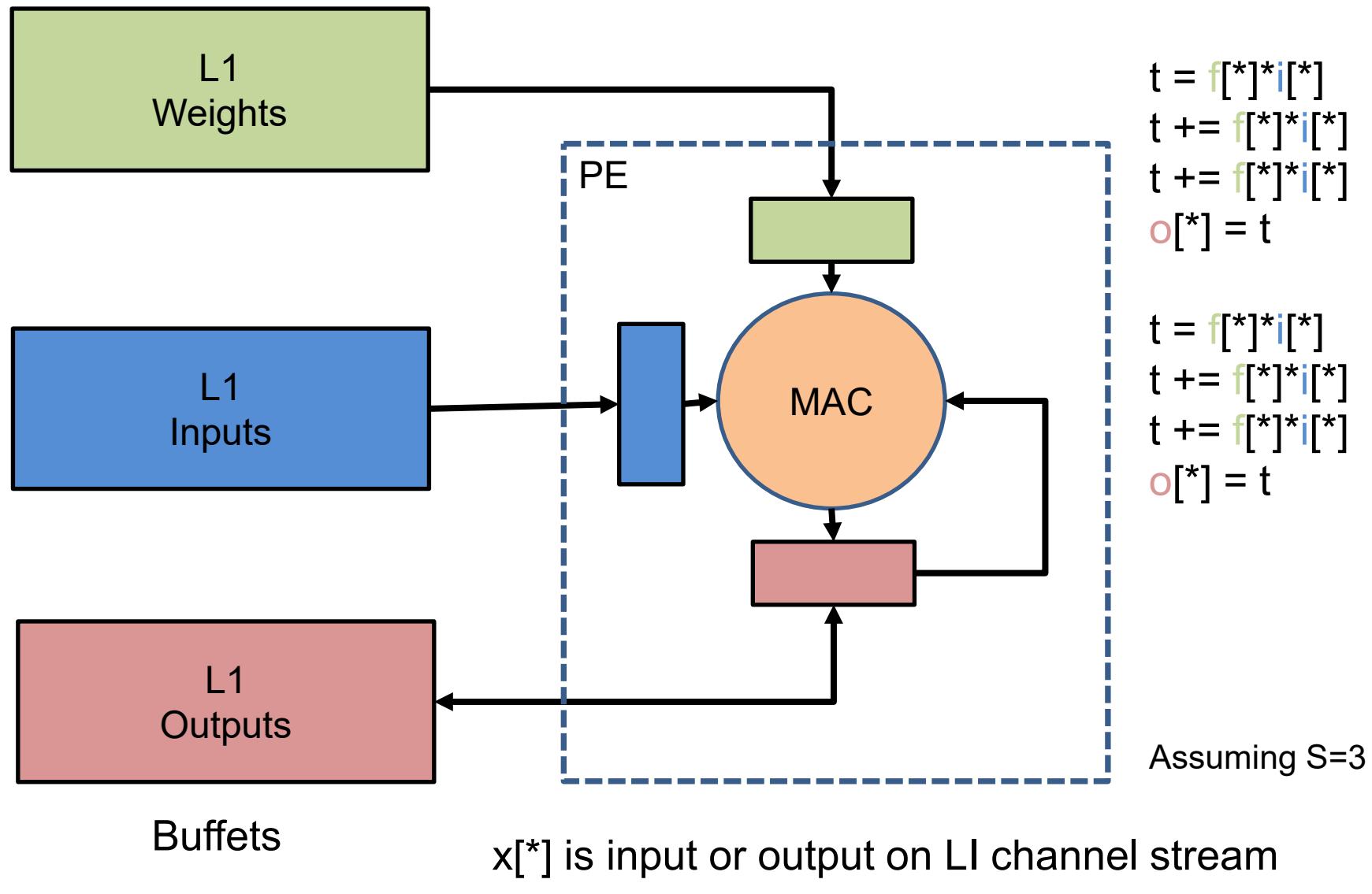
Single PE Output Stationary Flow



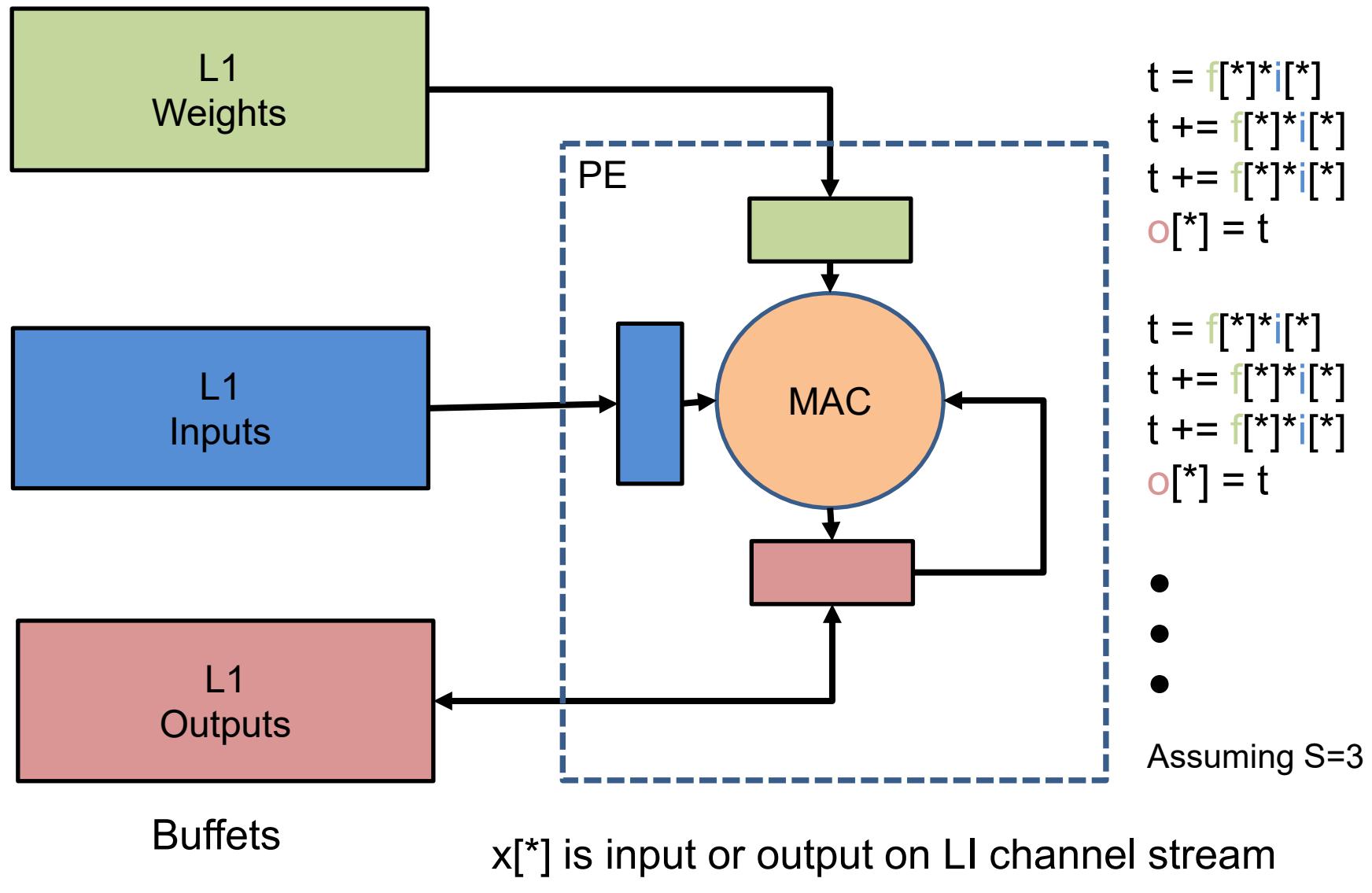
Single PE Output Stationary Flow



Single PE Output Stationary Flow



Single PE Output Stationary Flow



LI Channel-based Buffer vs Scratchpads

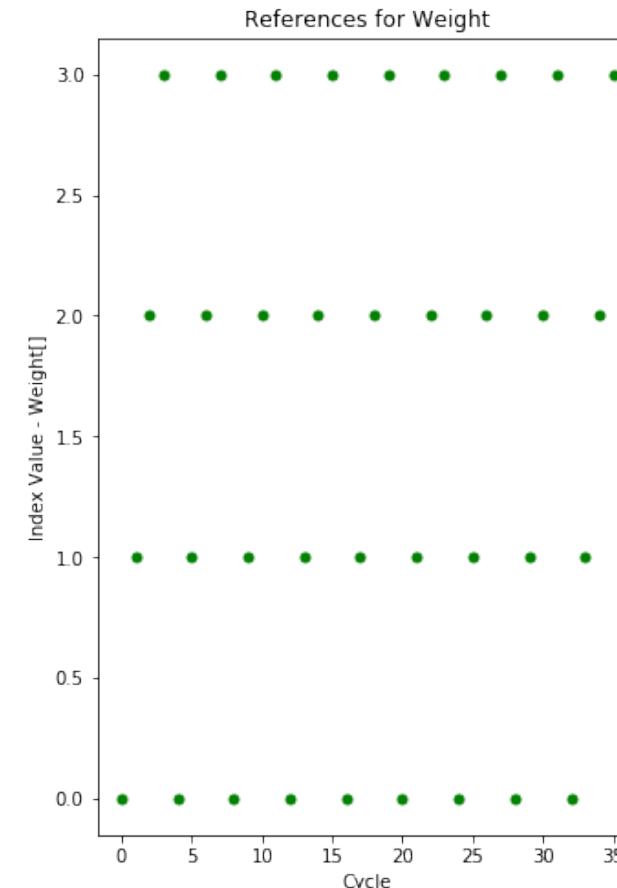
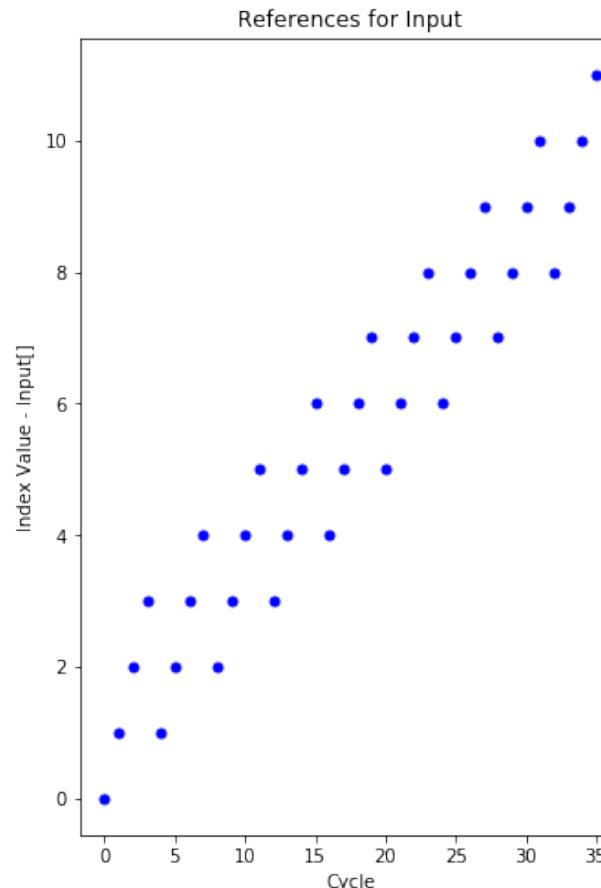
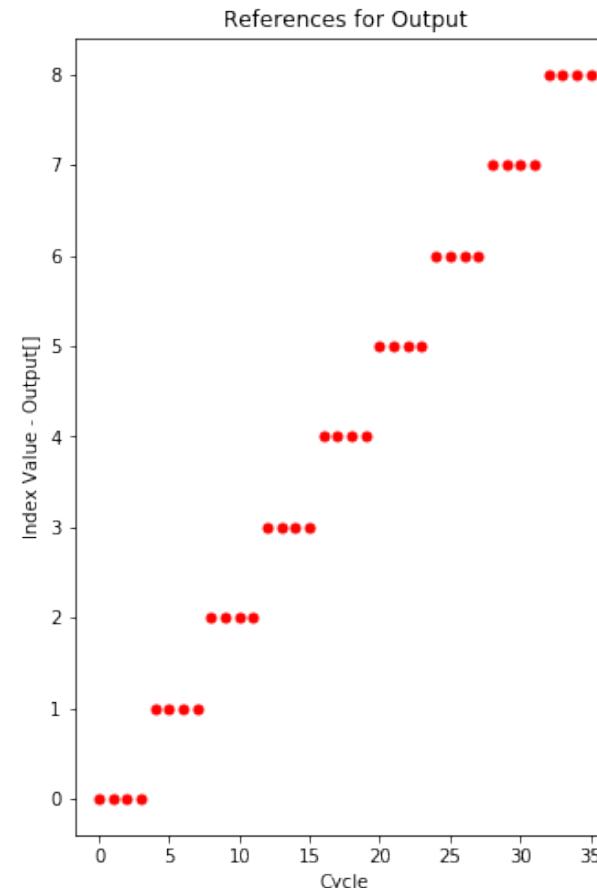
- PE does not generate addresses, i.e., no load or store address calculations
- Address generation is not serialized with arithmetic operations, e.g., as loads or stores
- PE does not need register target for each scratchpad request in flight
- If the channel operations are guaranteed never to block then the channel logic can be optimized away and the reads/writes can happen systolically.

Output Stationary – Reference Pattern

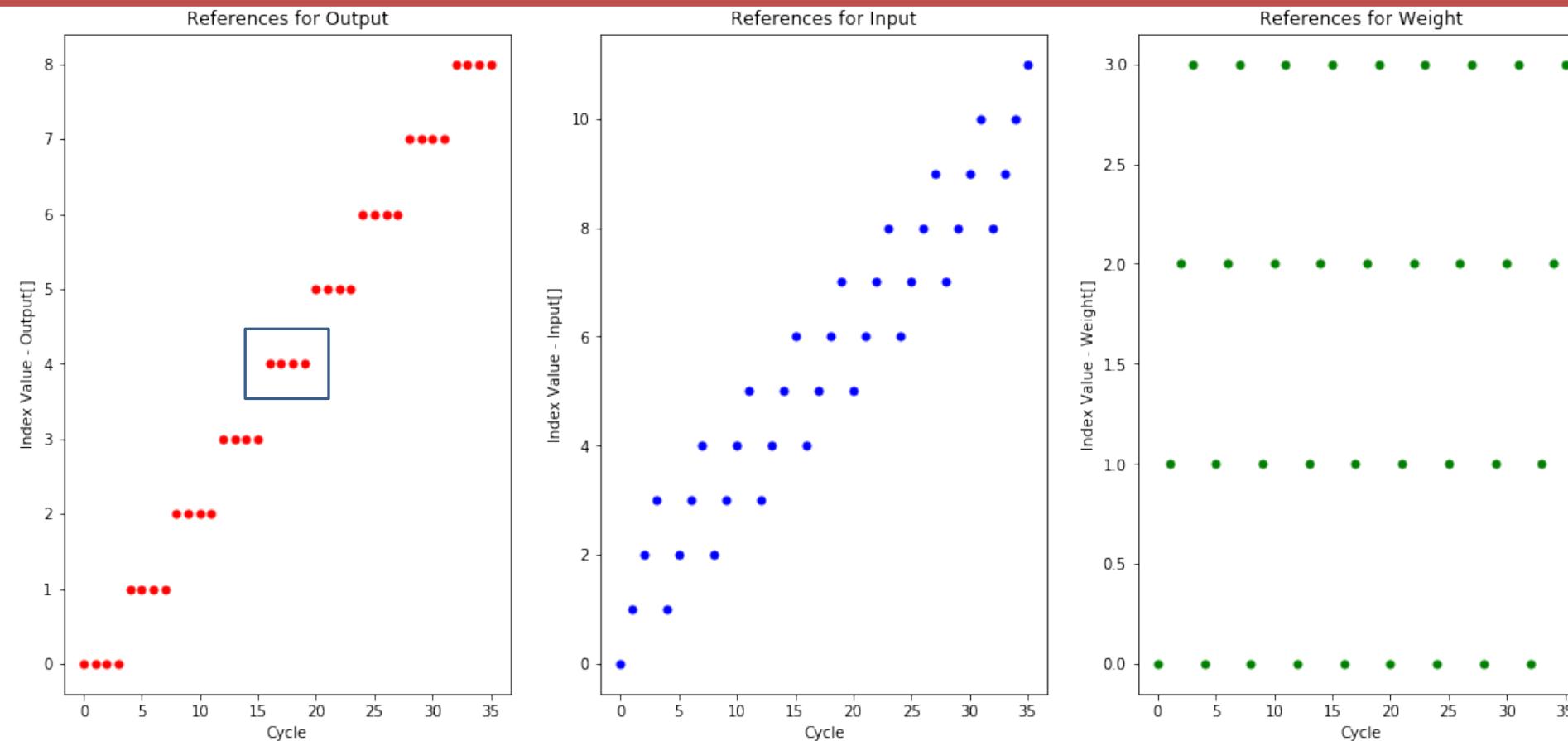
```
for q in [0, Q):
    for s in [0, S):
        o[q] += i[q+s]*f[s]
```

Layer Shape:

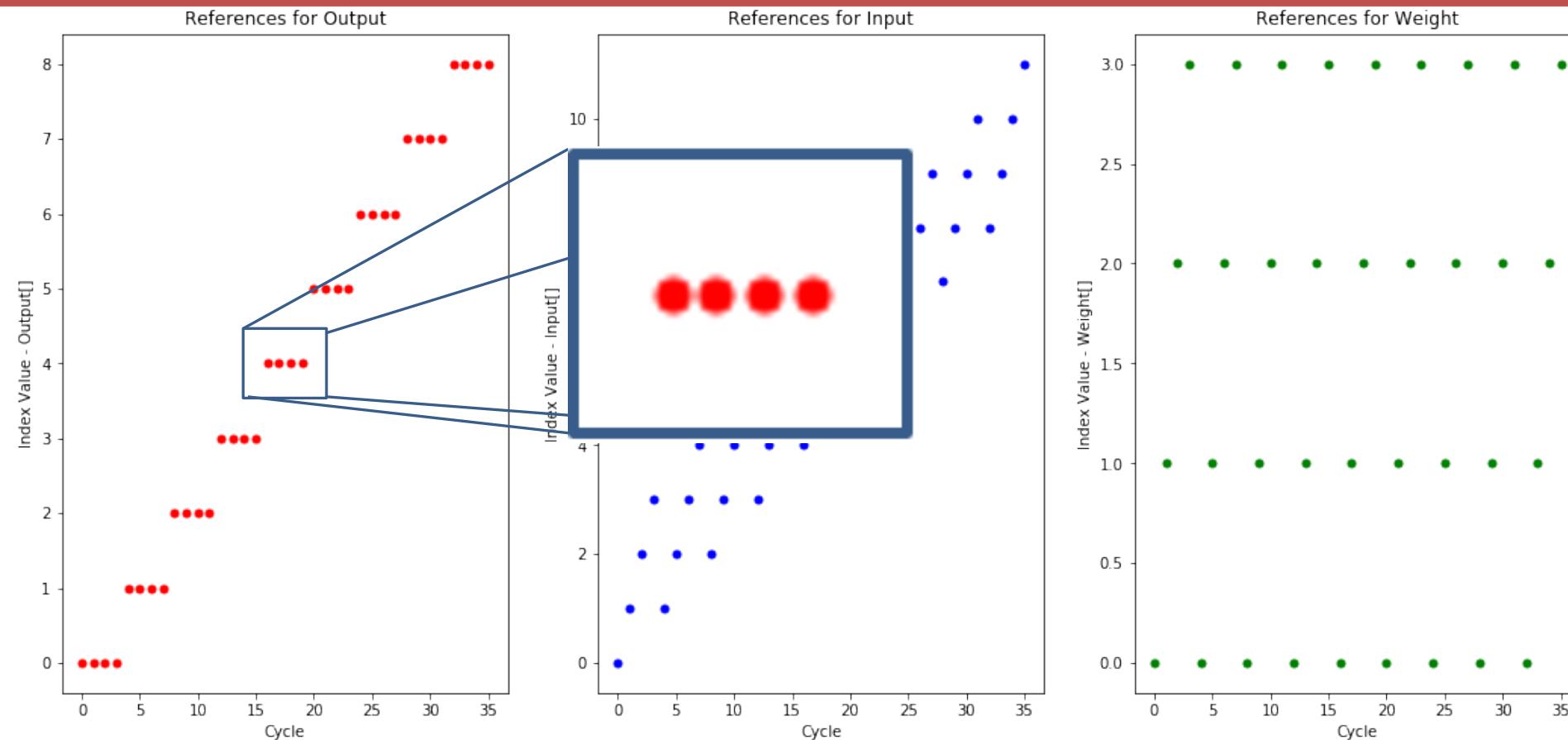
- $S = 4$
- $Q = 9$
- $W = 12$



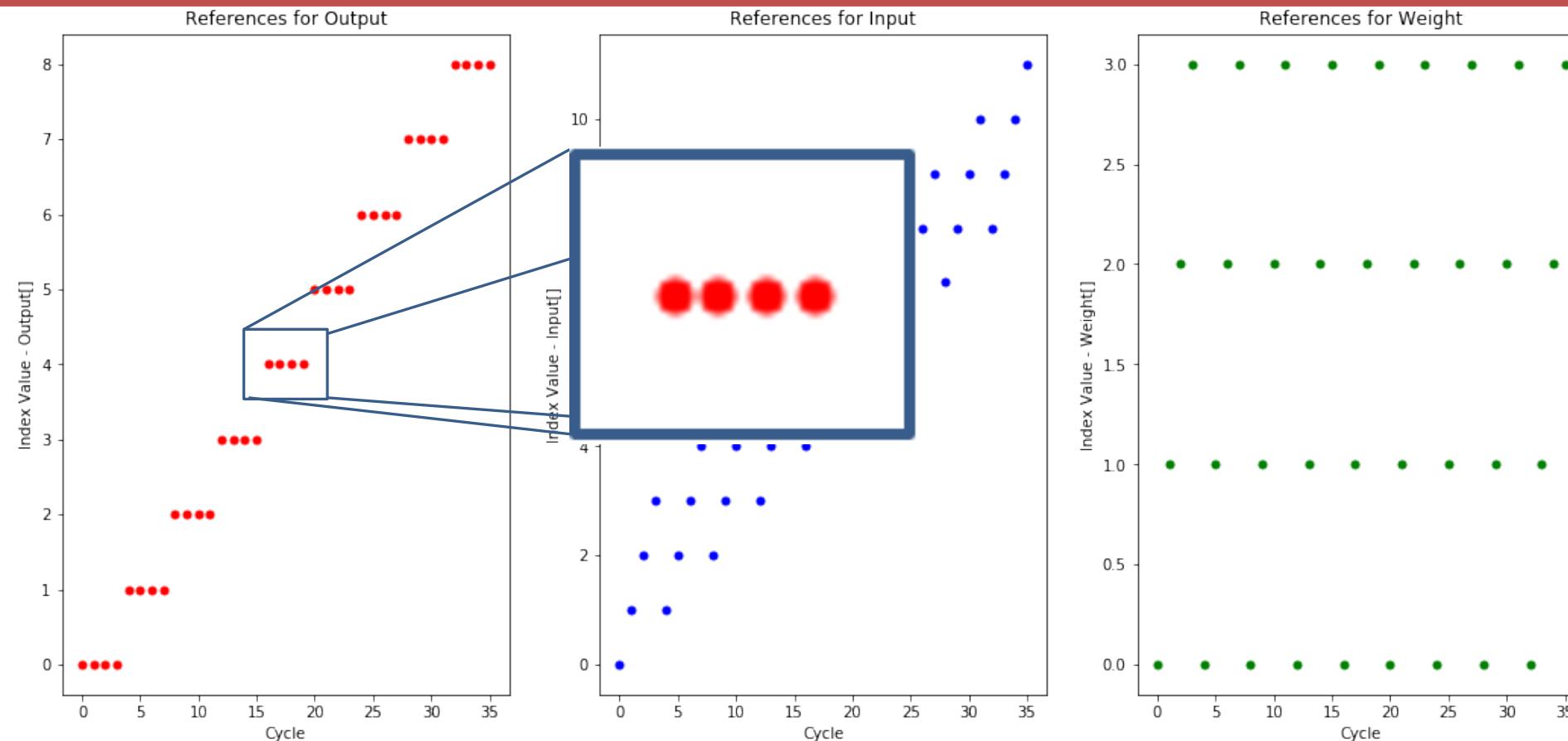
Output Stationary – Reference Pattern



Output Stationary – Reference Pattern



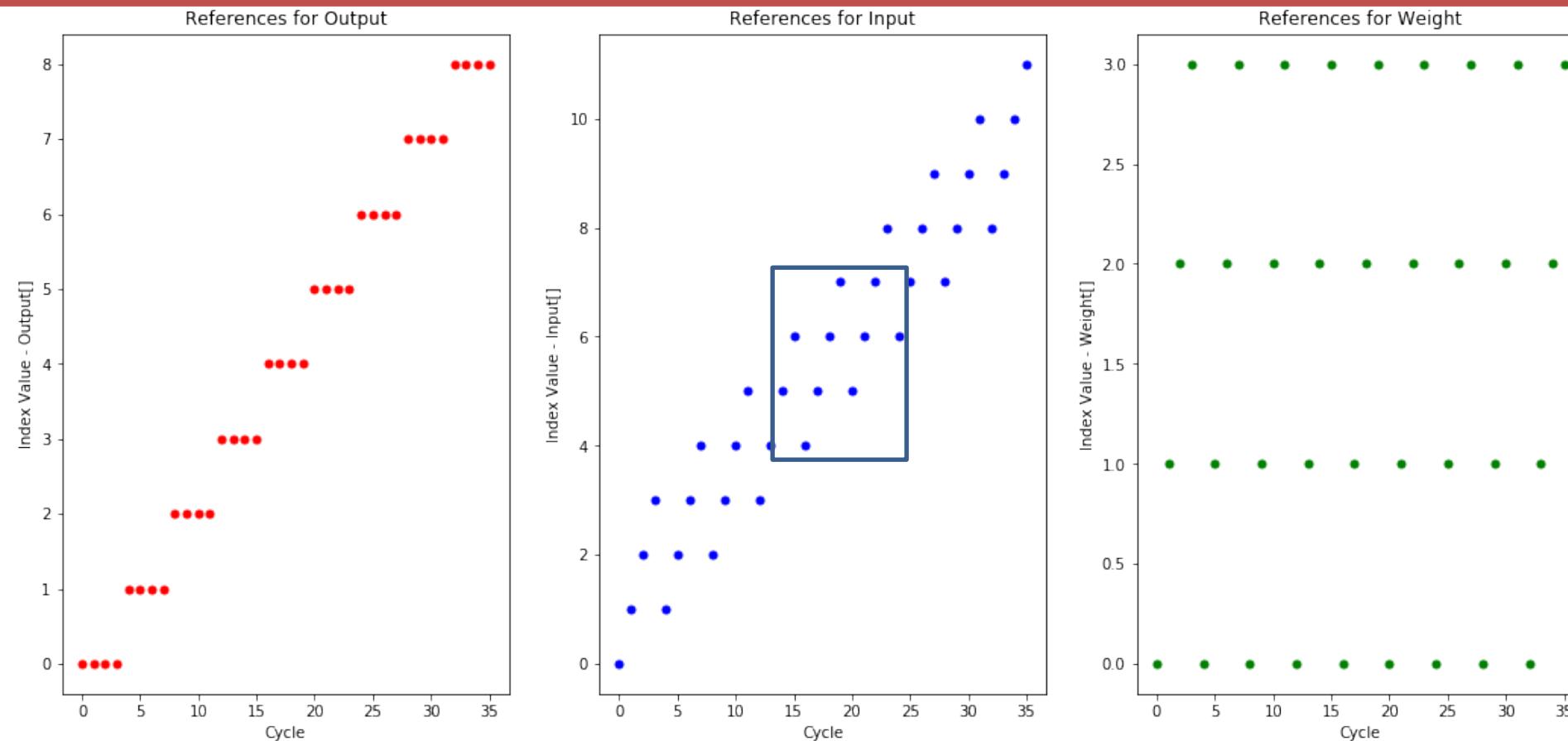
Output Stationary – Reference Pattern



Observations:

- Single **output** is reused many times (S)

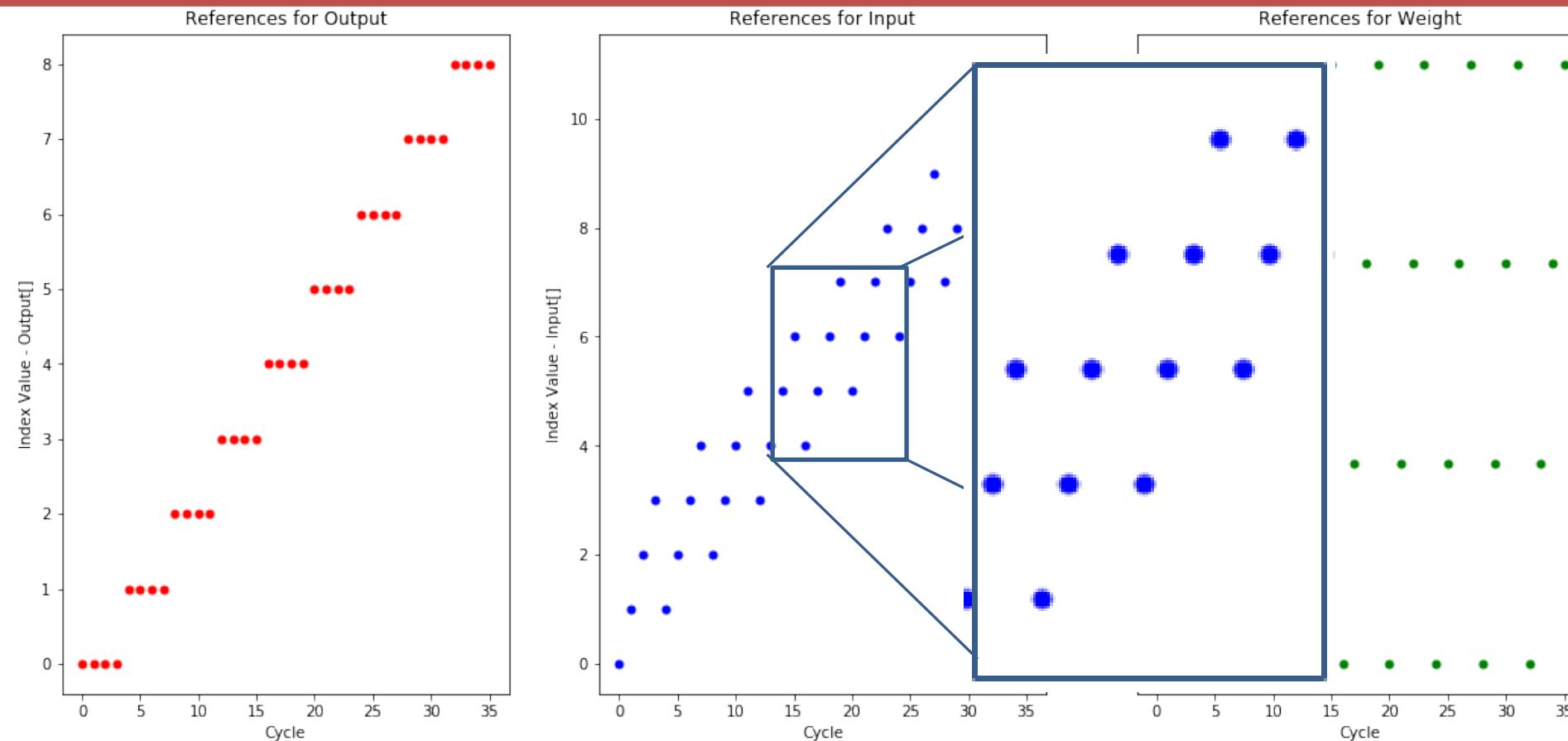
Output Stationary – Reference Pattern



Observations:

- Single **output** is reused many times (S)
- All **weights** reused repeatedly

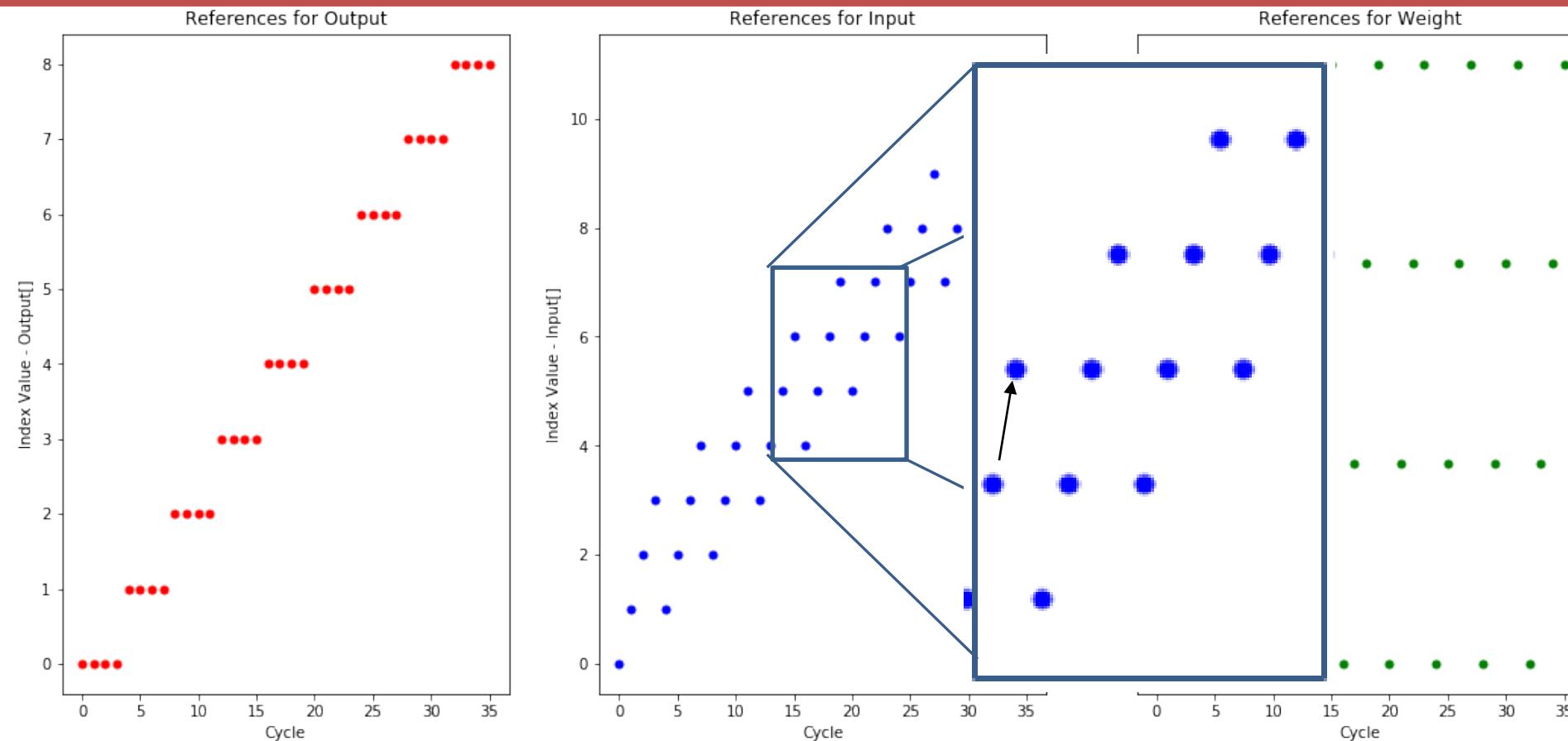
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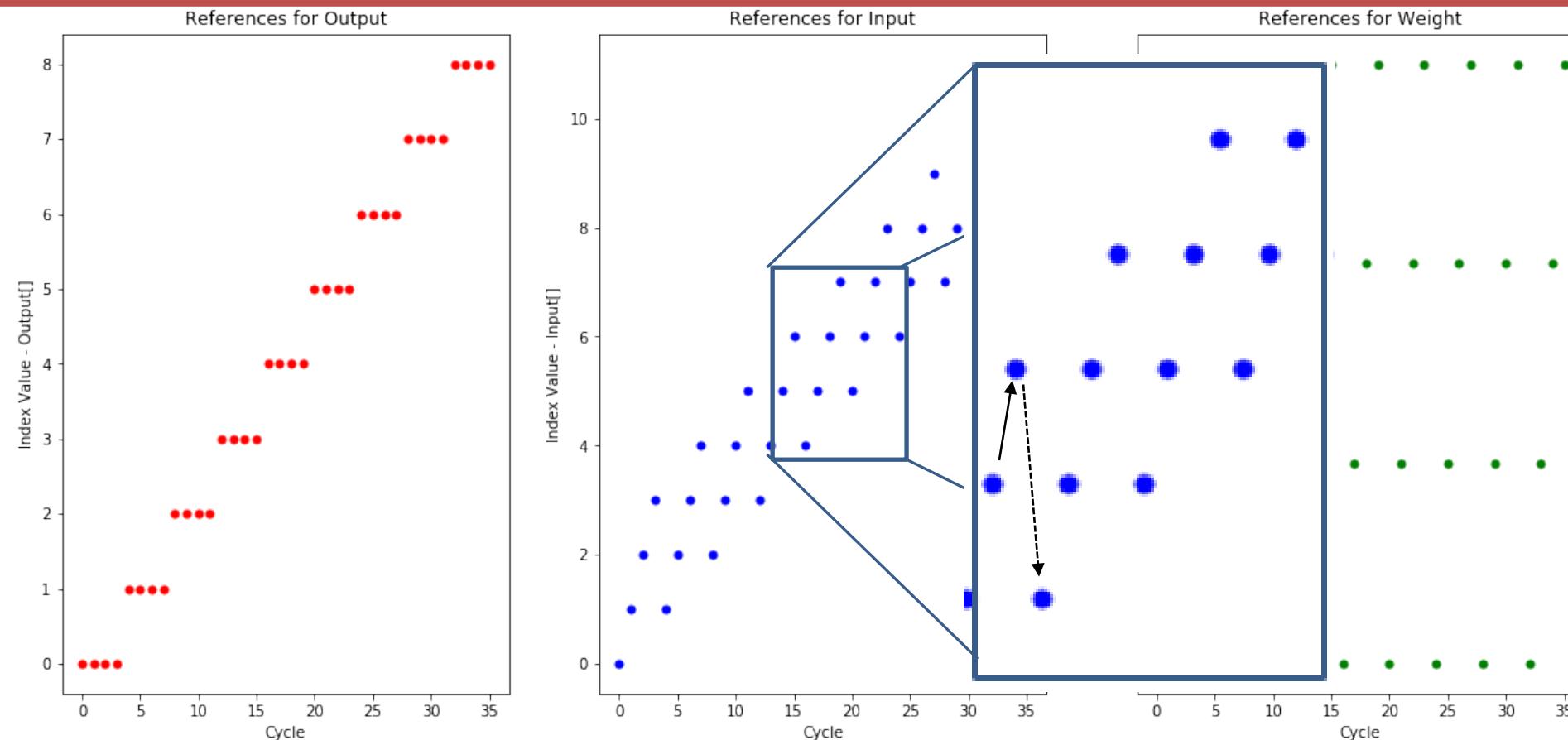
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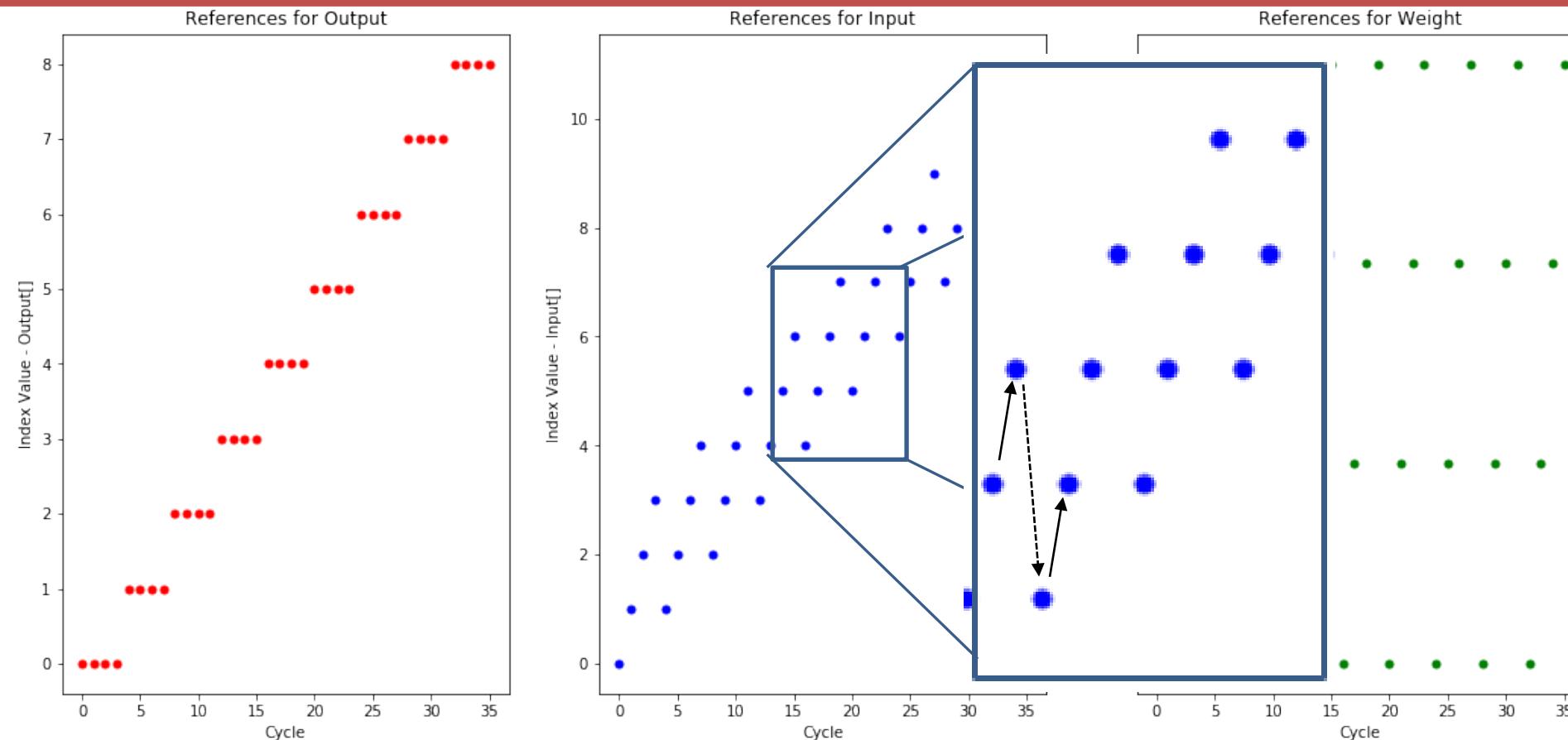
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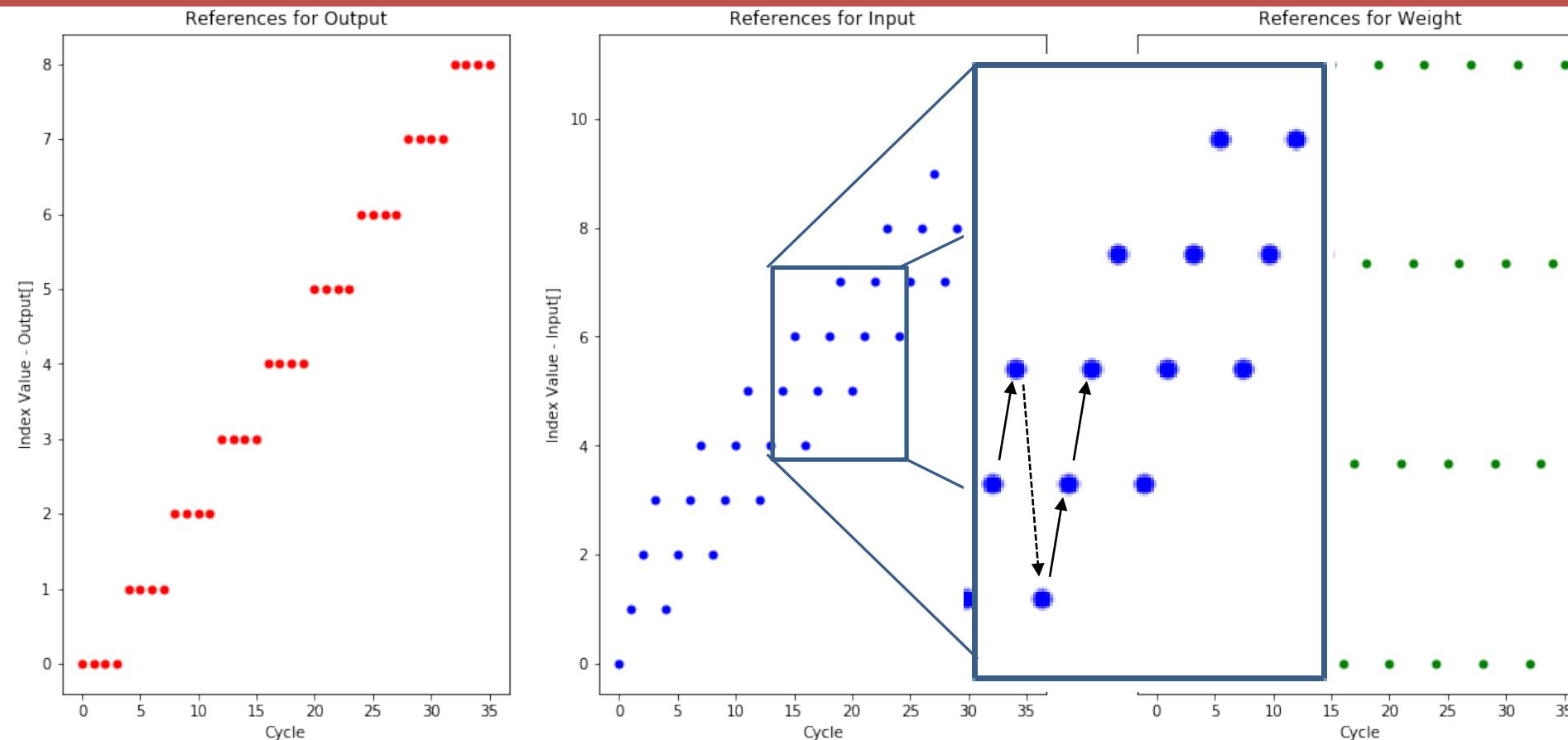
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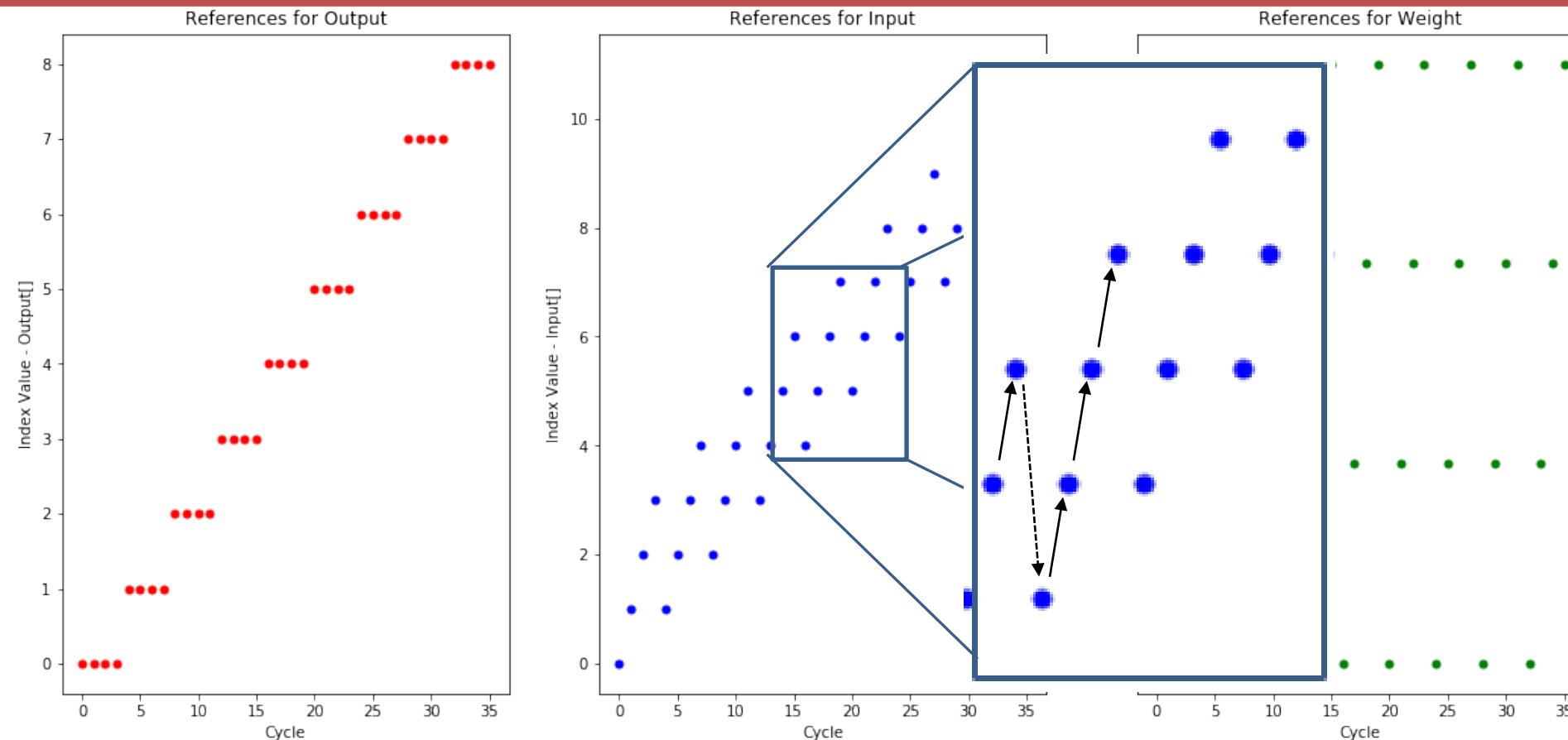
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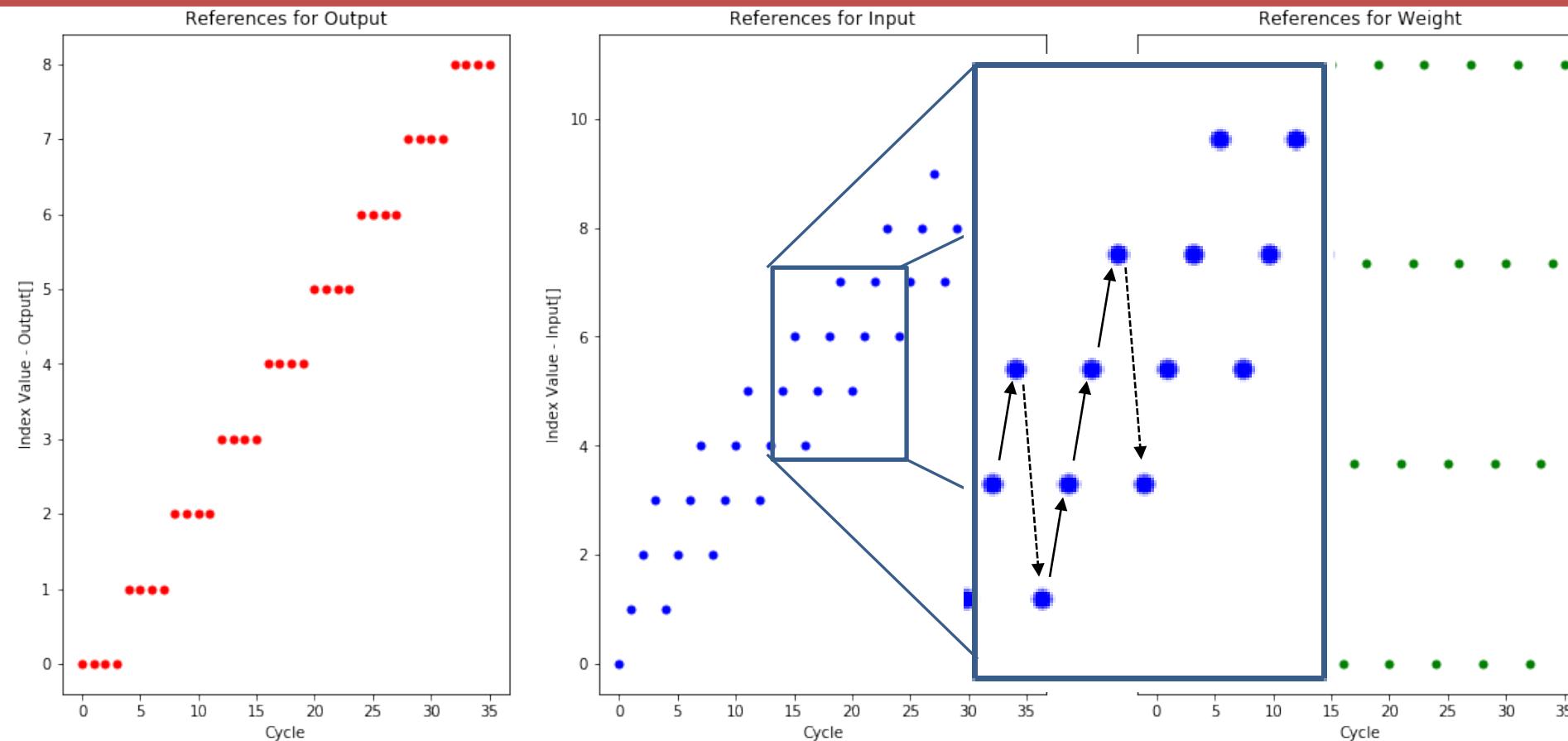
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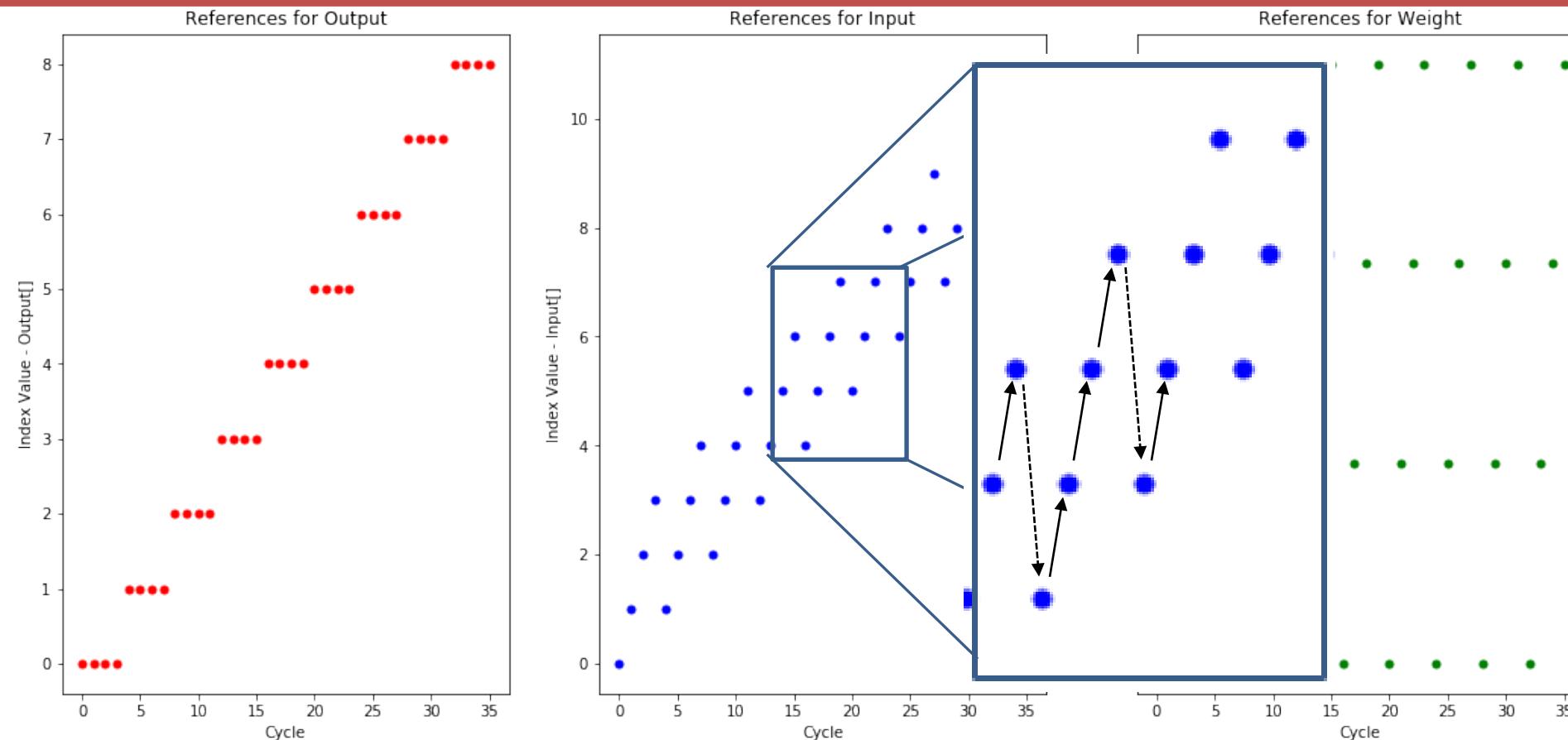
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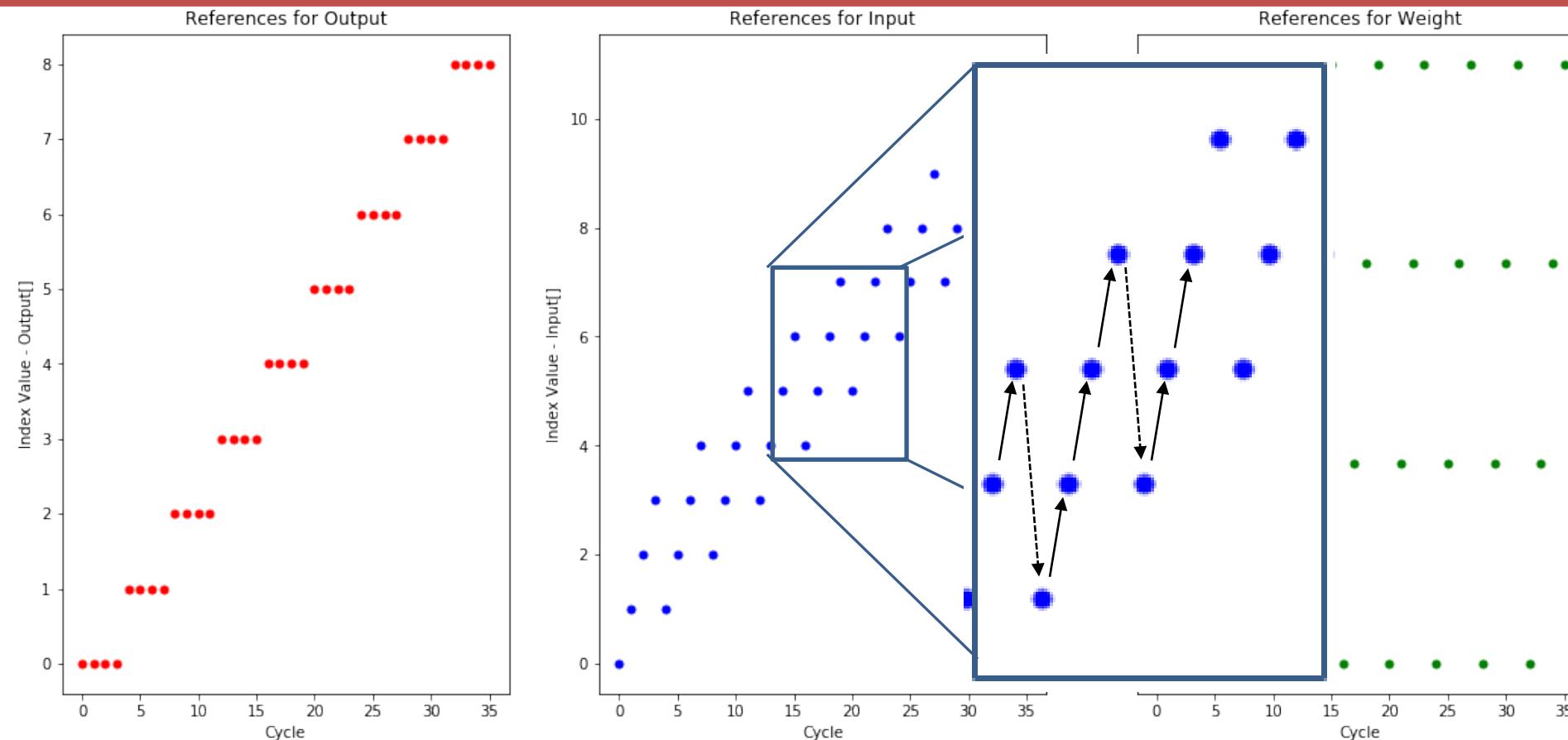
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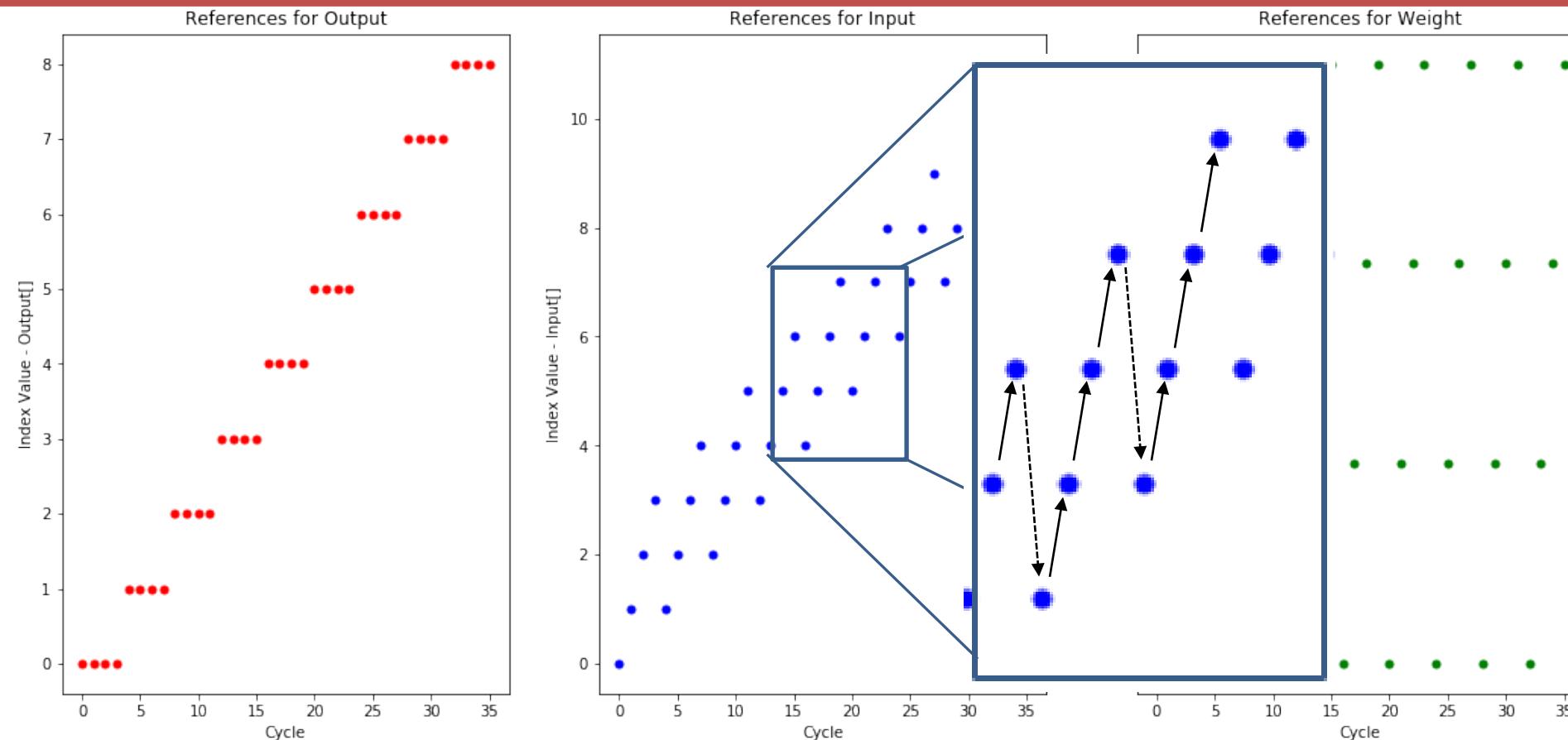
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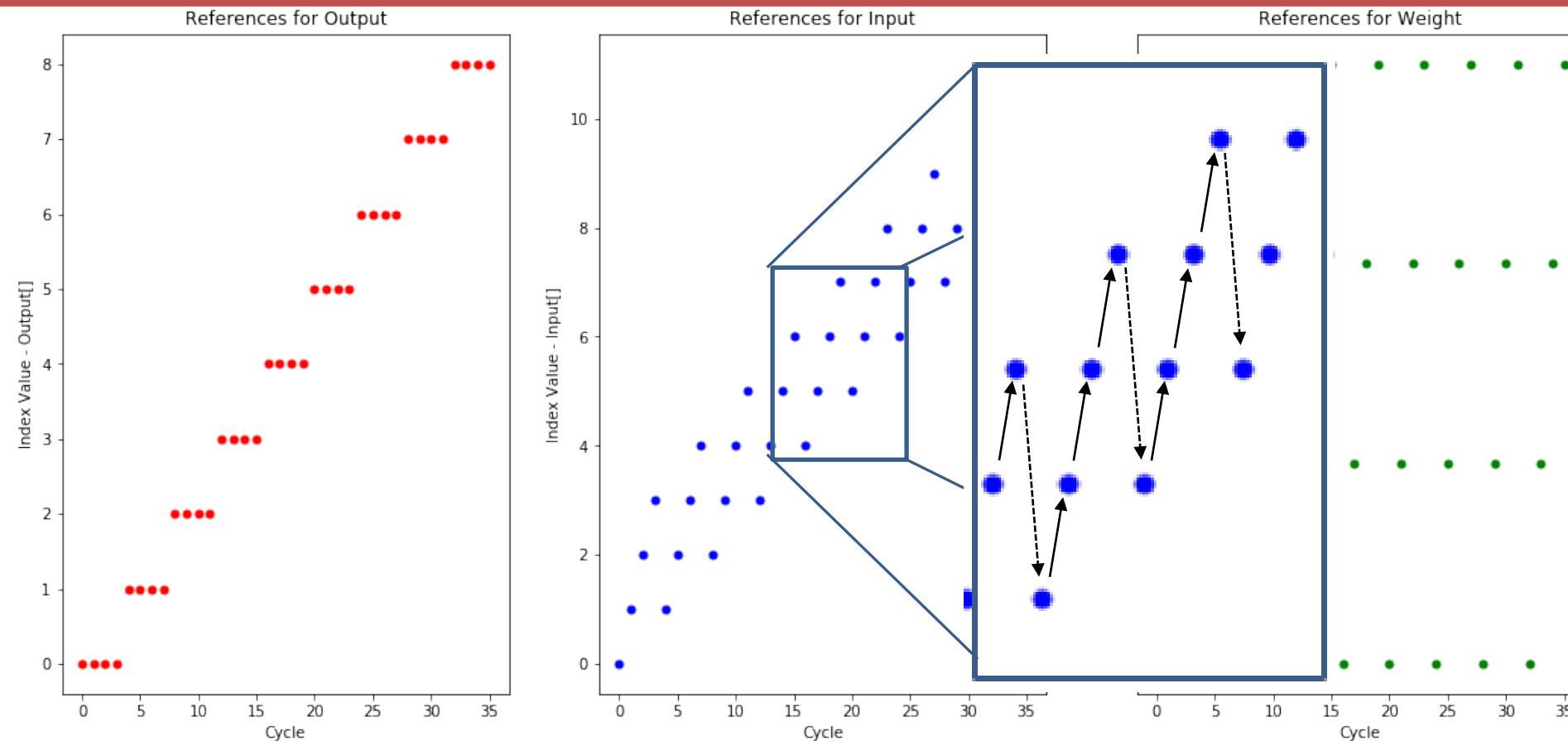
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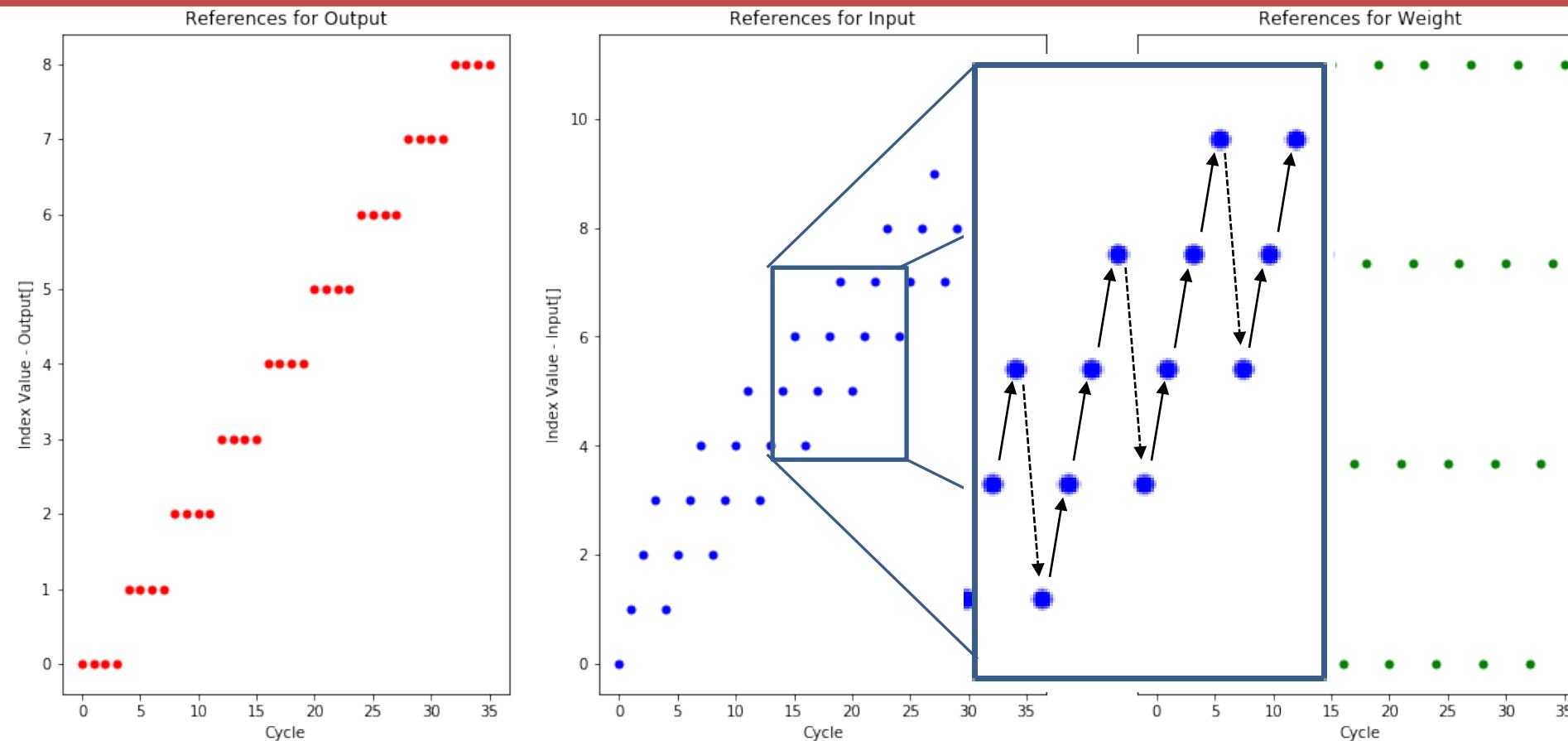
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Observations:

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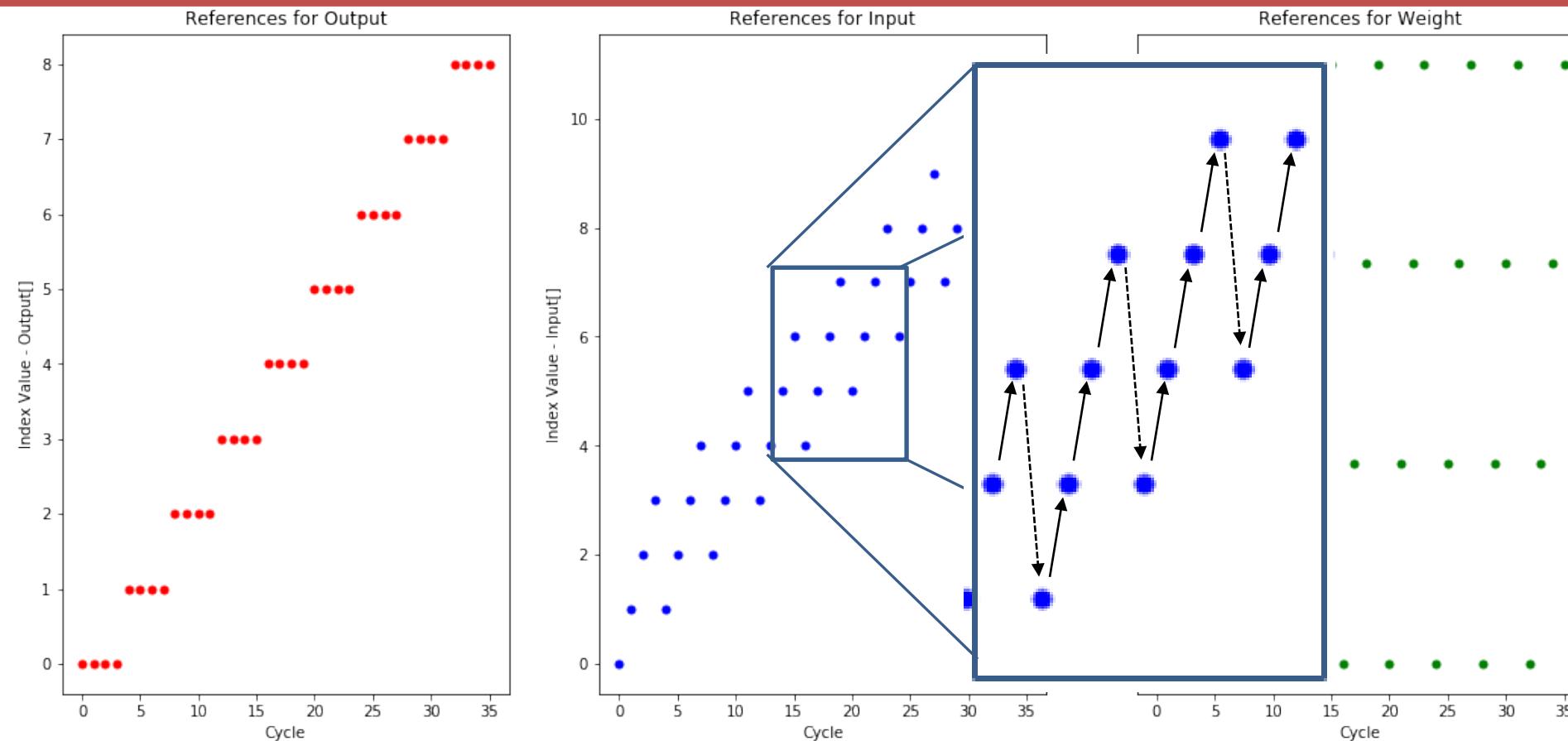
Output Stationary – Reference Pattern



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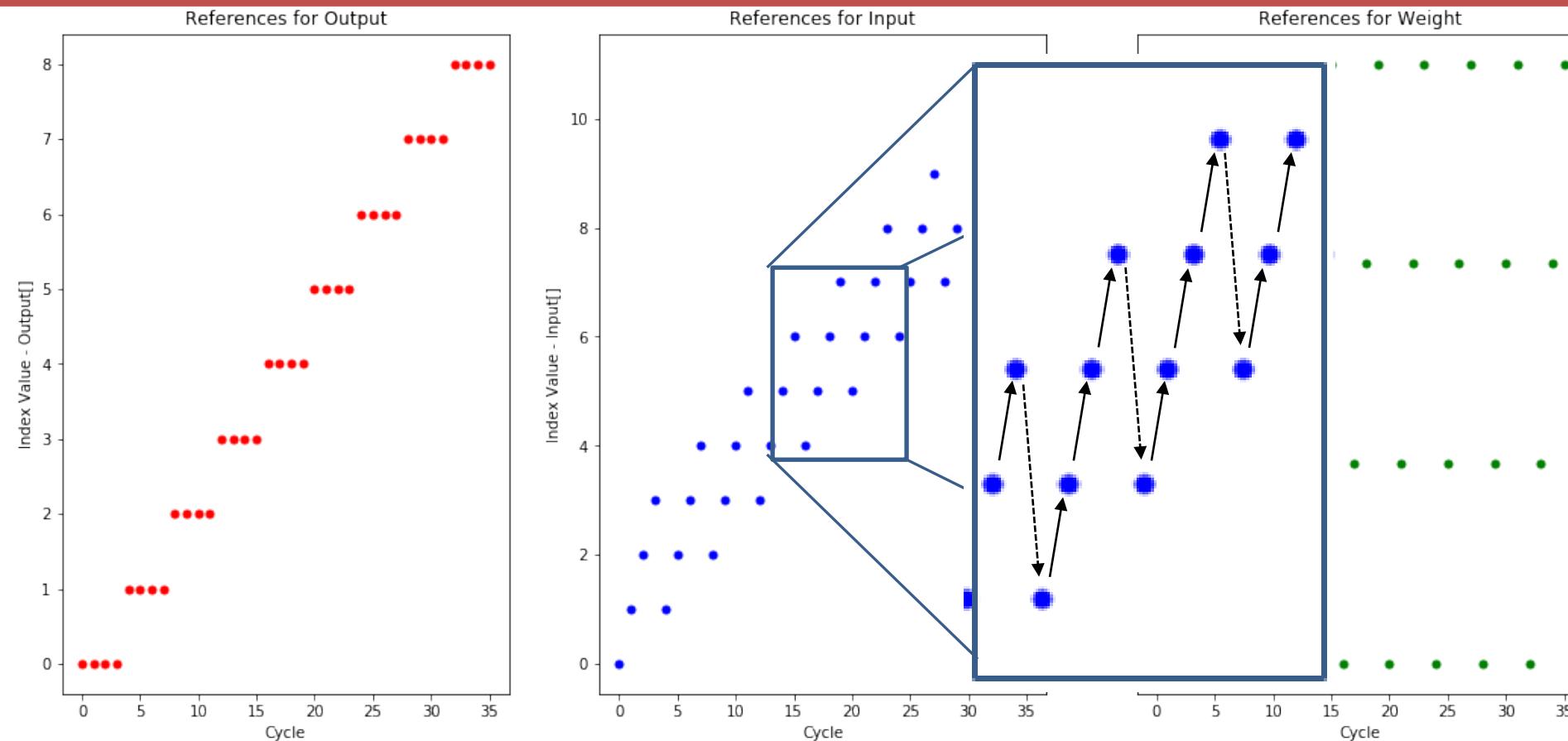
Output Stationary – Reference Pattern



Observations:

- Single **output** is reused many times (S)
- All **weights** reused repeatedly
- Sliding window of **inputs** (size = S)

Output Stationary – Reference Pattern



Observations:

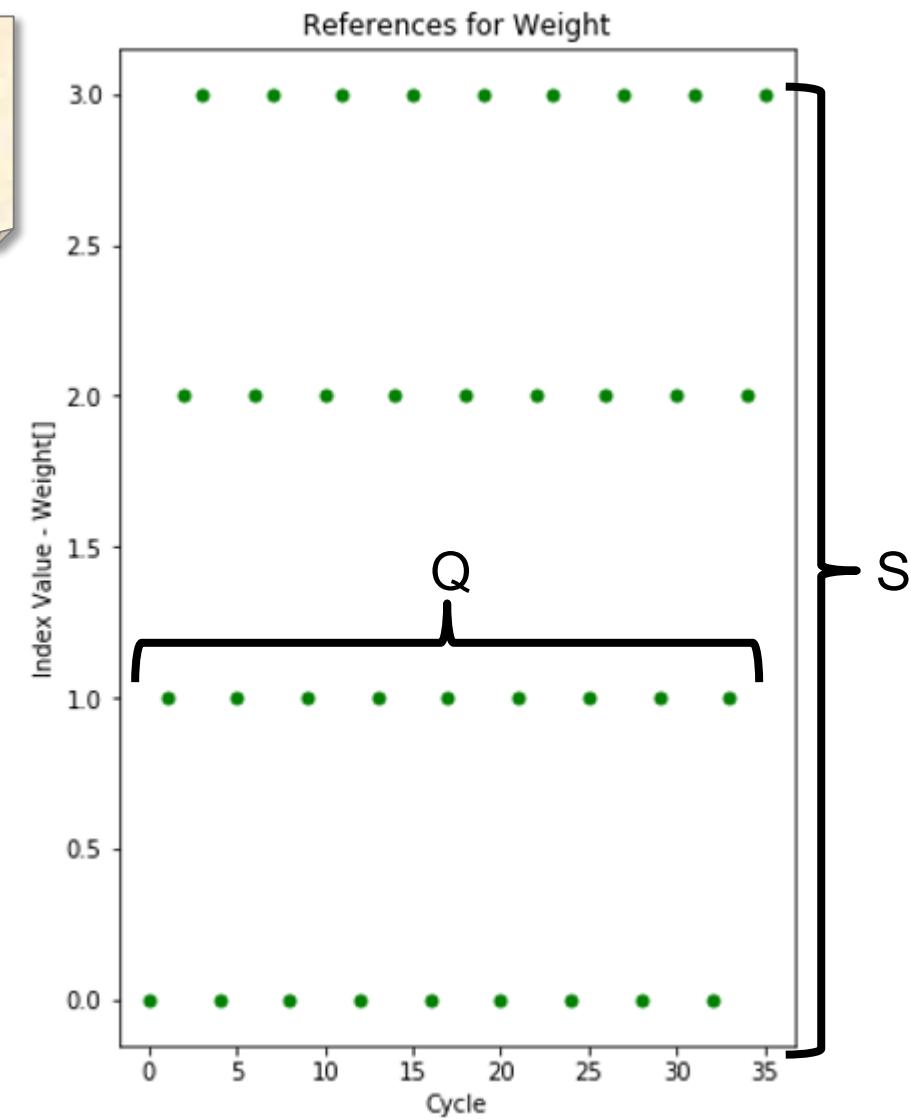
- Single **output** is reused many times (S)
- All **weights** reused repeatedly
- Sliding window of **inputs** (size = S)

L1 Data Accesses - Weights

```

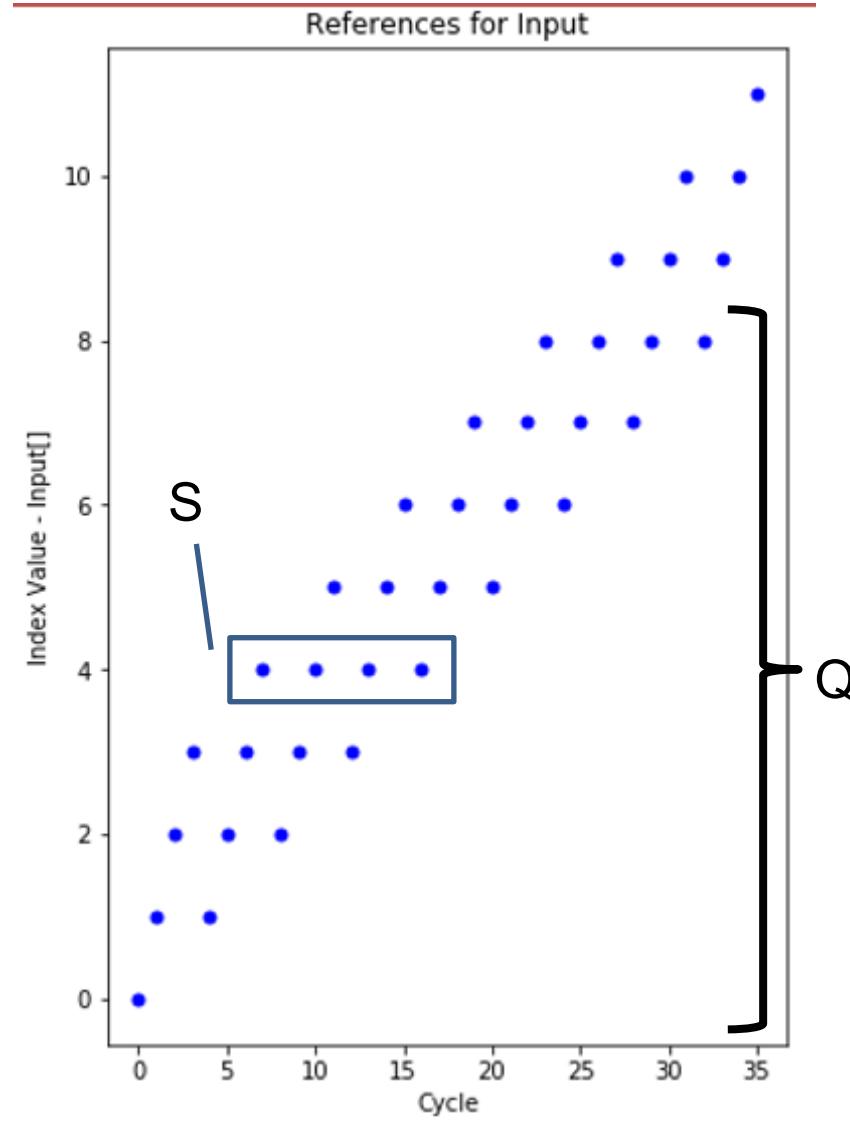
for q in [0, Q):
    for s in [0, S):
        o[q] += i[q+s]*f[s]
    
```

	OS
MACs	$Q*S$
Weight Reads	$Q*S$
Input Reads	
Output Reads	
Output Writes	



L1 Data Accesses - Inputs

	OS
MACs	Q^*S
Weight Reads	Q^*S
Input Reads	Q^*S
Output Reads	
Output Writes	

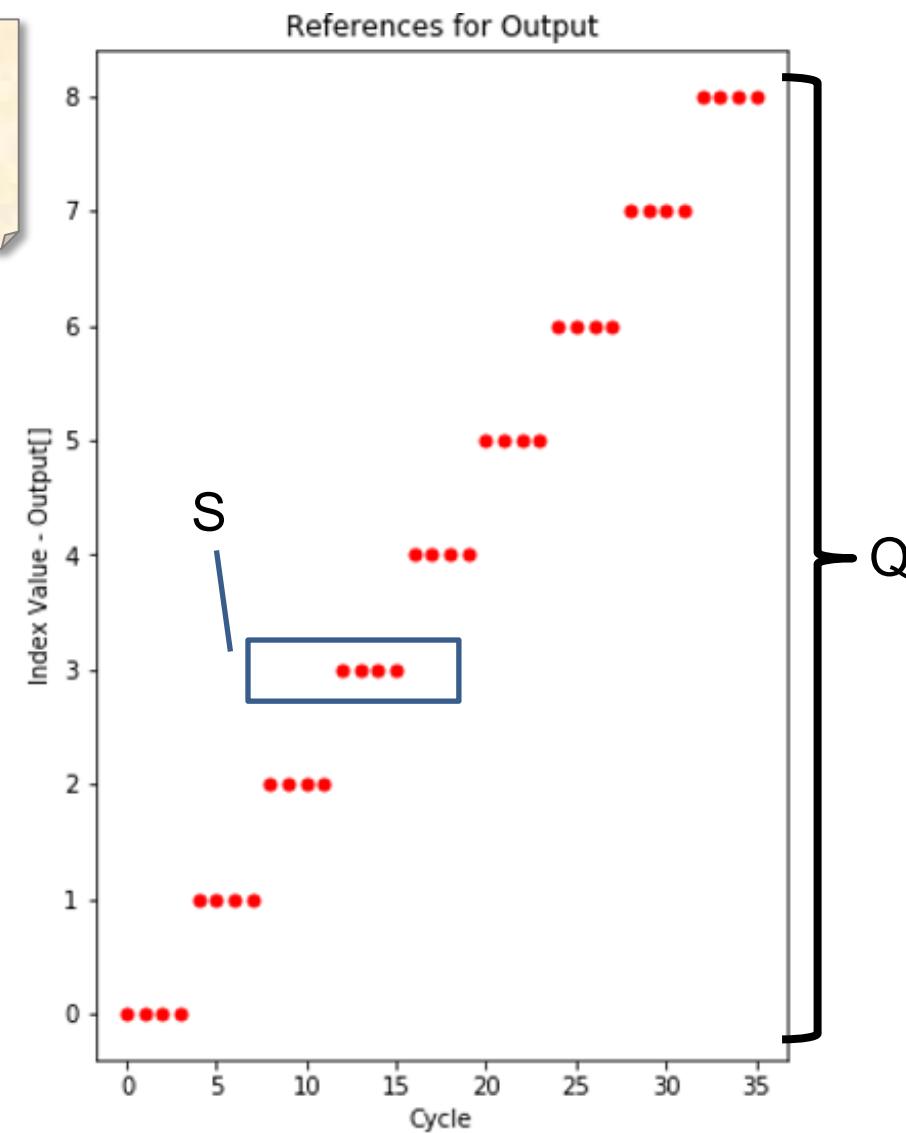


L1 Data Accesses - Outputs

```

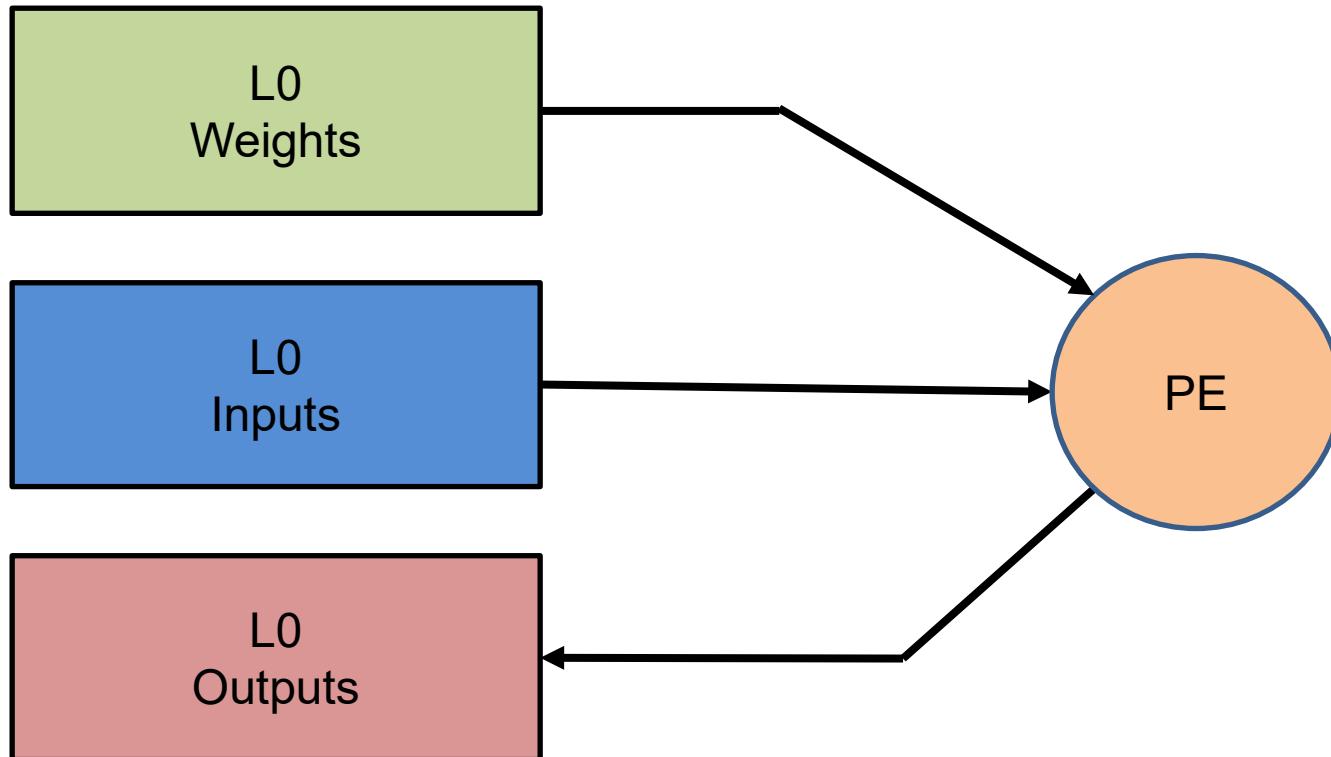
for q in [0, Q):
    for s in [0, S):
        o[q] += i[q+s]*f[s]
    
```

	OS
MACs	$Q * S$
Weight Reads	$Q * S$
Input Reads	$Q * S$
Output Reads	0
Output Writes	Q

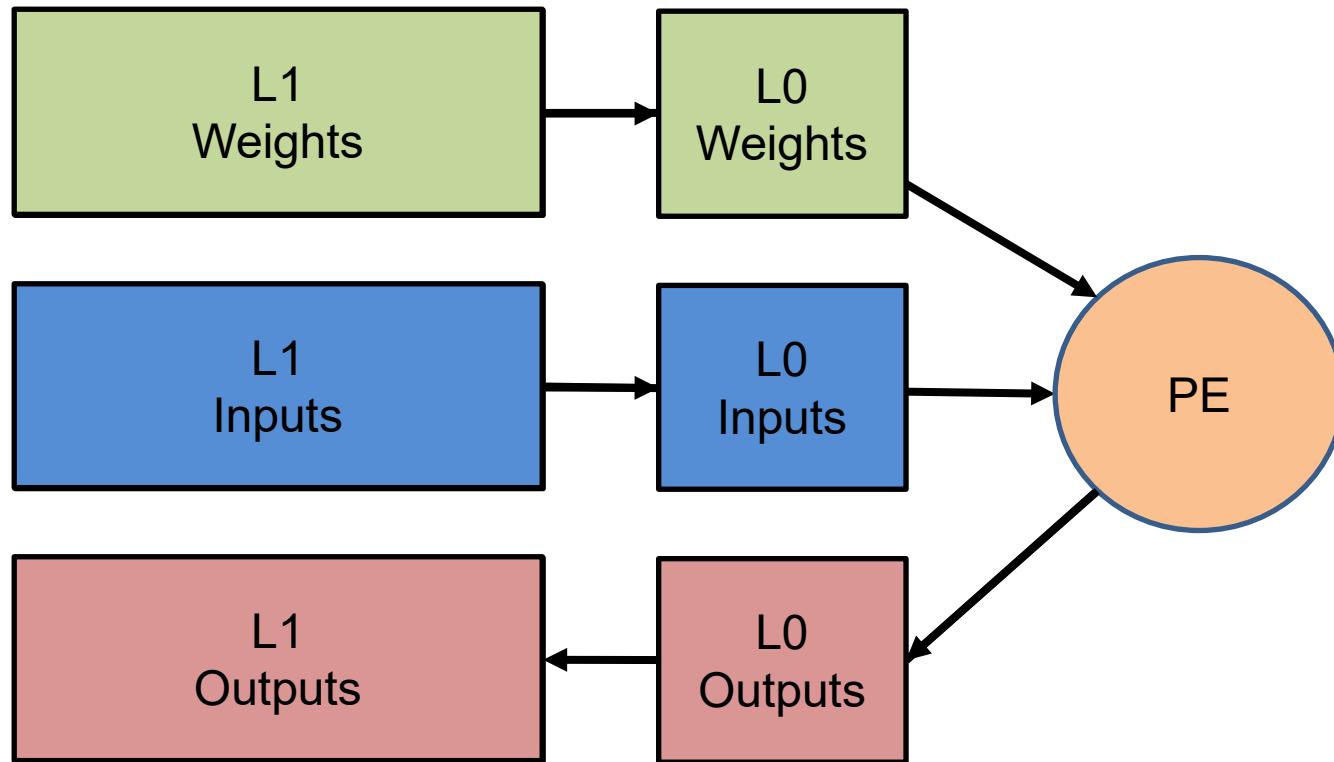


Intermediate Buffering

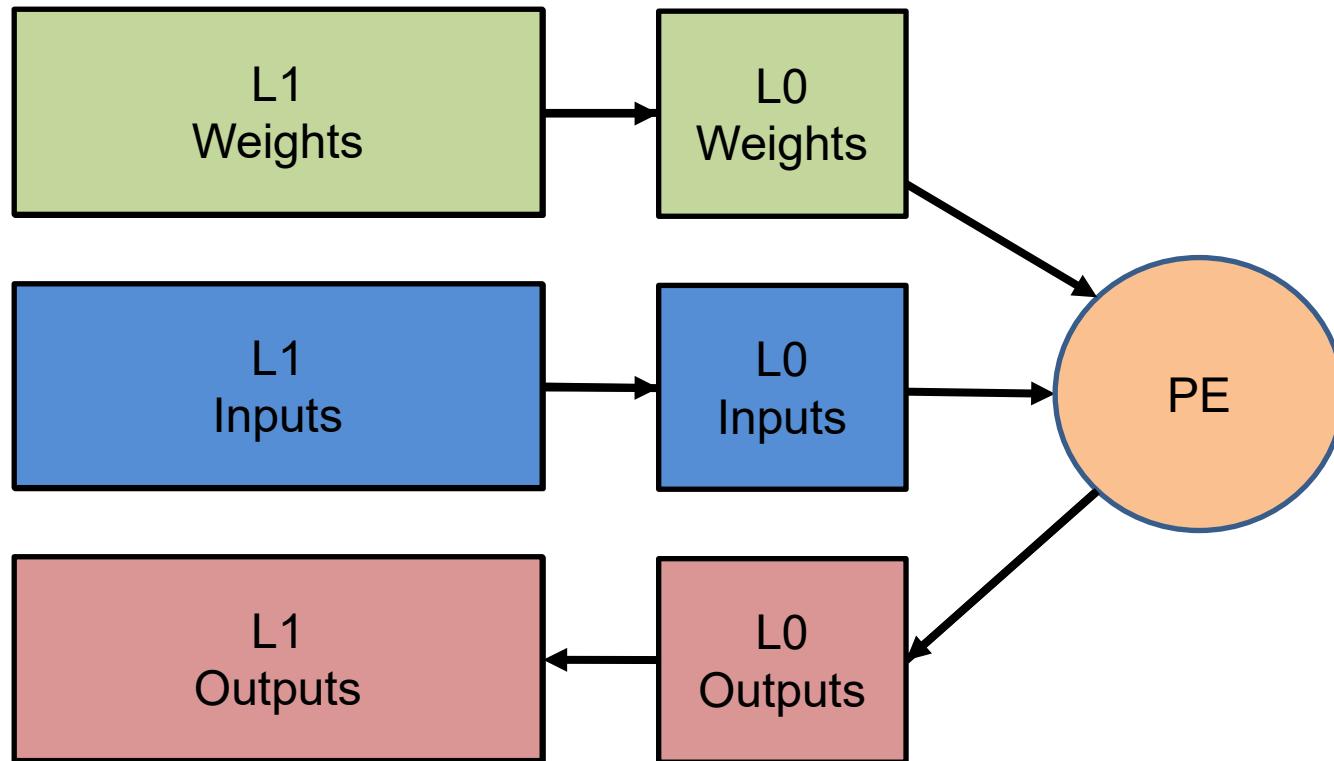
Intermediate Buffering



Intermediate Buffering

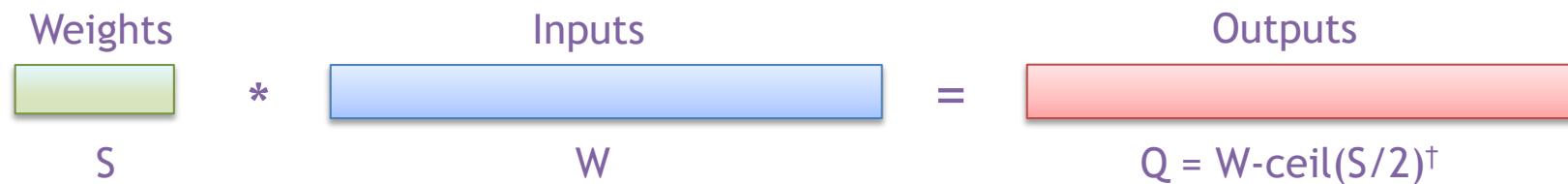


Intermediate Buffering



How will this be reflected
in the loop nest?

1-D Convolution – Buffered



```
int i[W];      # Input activations
int f[S];      # Filter Weights
int o[Q];      # Output activations

// Level 1
for q1 in [0, Q1):
    for s1 in 0, S1):
        // Level 0
        for q0 in [0, Q0):
            for s0 in [0, S0):
                o[q1*Q0+q0] += i[q1*Q0+q0 + s1*S0+s0]
                                * f[s1*S0+s0]
```

Note Q and S are factored out:
 $Q_0 \cdot Q_1 \cdot \dots$
 $S_0 \cdot S_1 \cdot \dots$

Note Q and S are factored so:
 $Q_0 * Q_1 = Q$
 $S_0 * S_1 = S$

[†] Assuming: ‘valid’ style convolution

Buffer sizes

```
// Level 1
for q1 in 0, Q1):
    for s1 in [0 to S1):
        // Level 0
        for q0 in [0, Q0):
            for s0 in [0, S0):
                o[q1*Q0+q0] += i[q1*Q0+q0 + s1*S0+s0]* f[s1*S0+s0];
}
```

Constant over each level 1 iteration

- Level 0 buffer size is volume needed in each Level 1 iteration.
- Level 1 buffer size is volume needed to be preserved and re-delivered in future (usually successive) Level 1 iterations.
- A legal assignment of loop limits will fit into the hardware's buffer sizes

Buffered – 1D Convolution Einsum

$$O_q = I_{q+s} \times F_s$$

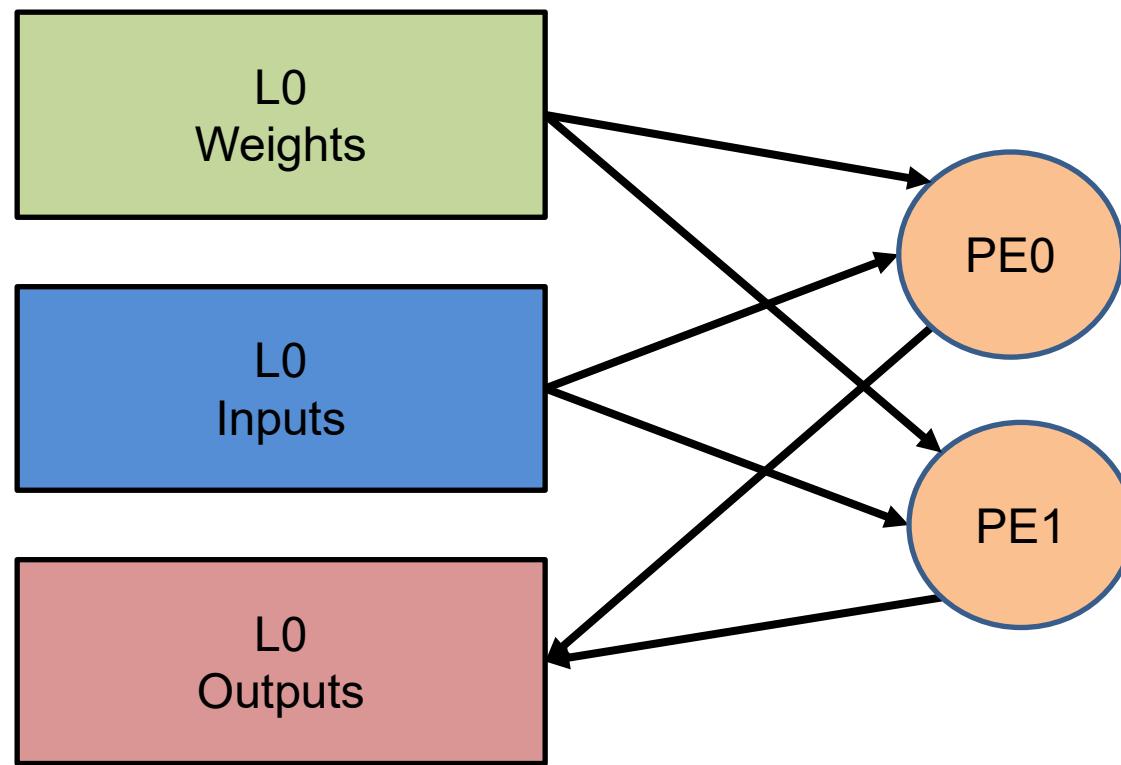
Split: S by S0 and Q by Q0

$$O_{q1*Q0+q0} = I_{q1*Q0+q0+s1*S0+s0} \times F_{S*S0+s0}$$

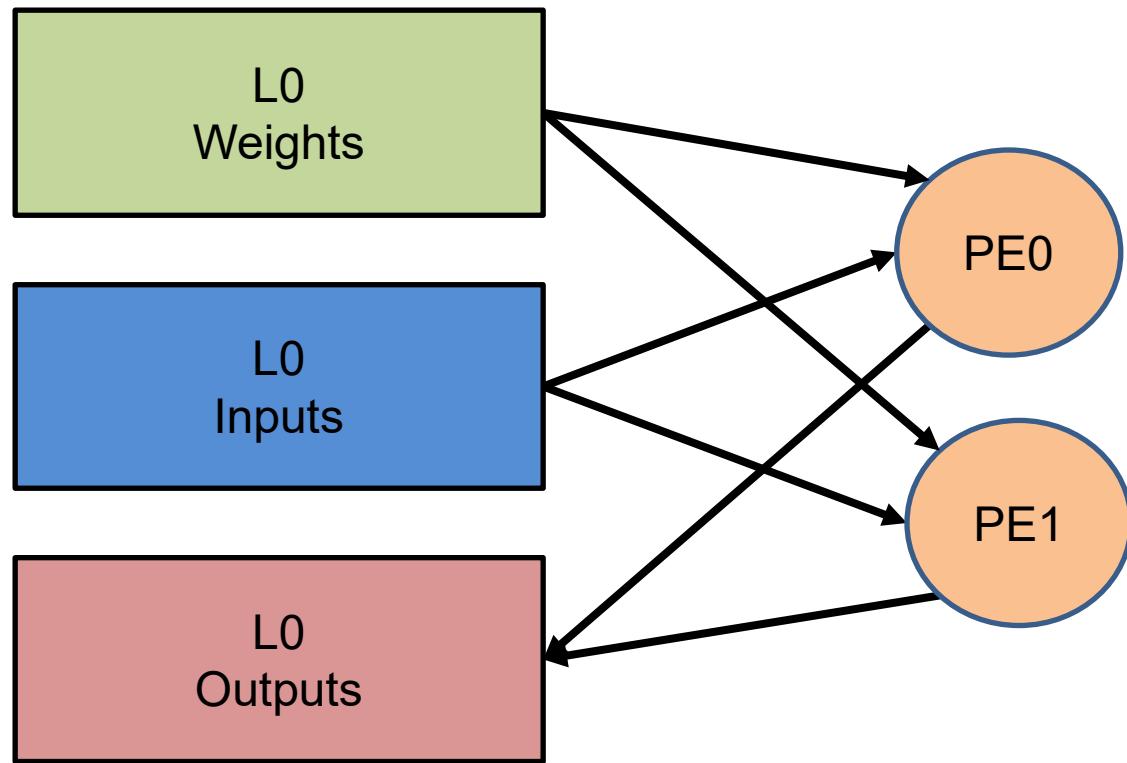
Traversal order (fastest to slowest): S0, Q0, S1, Q1

Spatial Mapping

Spatial PEs



Spatial PEs



How will this be reflected
in the loop nest?

1-D Convolution – Spatial

$$\begin{array}{ccc} \text{Weights} & & \text{Inputs} \\ \textcolor{lightgreen}{S} & * & \textcolor{lightblue}{W} \\ & & = \\ & & \text{Outputs} \\ & & \textcolor{lightred}{Q} = W \cdot \text{ceil}(S/2)^{\dagger} \end{array}$$

```
int i[W];      # Input activations
int f[S];      # Filter Weights
int o[Q];      # Output activations
```

```
// Level 1
parallel-for q1 in [0, Q1):
    parallel-for s1 in [0, S1):
        // Level 0
        for s0 in S0):
            for q0 in 0, Q0):
                o[q1*Q0+q0] += i[q1*Q0+q0 + s1*S0+s0]
                                * f[s1*S0+s0];
    }
```

Note:
 $Q_0 \cdot Q_1 = Q$
 $S_0 \cdot S_1 = S$

$Q_1 = 1 \Rightarrow q_1 = 0$

[†] Assuming: ‘valid’ style convolution

1-D Convolution – Spatial

$$\begin{array}{ccc} \text{Weights} & & \text{Inputs} \\ \textcolor{lightgreen}{S} & * & \textcolor{lightblue}{W} \\ & & = \\ & & \text{Outputs} \\ & & \textcolor{lightred}{Q} = W \cdot \text{ceil}(S/2)^{\dagger} \end{array}$$

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parallel for q1 in [0, Q1]:
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        // Level 0
        for s0 in S0):
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                o[q1*Q0+q0] += i[q1*Q0+q0 + s1*S0+s0]
                                * f[s1*S0+s0];
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$Q_1 = 1 \Rightarrow q1 = 0$

[†] Assuming: ‘valid’ style convolution

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                o[q1*Q0+q0] += i[q1*Q0+q0 + s1*S0+s0]
                                * f[s1*S0+s0];
    }
```

Note:
 $Q_0 \cdot Q_1 = Q$
 $S_0 \cdot S_1 = S$

$Q_1 = 1 \Rightarrow q_1 = 0$

$S_0 = 1, S_1 = 2$

[†] Assuming: ‘valid’ style convolution

1-D Convolution – Spatial



```
int i[W];      # Input activations  
int f[S];      # Filter Weights  
int o[Q];      # Output activations
```

```
// Level 1
parallel-for s1 in [0, S1):
    // Level 0
    for s0 in [0, S0):
        for q in [0, Q):
            o[q] += i[q+s1*s0+s0] * f[s1*s0+s0]
```

Note:
 $Q_0^* Q_1 = Q$
 $S_0^* S_1 = S$

[†] Assuming: ‘valid’ style convolution

Spatial – 1D Convolution Einsum

$$O_q = I_{q+s} \times F_s$$

Split: S by S0

$$O_q = I_{q+s1*s1+s0} \times F_{s*s0+s0}$$

Traversal order (fastest to slowest): S0, Q

Spatial – 1D Convolution Einsum

$$O_q = I_{q+s} \times F_s$$

Split: S by S0

$$O_q = I_{q+s1*s1+s0} \times F_{s*s0+s0}$$

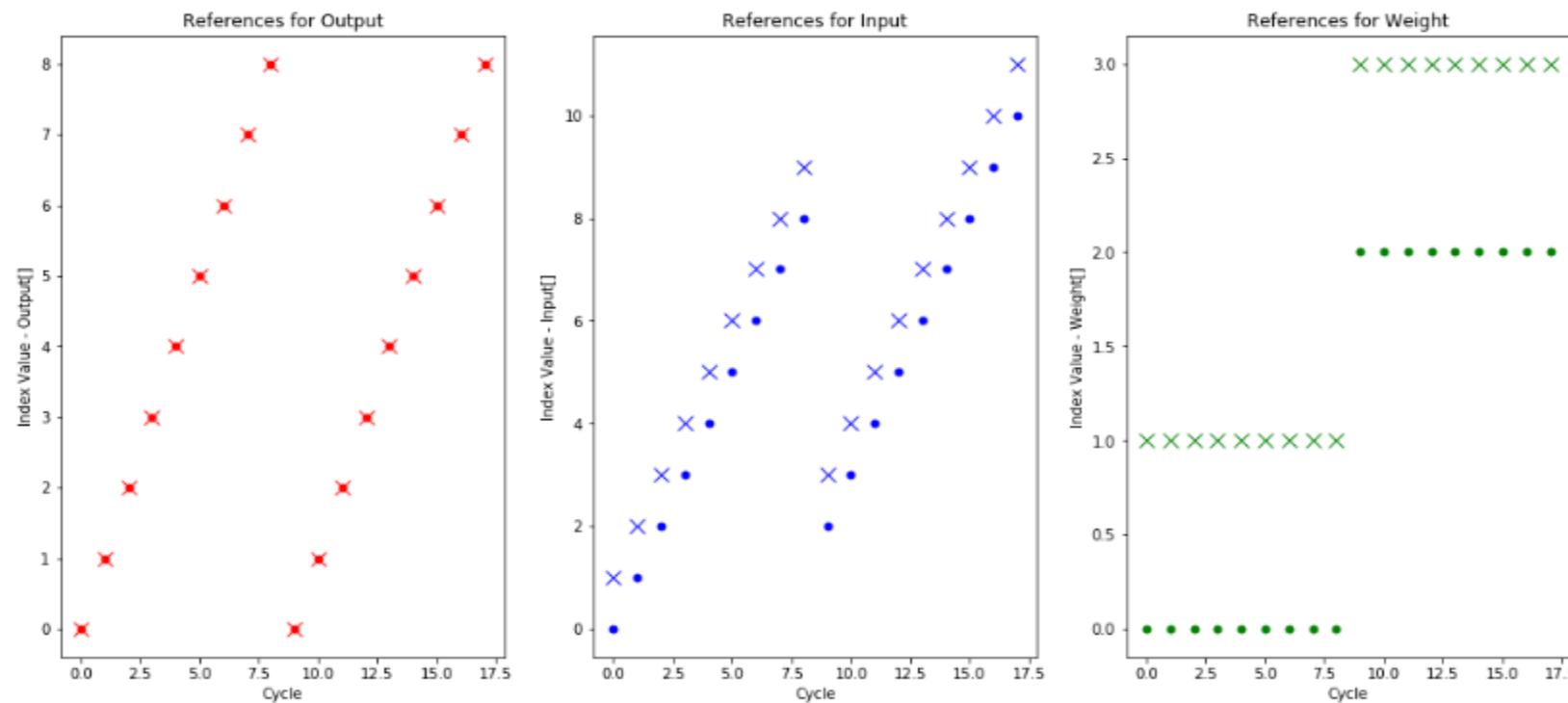
Traversal order (fastest to slowest): S0, Q

Parallel: S1

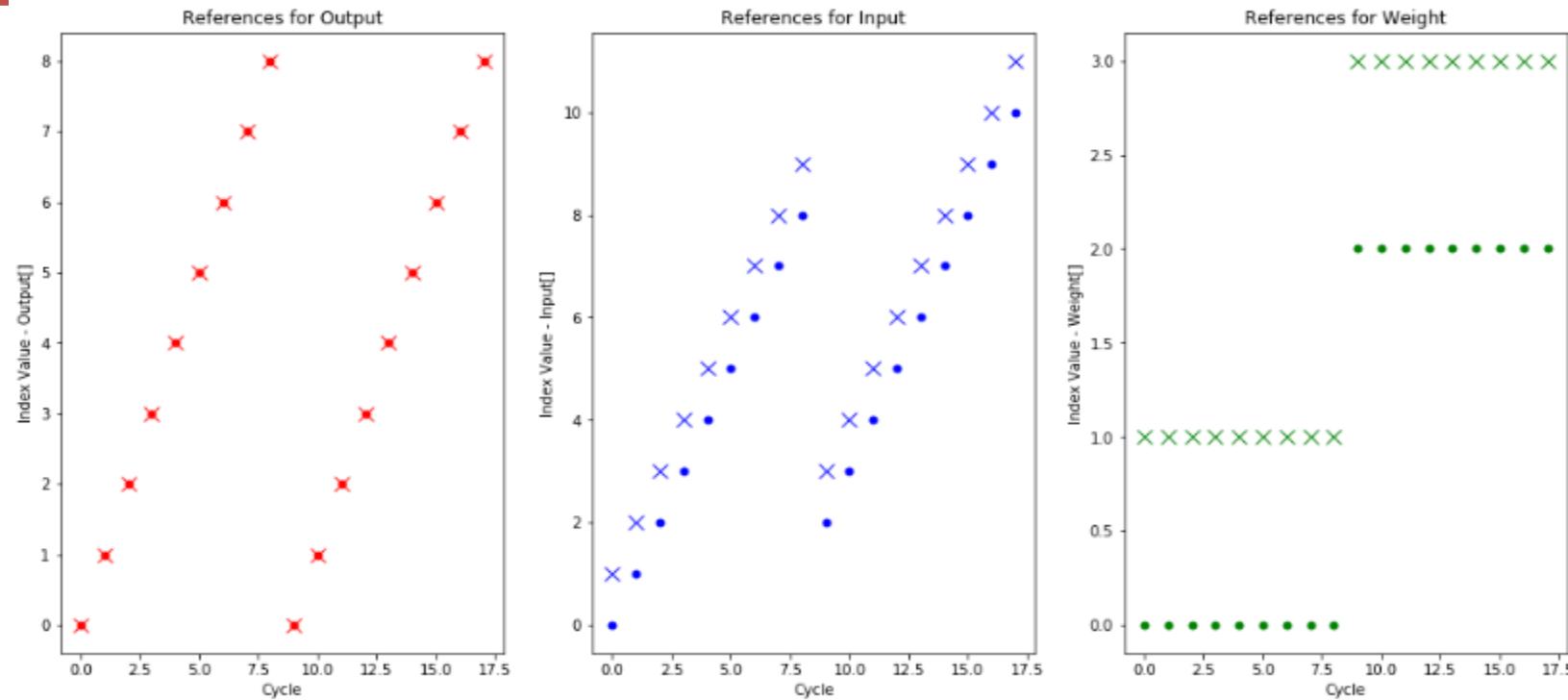
Spatial Weight Stationary References

Shape: - $S = 4$
- $Q = 9$
- $W = 12$

Loop limits: - $S_1 = 2$
- $S_0 = 2$
- $Q_0 = 9$

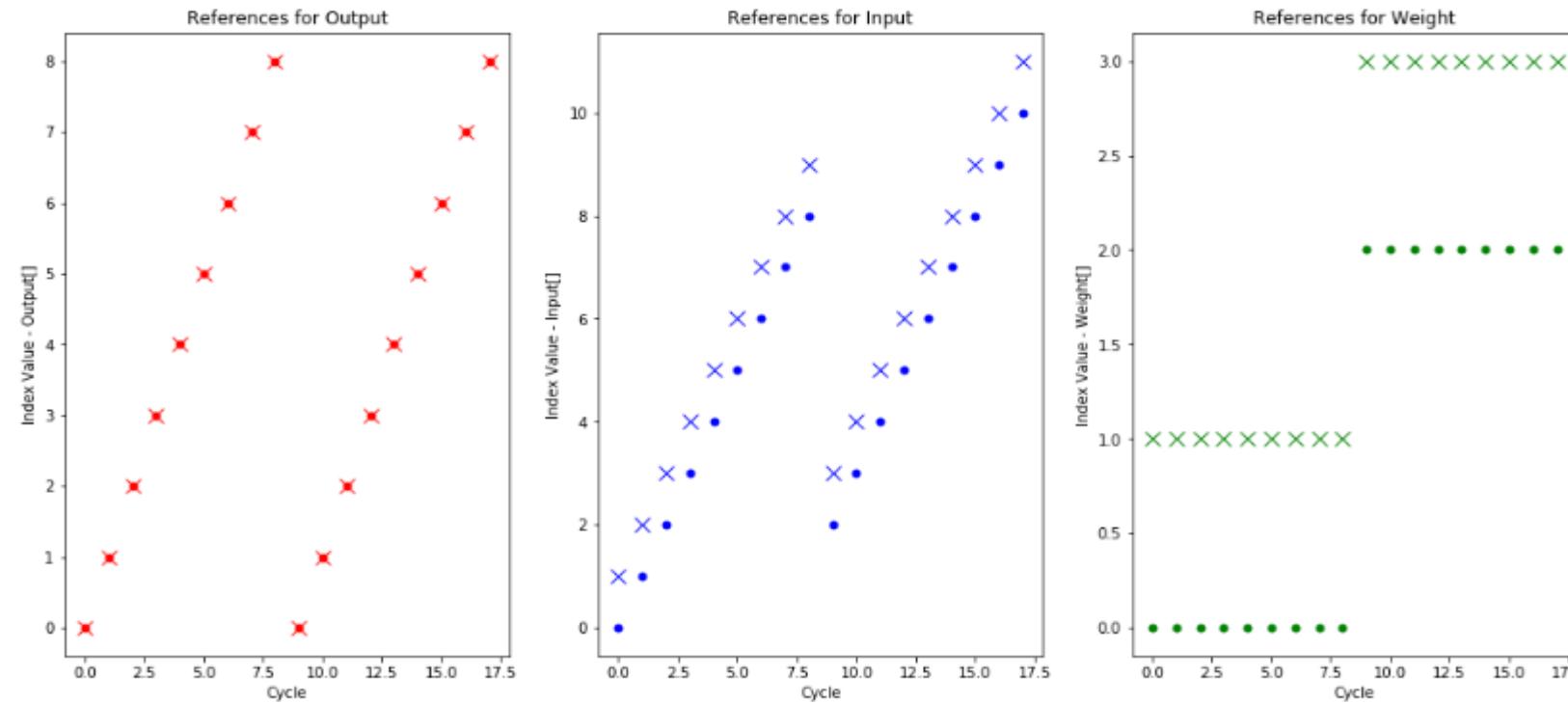


Spatial Weight Stationary References



$$\begin{aligned} S1 &= 2 \\ S0 &= 2 \\ Q0 &= 9 \end{aligned}$$

Spatial Weight Stationary References



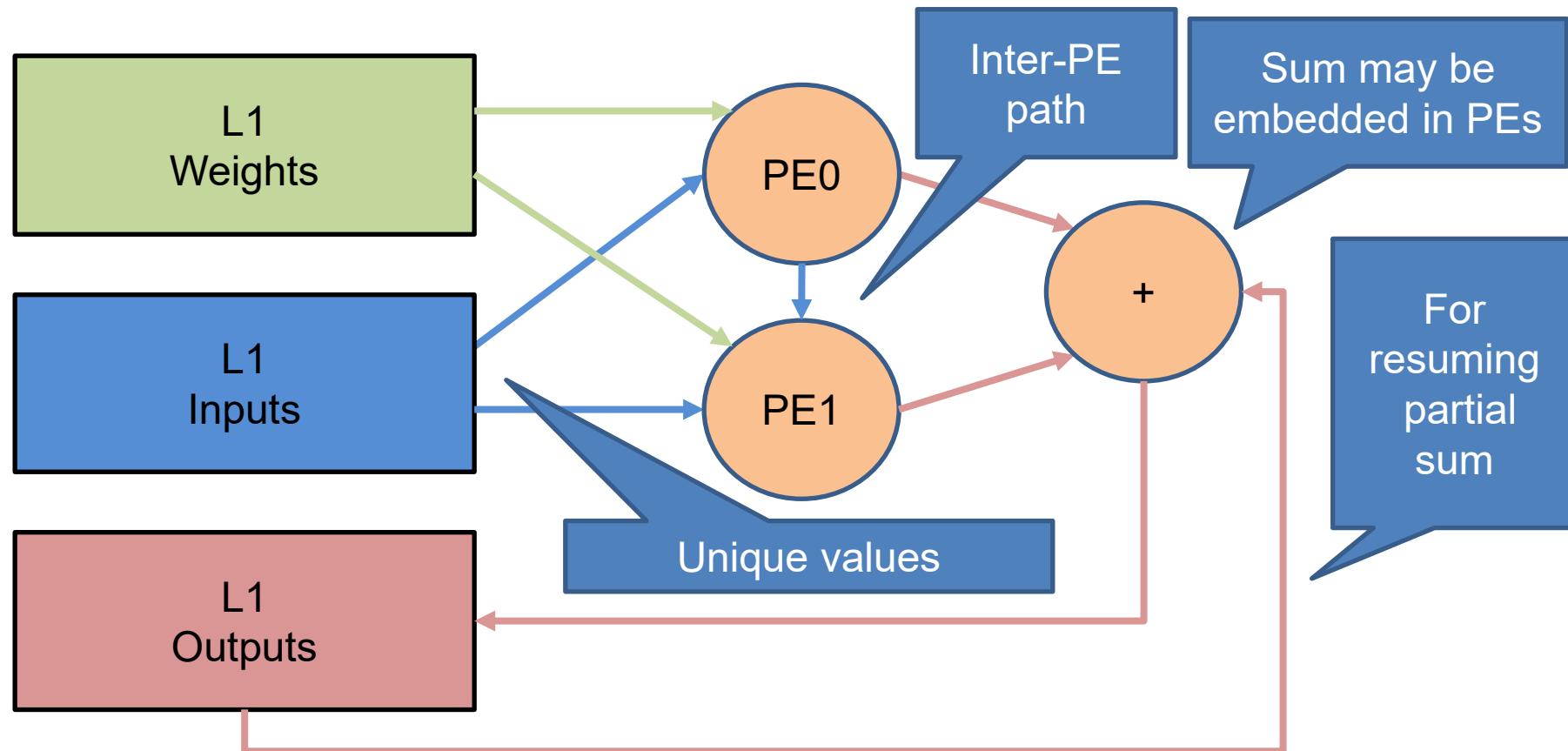
- Number of cycles half that of before
- Single **weight** per PE used for a long time (unicast to each PE)
- Inputs** reused in next cycle (opportunity for inter-PE communication)
- Inputs** are also reused after a long interval, implying a large window (Q)
- Partial sums** are reused in same cycle (opportunity for spatial sum)
- Partial sums** reused after a long interval, very large window (size = Q)

$$S1 = 2$$

$$S0 = 2$$

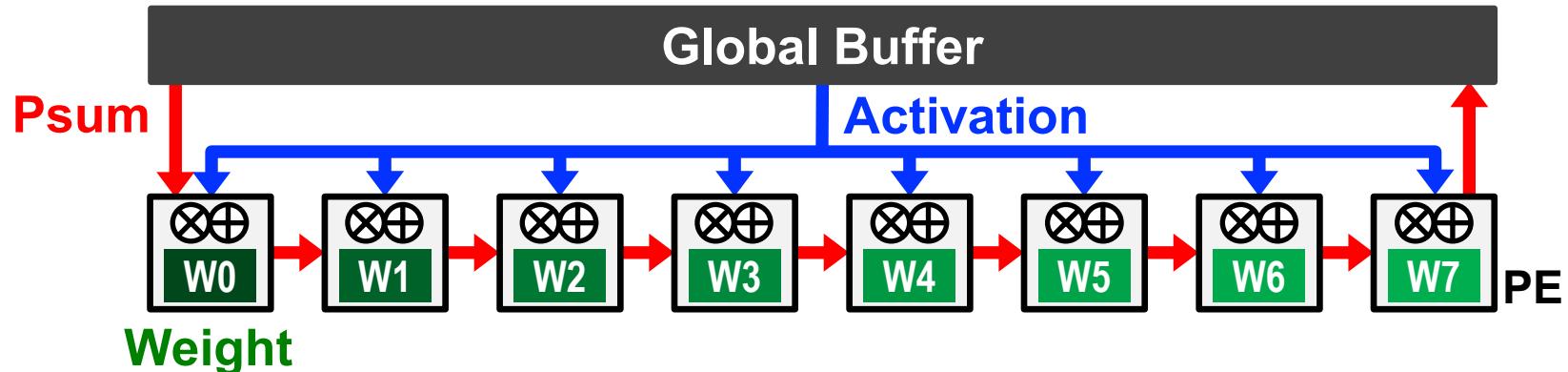
$$Q0 = 9$$

Spatial PEs



What if hardware cannot do a spatial sum?

Weight Stationary (WS)



- Note that activations are multi-cast.
- To achieve this behavior we need to “skew” the activity in the PEs so instead of needing activation in adjacent cycles there are needed in the same cycle!

Buffered – 1D Convolution Einsum

$$O_q = I_{q+s} \times F_s$$

Split: S by S0

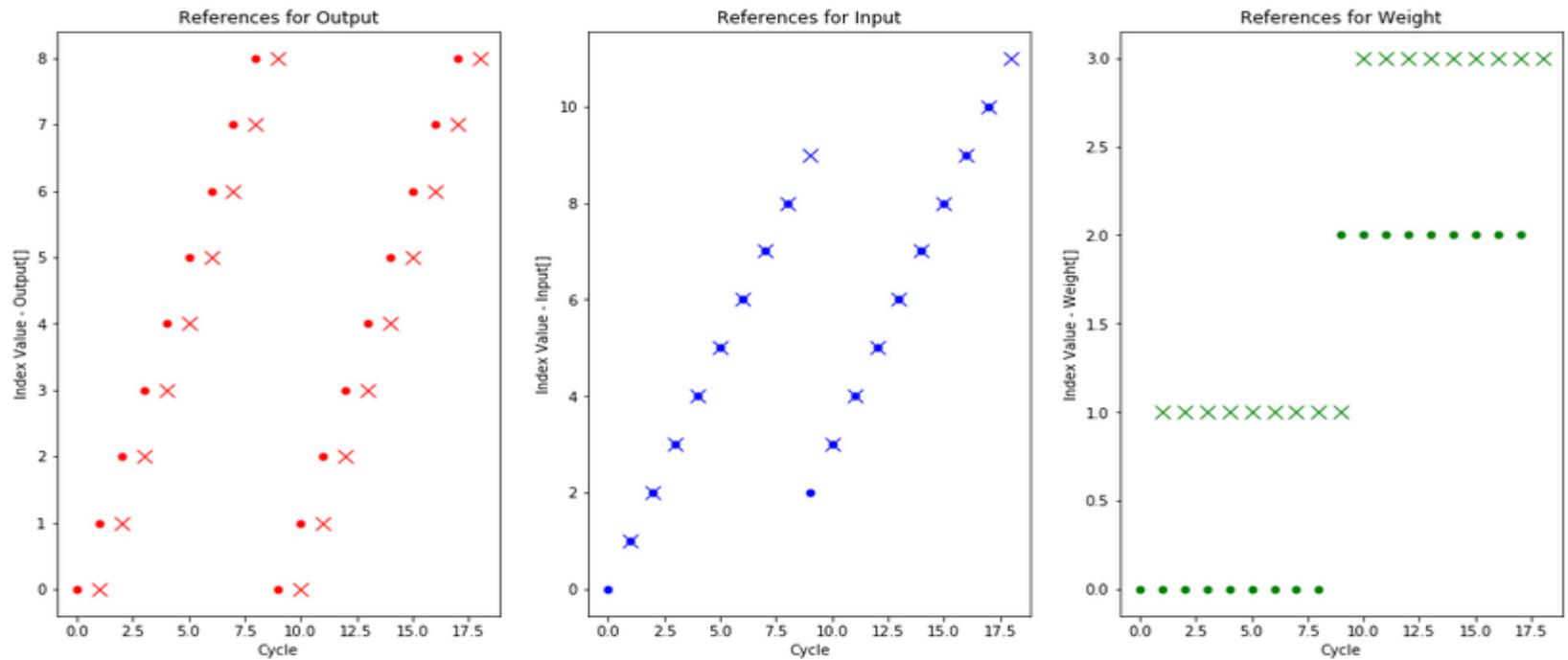
$$O_q = I_{q+s1*s1+s0} \times F_{s*s0+s0}$$

Traversal order (fastest to slowest): S0, Q

Parallel: S1

Time Skew: +s1

Spatial Weight Stationary (Skewed)



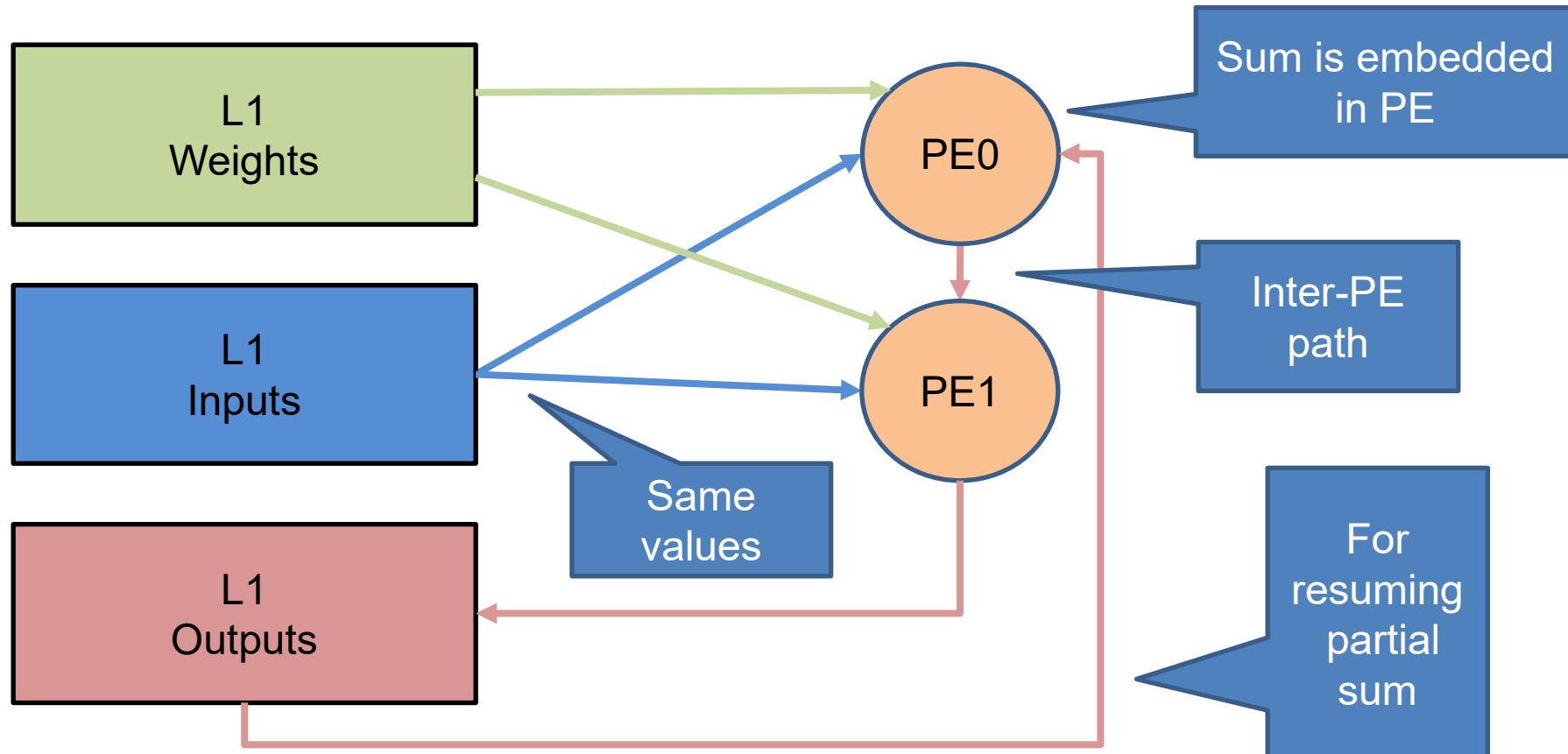
$$S1 = 2$$

$$S0 = 2$$

$$Q0 = 9$$

- Single **weight** per PE used for a long time (unicast to each PE)
- **Inputs** used **simultaneously** at both PEs (opportunity for multicast)
- **Inputs** are also reused after a long interval, implying a large window (Q)
- **Partial sums** are reused are **reused in adjacent cycles** in adjacent PEs
opportunity for inter-PE communication and temporal sum
- **Partial sums** reused after a long interval, very large window (size = Q)

Spatial PEs



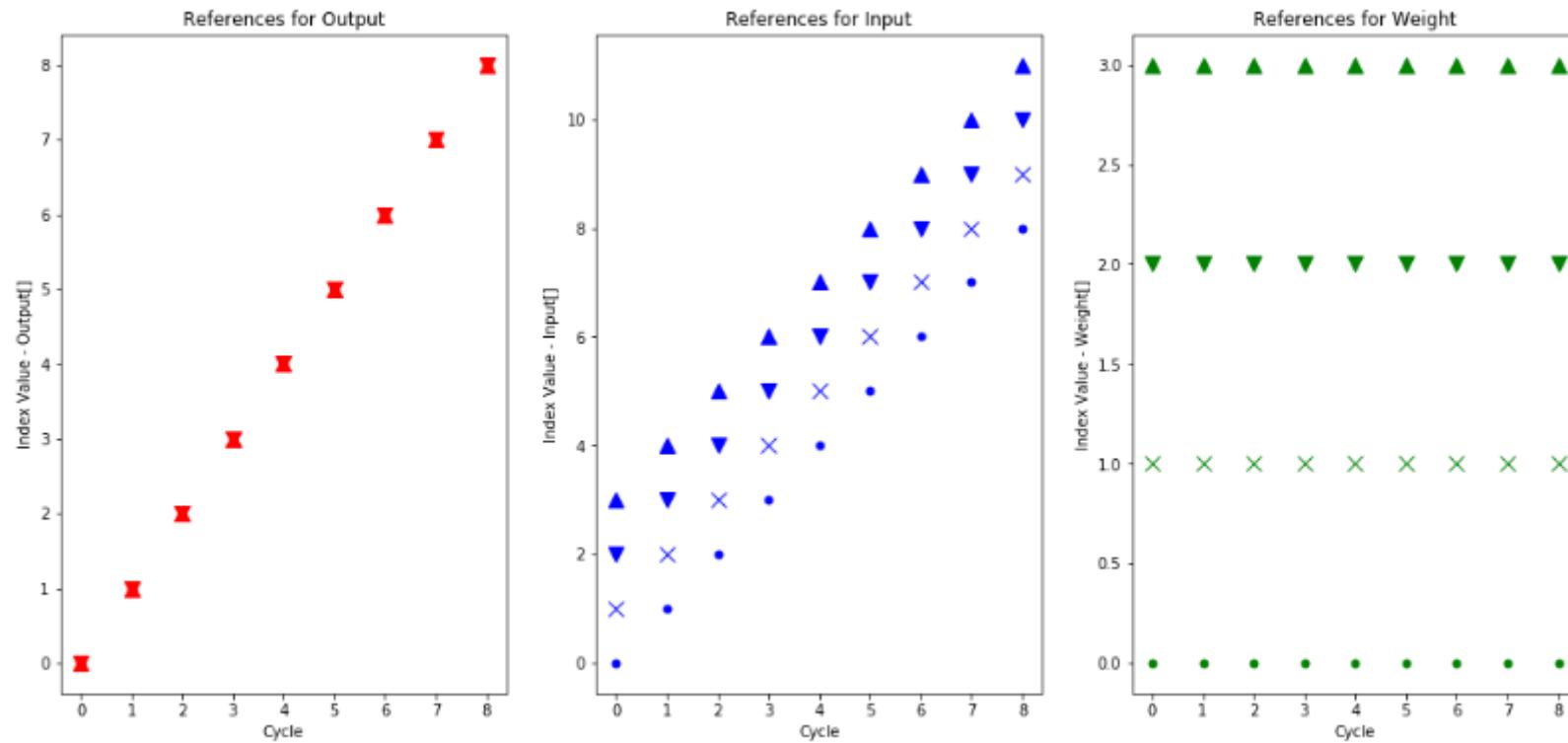
Weights are still unique, but note lower bandwidth and bursty demand.

Is there a way to avoid the large input and psum window?

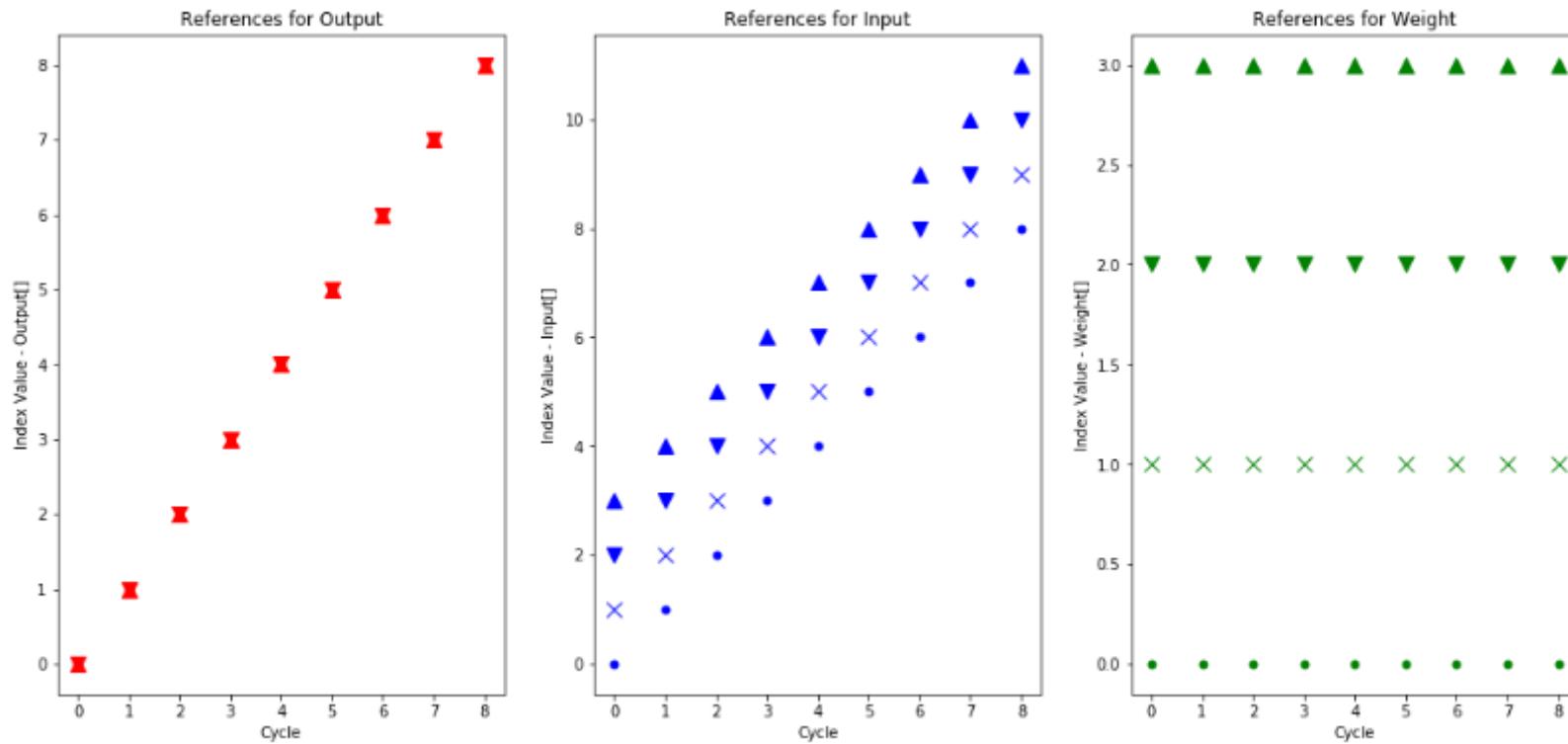
With $S1 = S$

Shape: - $S = 4$
 - $Q = 9$
 - $W = 12$

Loop limits: - $S1 = 4$
 - $S0 = 1$
 - $Q0 = 9$



With $S1 = S$



Mapping Process

Mapping

Definition: selecting the placement and scheduling in space and time of every operation (including delivering the appropriate operands) required for a DNN computation onto the hardware function units of the accelerator.

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Steps: Within the constraints of the hardware, select for each level of the storage hierarchy:

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- A dataflow (**for** loop order)

Mapping

Definition: selecting the placement and scheduling in space and time of every operation (including delivering the appropriate operands) required for a DNN computation onto the hardware function units of the accelerator.

Steps: Within the constraints of the hardware, select for each level of the storage hierarchy:

- A dataflow (**for** loop order)
- A partitioning (**for** loop limits) both spatial and temporal

Mapping

Definition: selecting the placement and scheduling in space and time of every operation (including delivering the appropriate operands) required for a DNN computation onto the hardware function units of the accelerator.

Steps: Within the constraints of the hardware, select for each level of the storage hierarchy:

- A dataflow (**for** loop order)
- A partitioning (**for** loop limits) both spatial and temporal
- Other behavioral details..., e.g., bypassing

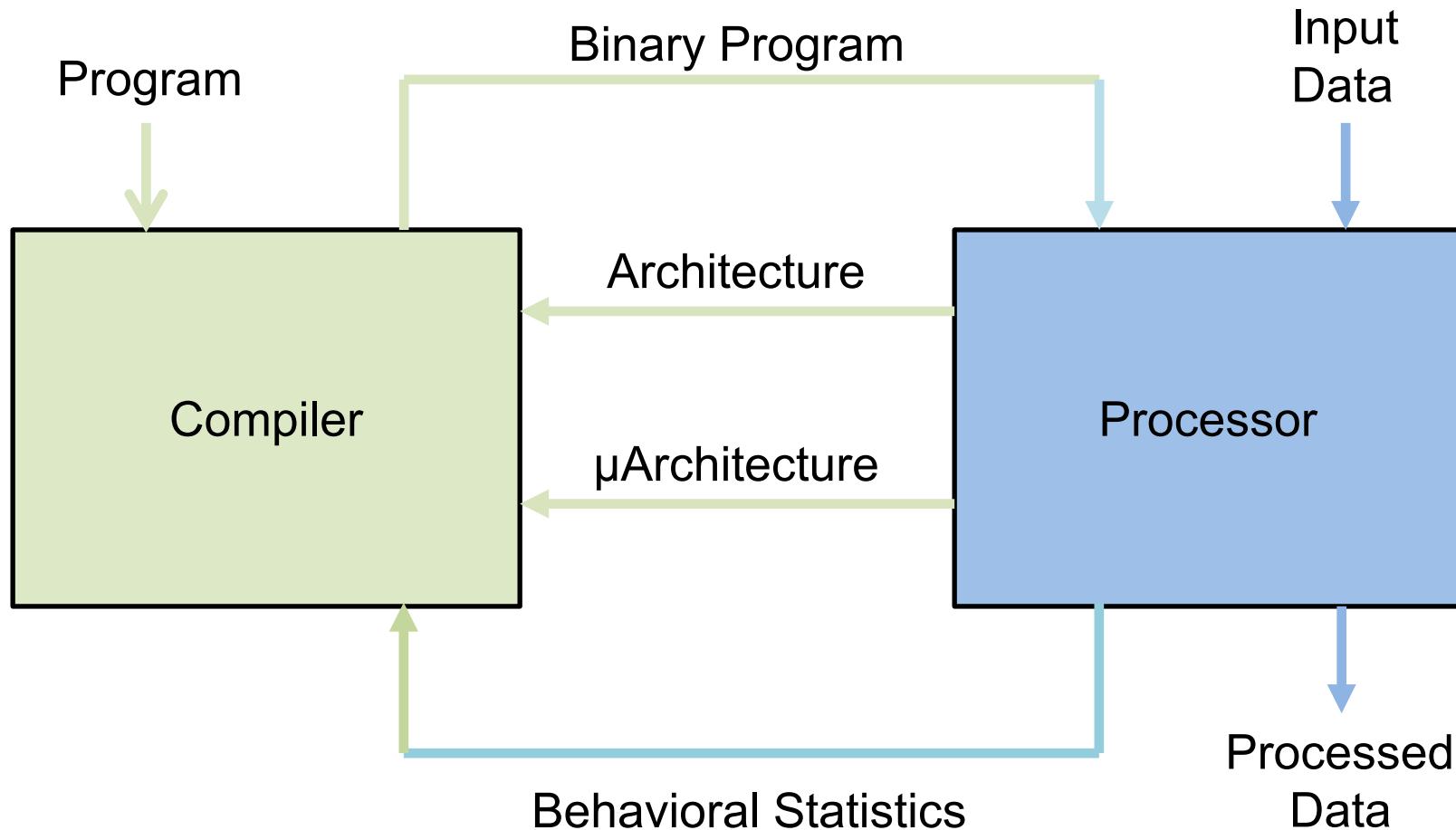
Mapping

Definition: selecting the placement and scheduling in space and time of every operation (including delivering the appropriate operands) required for a DNN computation onto the hardware function units of the accelerator.

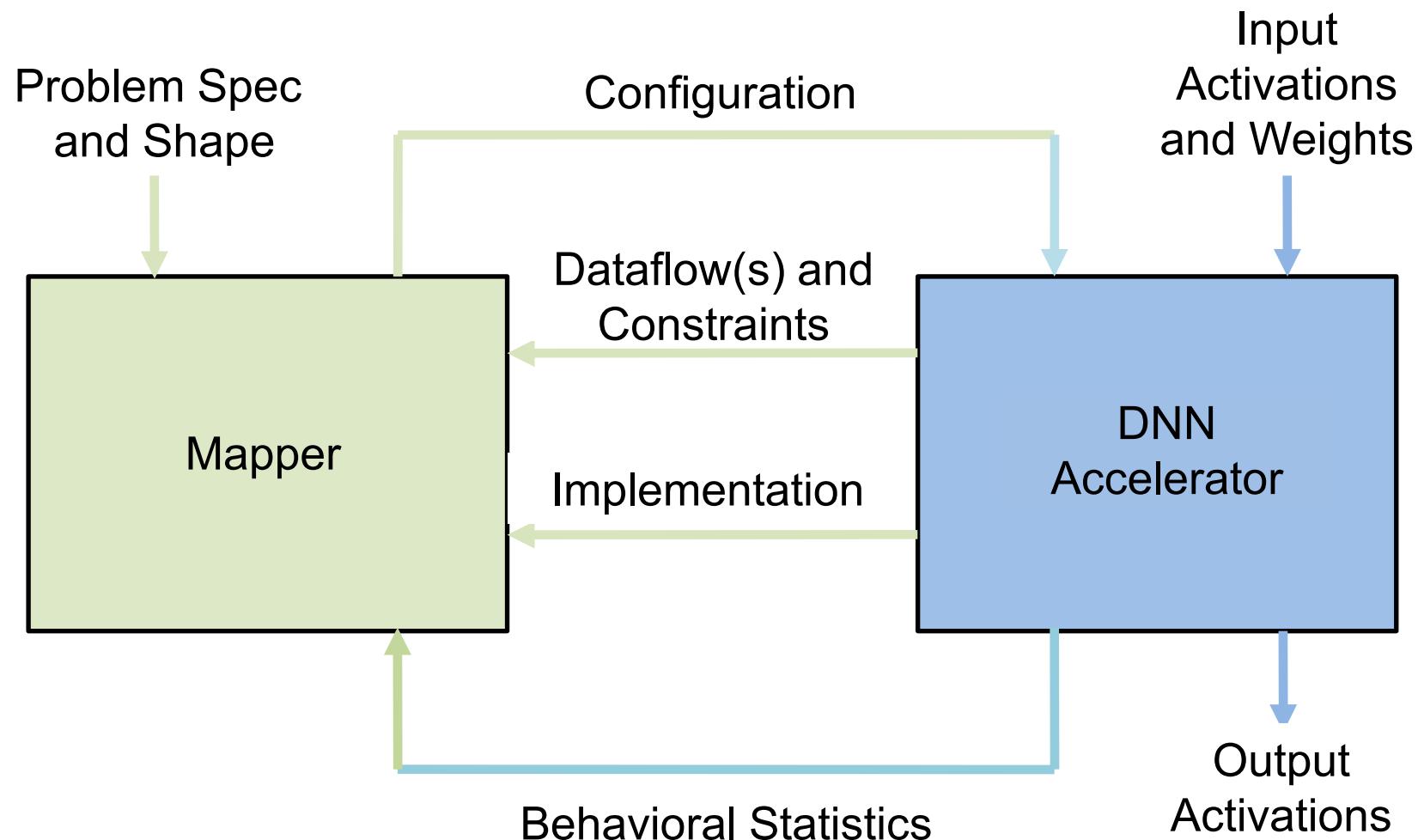
Steps: Within the constraints of the hardware, select for each level of the storage hierarchy:

- A dataflow (**for** loop order)
- A partitioning (**for** loop limits) both spatial and temporal
- Other behavioral details..., e.g., bypassing
- A binding computation to specific hardware units

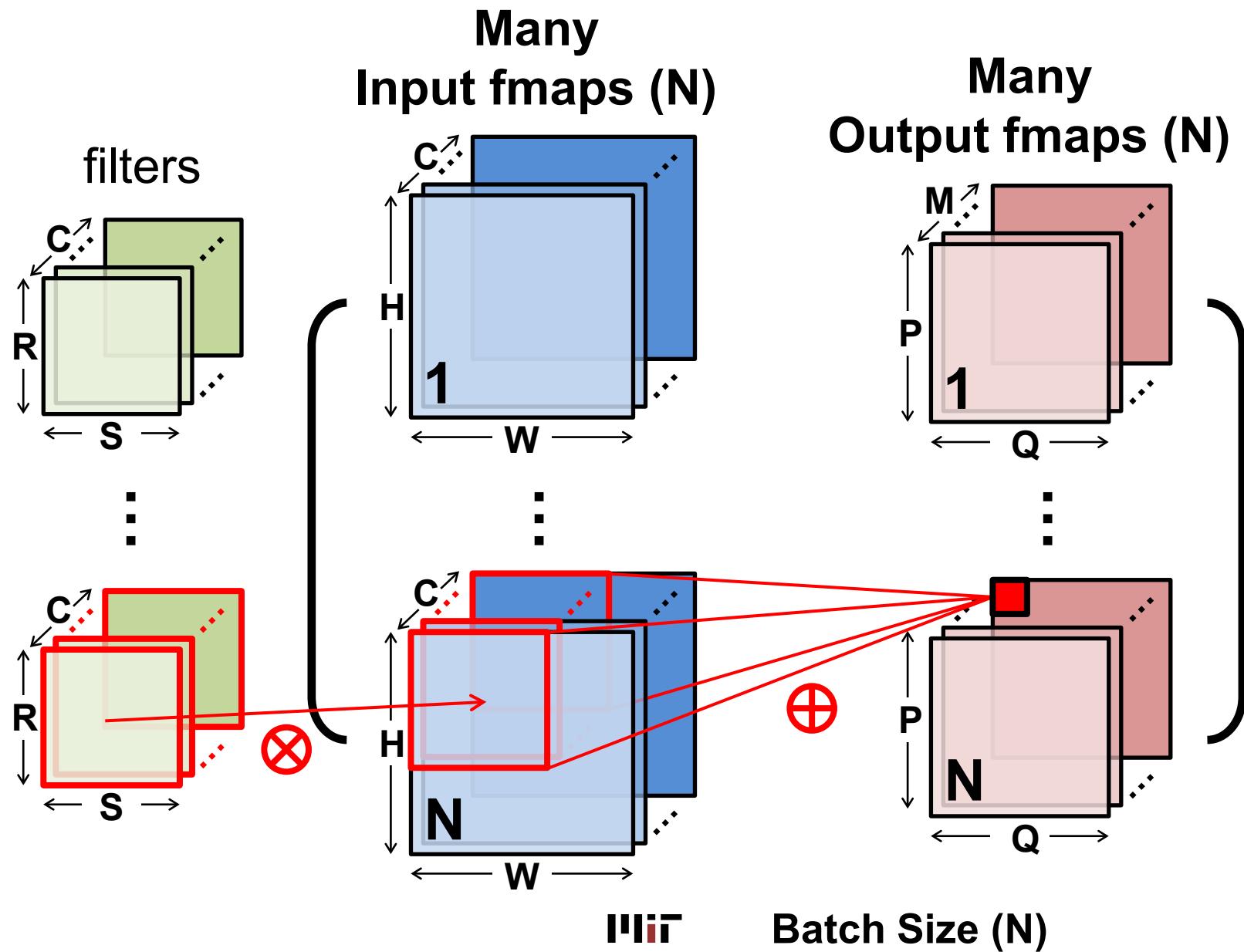
CPU Compute Model



DNN Compute Model

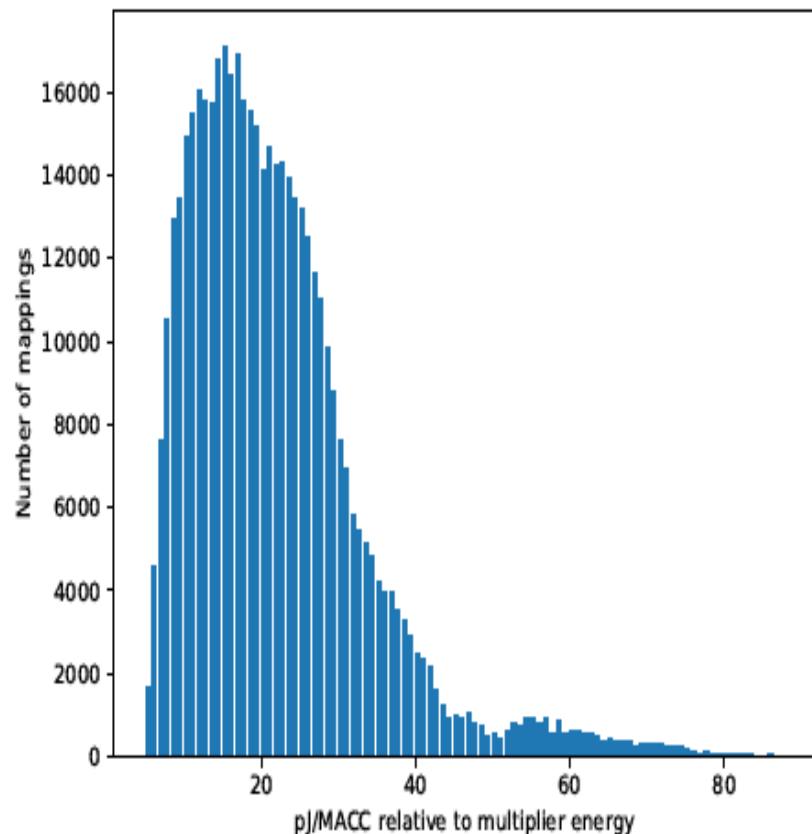


Convolution (CONV) Layer



Mapping Choices

Energy-efficiency of peak-perf mappings of a single problem



480,000 mappings shown

Spread: 19x in energy efficiency

Only 1 is optimal, 9 others within 1%

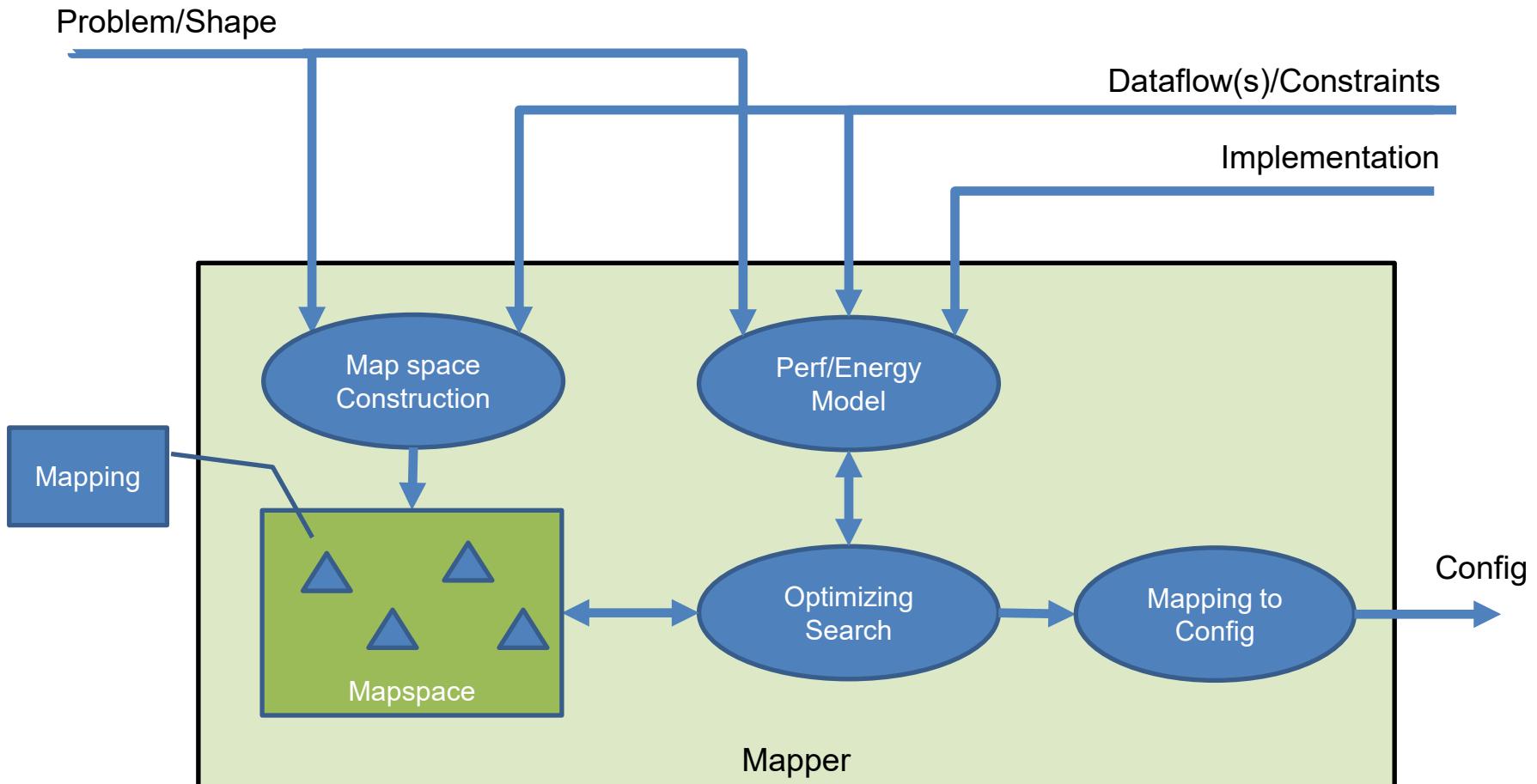
A **model** needs a **mapper** to evaluate a DNN workload on an architecture

6,582 mappings have min. DRAM accesses but vary 11x in energy efficiency

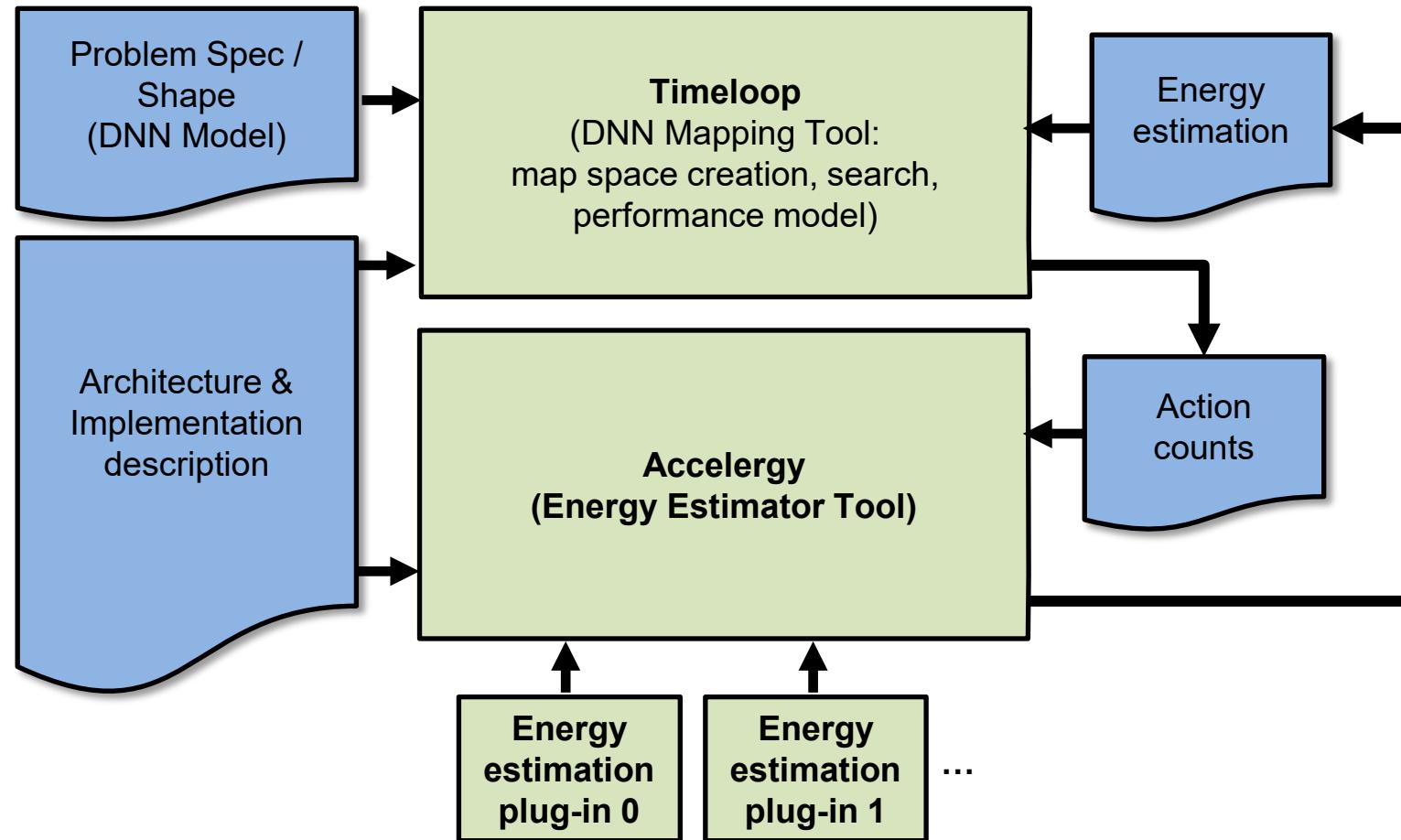
A **mapper** needs a good cost **model** to find an optimal mapping

Source: Parashar, Timeloop

Mapper Organization

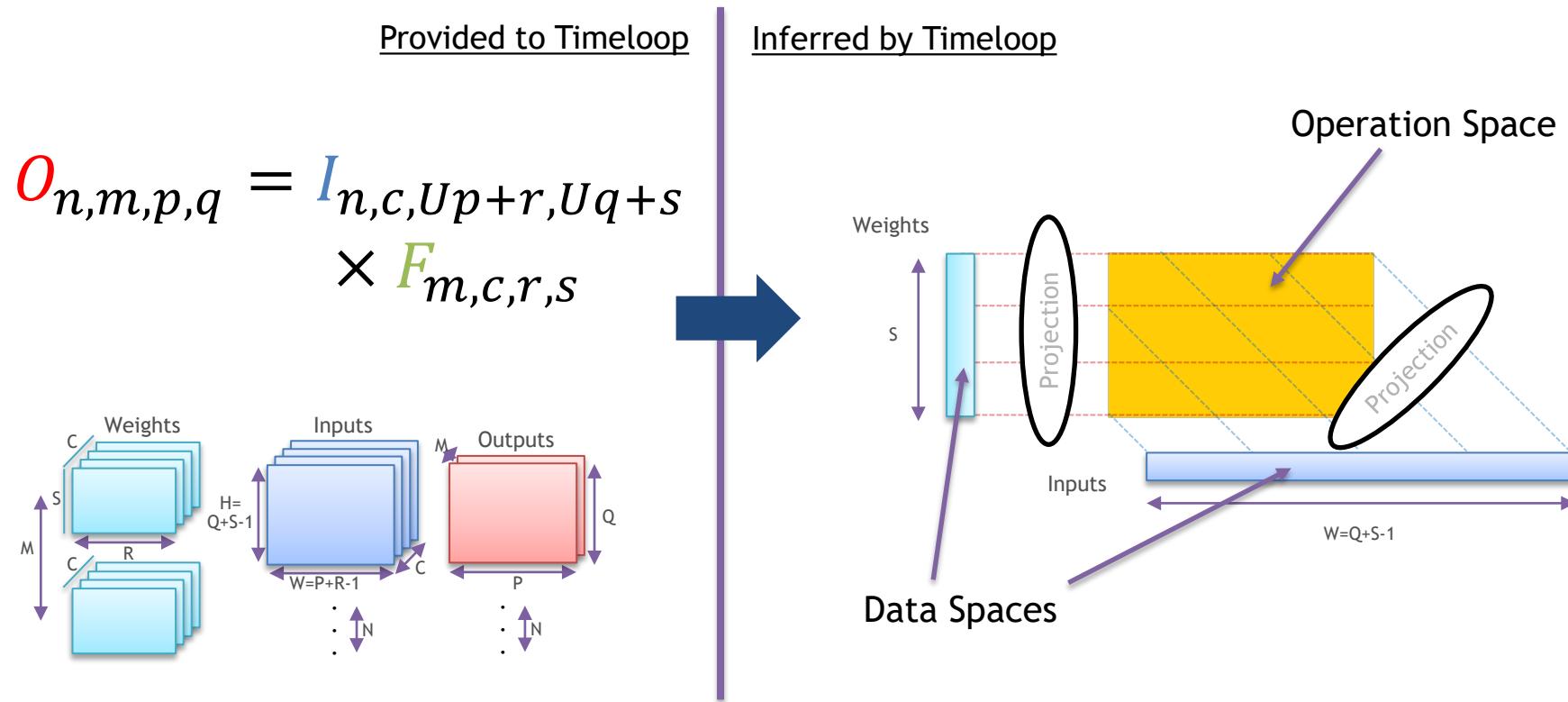


Timeloop Accelergy



Workload Specification

- Deep Loop Nest



Source: Parashar, Timeloop

Architecture Specifications

- Temporal reuse features
 - Number of buffer levels and buffer sizes
 - Buffer bypassing capabilities
 - Network topology
 - ...
- Parallelism and spatial reuse features
 - Topology of spatial fractures
 - Multicast capabilities
 - Inter-PE network, e.g., spatial sum and forwarding reuse
 - ...
- Constraints
 - Index sequence restrictions, e.g., allowable strides
 - Fixed level 0 loop nest, e.g., fixed vector width
 - Fixed level 1 spatial mappings, e.g., input/output channel array
 - ...

Determines the legal mappings:
loop permutations (dataflows) and associated loop limits

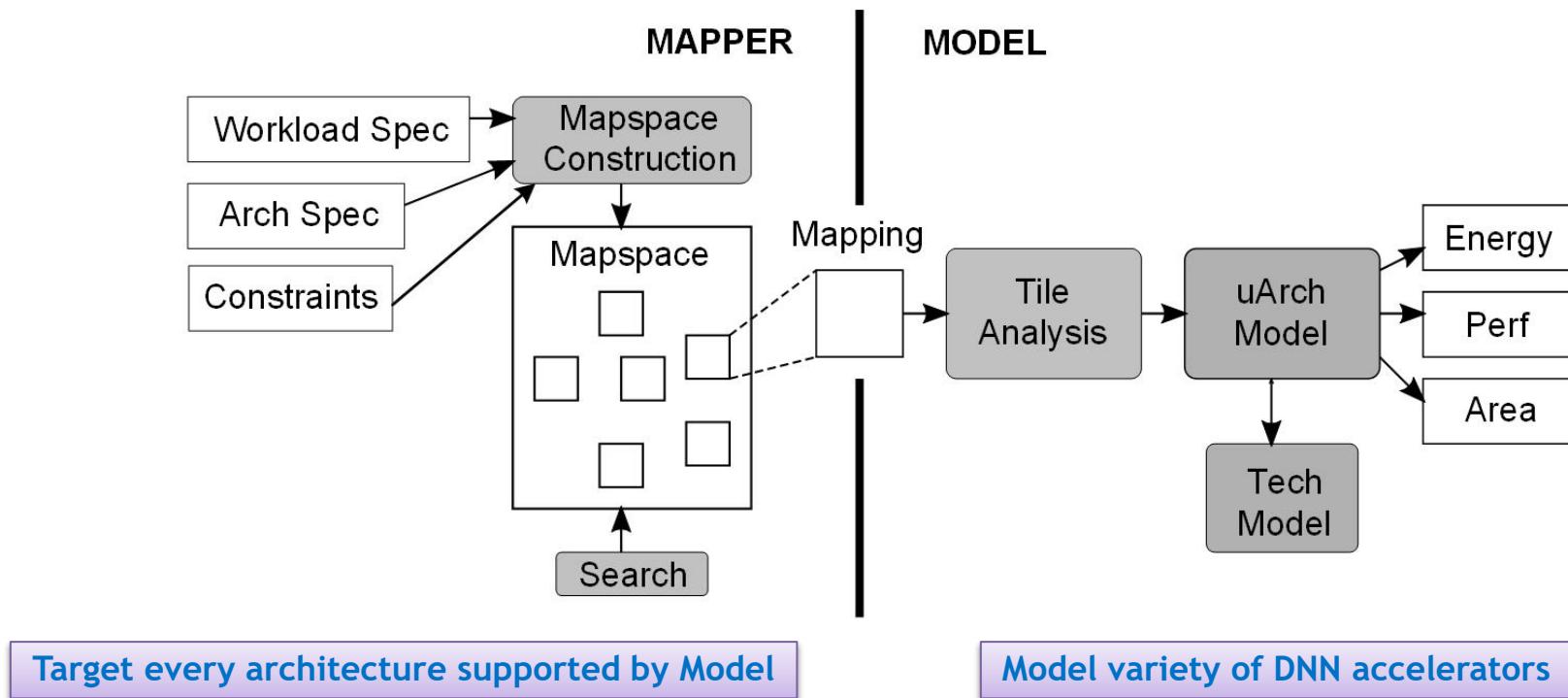
Implementation Specifications

- Buffer bandwidth and latency
- Buffer port and banking organization
- PE vectorization
- Network bandwidth and latency, e.g., router costs
- Shared or per-datatype network links
- ...

Determines the latency and energy consumption of a mapping.

Timeloop

- Tool for Evaluation and Architectural Design-Space Exploration of DNN Accelerators



Source: Parashar, Timeloop

Next Lecture: Calculating Data Movement

Thank you!