Accelerators (II)

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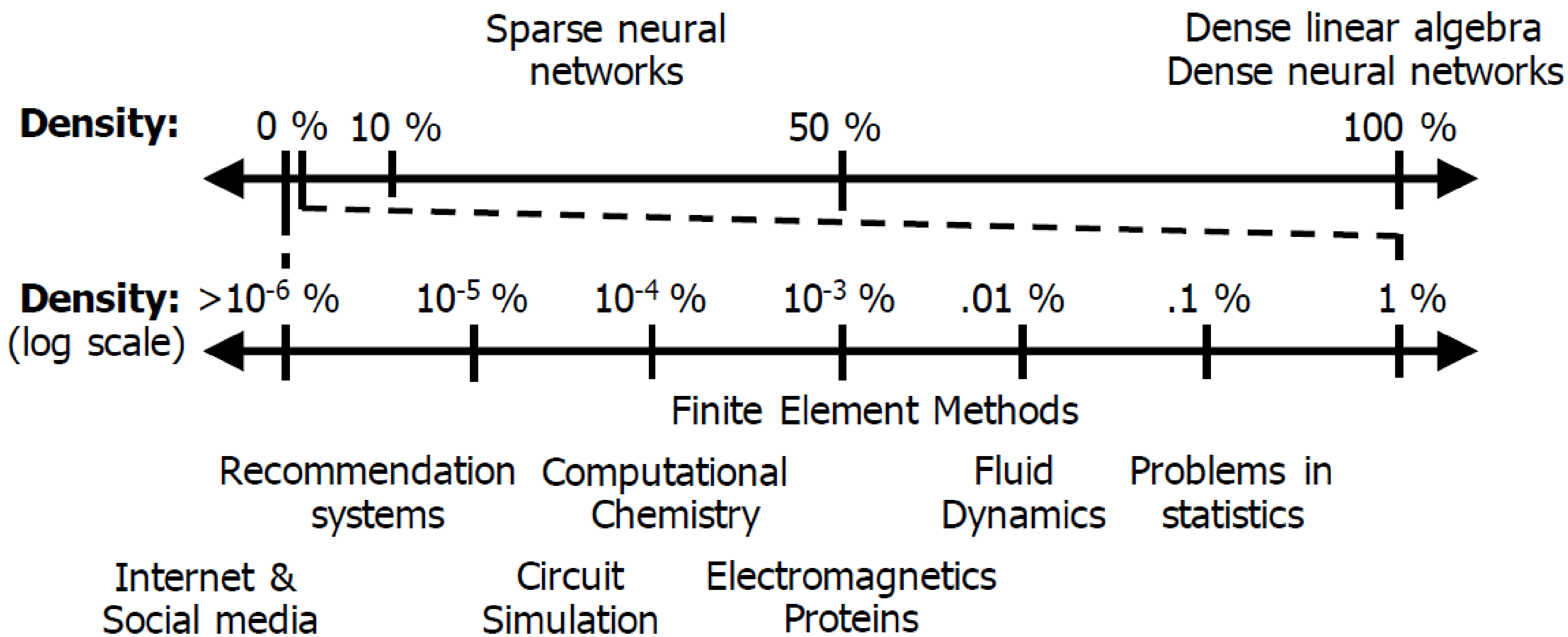
Many problems use Sparse Tensors



[Extensor, Hegde, et.al., MICRO 2019]

Exploiting Sparsity

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Sparse data can be compressed

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Exploiting Sparsity

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Can save space and energy by avoiding manipulation of zero values

$$\boldsymbol{anything \times 0 = 0}$$

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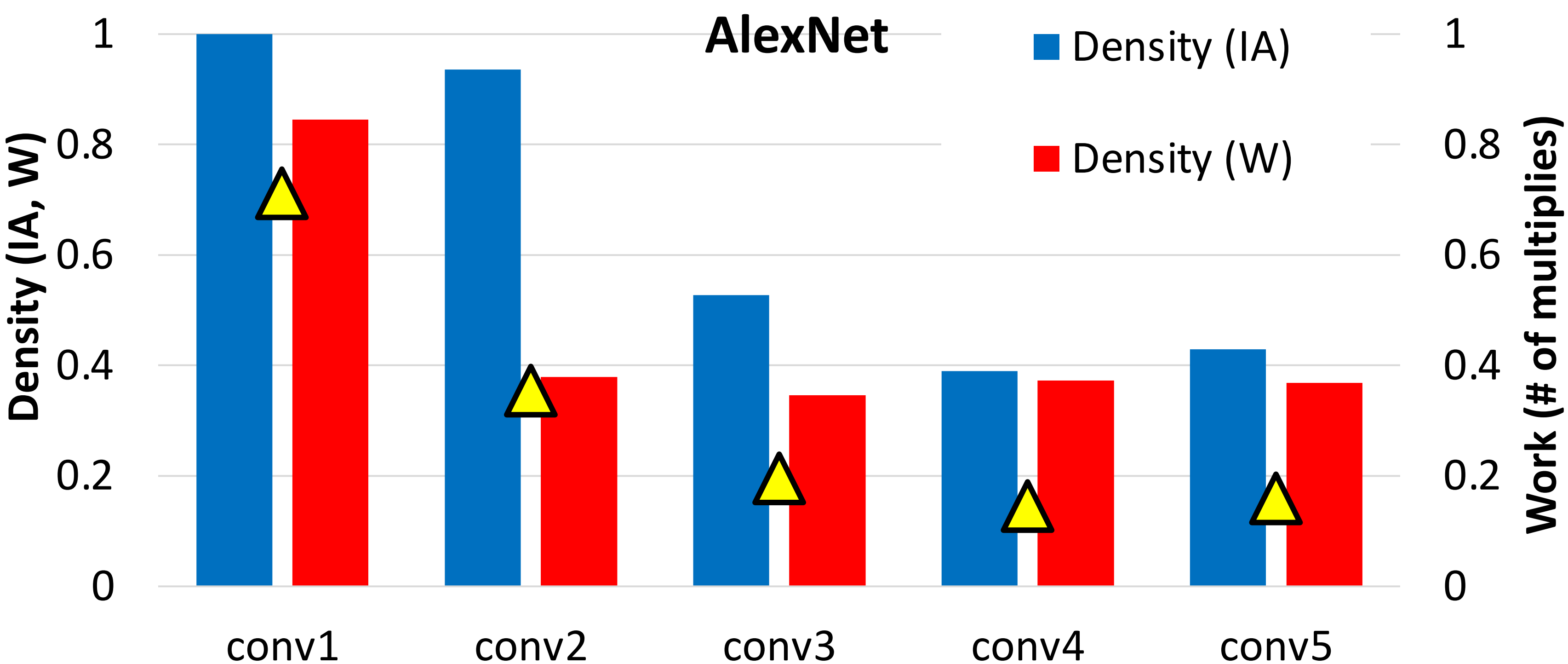
Can save time and energy by avoiding fetching unnecessary operands and avoiding **ineffectual** computations

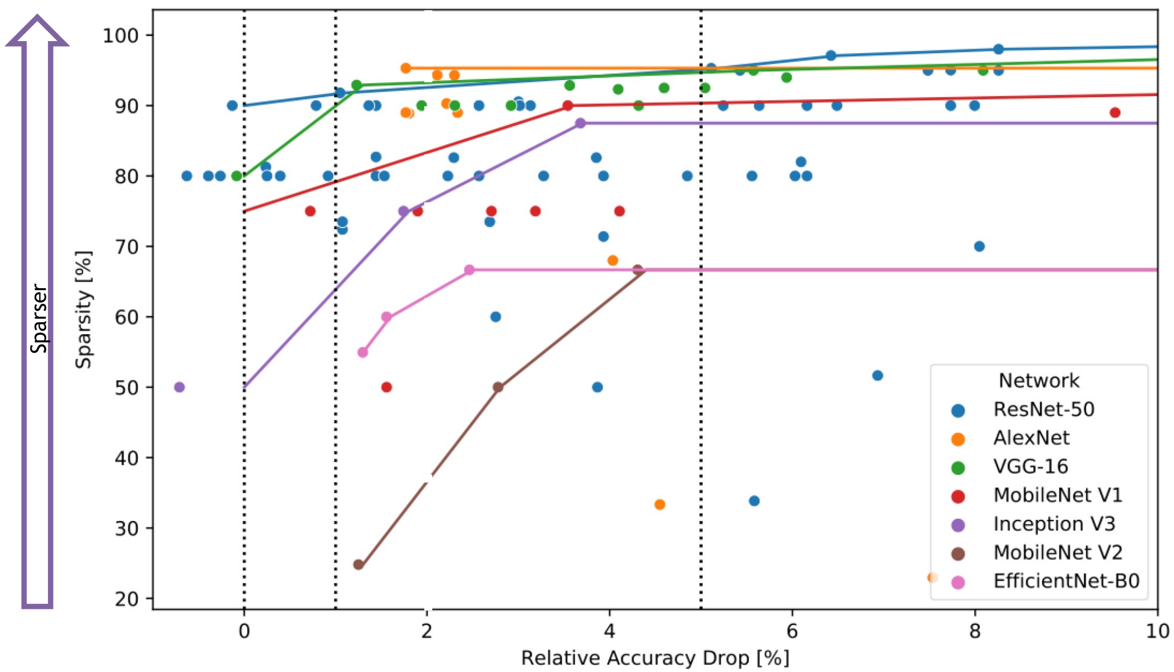
Motivation in DNNs

- Leverage CNN sparsity to improve energy-efficiency



Exploitable Sparsity

Acceptable sparsity depends on target task and error tolerance



Hoefler et al. arXiv, 2021

**MLPerf error tolerance*

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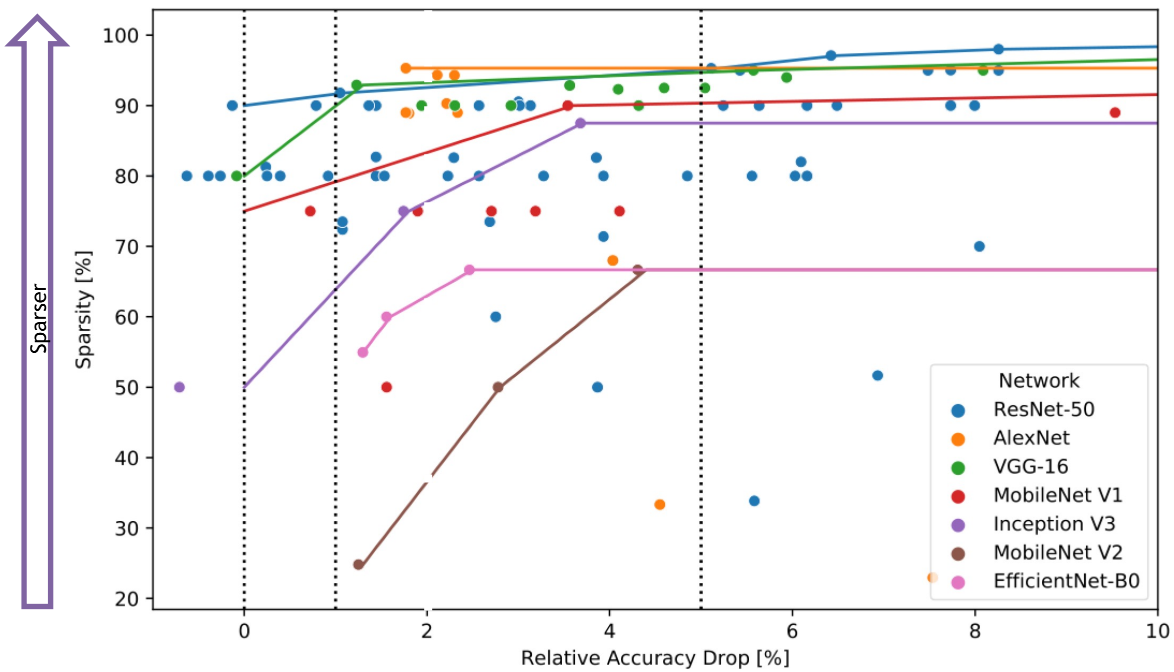

Hoefler et al. arXiv, 2021

Error Tolerance

	≤0%	≤1%*	≤2%
ResNet-50	~90%	~90%	~91%
AlexNet			~93%
VGG-16	~80%	~88%	~92%
MobileNet V1	~72%	~79%	~82%
Inception V3	~50%	~62%	~73%
EfficientNet-B0			~52%
MobileNet V2			~25%

**MLPerf error tolerance*

Hardware Sparse Acceleration Features

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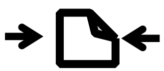

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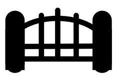

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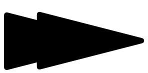

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Explicitly eliminate ineffectual storage accesses and computes by skipping the cycle to save energy and time

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What is the chosen format?

Do all tensors share the same format?

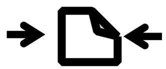
When is a storage access gated?

How much is the compute able to skip ahead?

At which storage level is the skipping performed?

What is the criteria for skipping?

Hardware Sparse Acceleration Features



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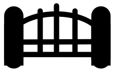

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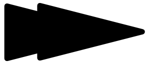

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1-D Output-Stationary Convolution

† Assuming: ‘valid’ style convolution

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1-D Output-Stationary Convolution


```
int i[W];       # Input activations
int w[S];       # Filter weights
int o[Q];       # Output activations

for q in [0..Q):
       for s in [0…S):
            o[q] += i[q+s]*f[s];
}}
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What opportunity(ies) exist if some of the filter weights are zero?

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What opportunity(ies) exist if some of the filter weights are zero?

Can avoid reading operands, doing multiply and updating output

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for q in [0..Q):
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            o[q] = i[q+s]*w[r];
}}
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1-D Output-Stationary Convolution

Weights		Inputs		Outputs
8 0 6	*		=	
R		W		E = W-ceil(R/2)†

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int i[W];      # Input activations
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for q in [0..Q):
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              if (!f[s]) o[q] += i[q+s]*f[r];
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What did we save using the conditional execution?

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What did we save using the conditional execution? Energy

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What did we save using the conditional execution? Energy

What didn't we save using the conditional execution?

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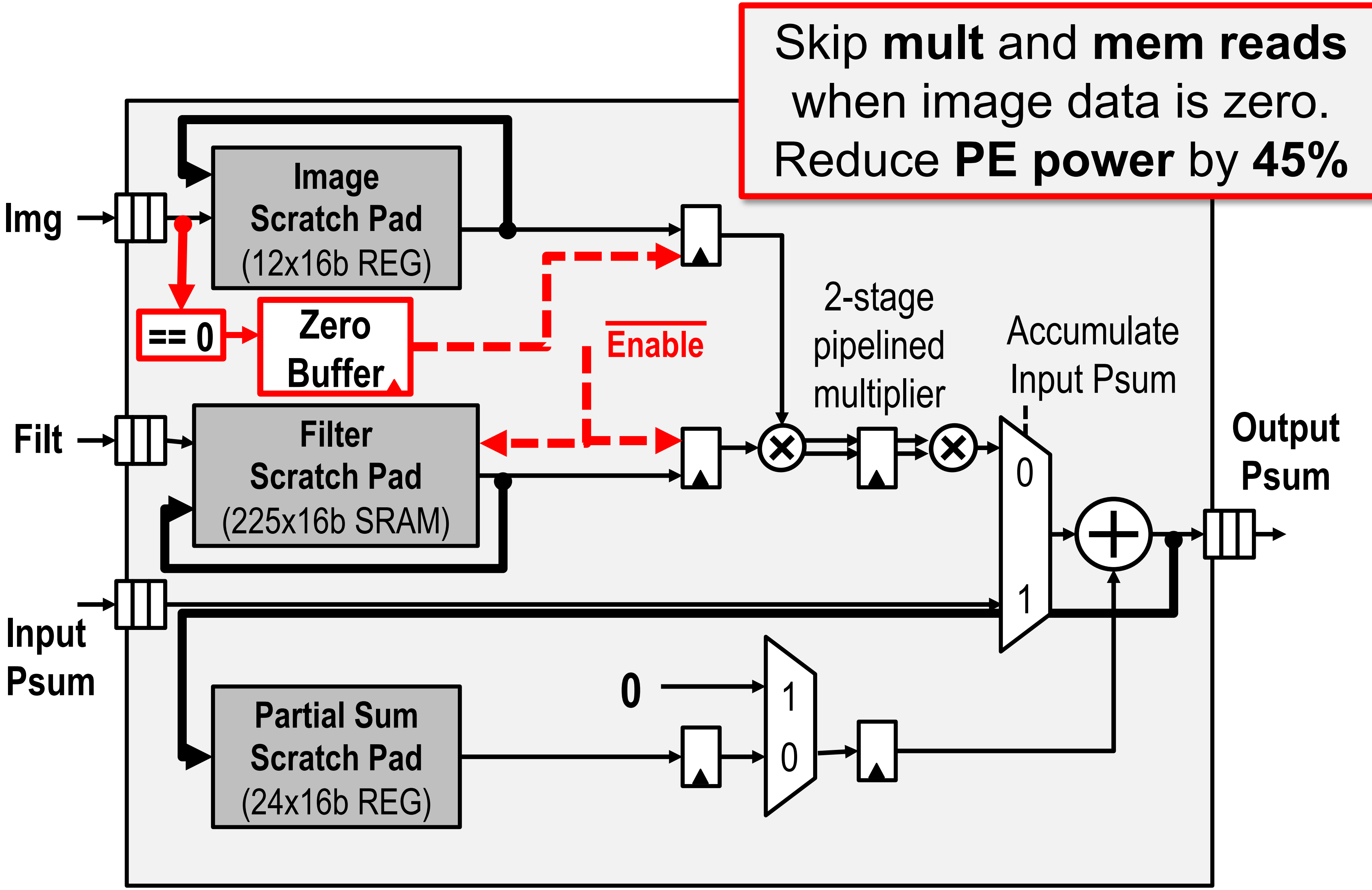
Eyeriss – Clock Gating



Sparse Tensor Representation

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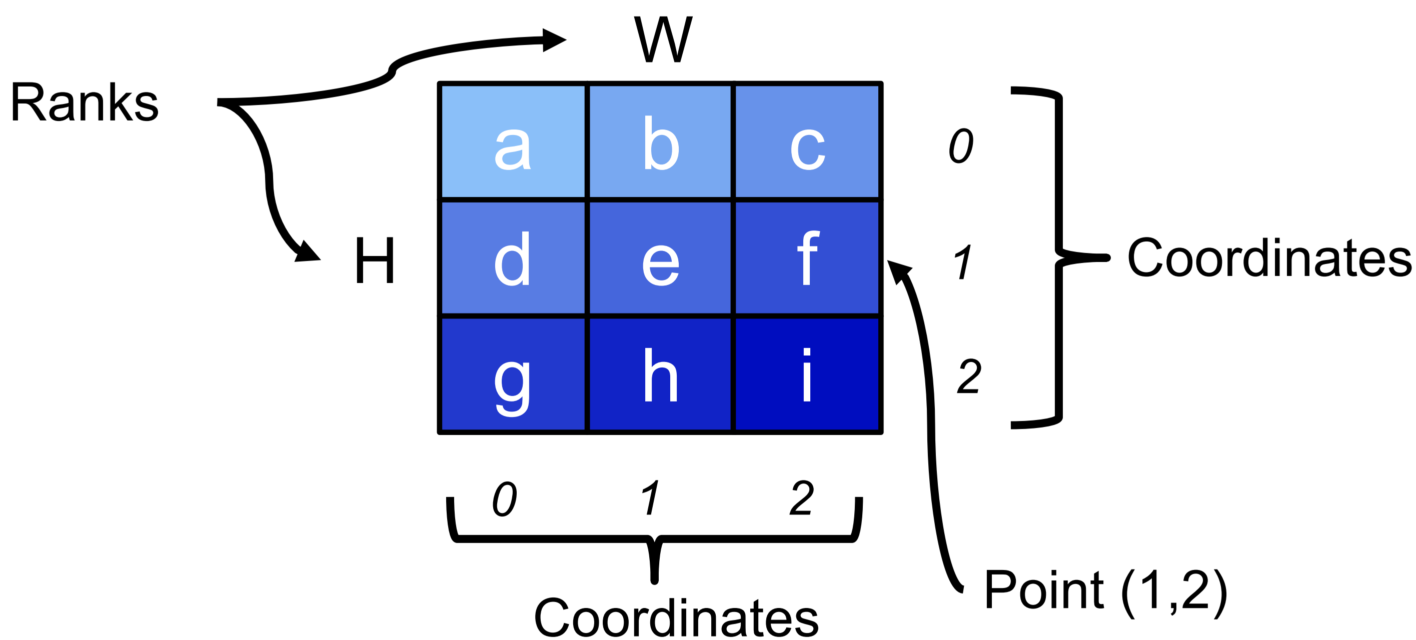
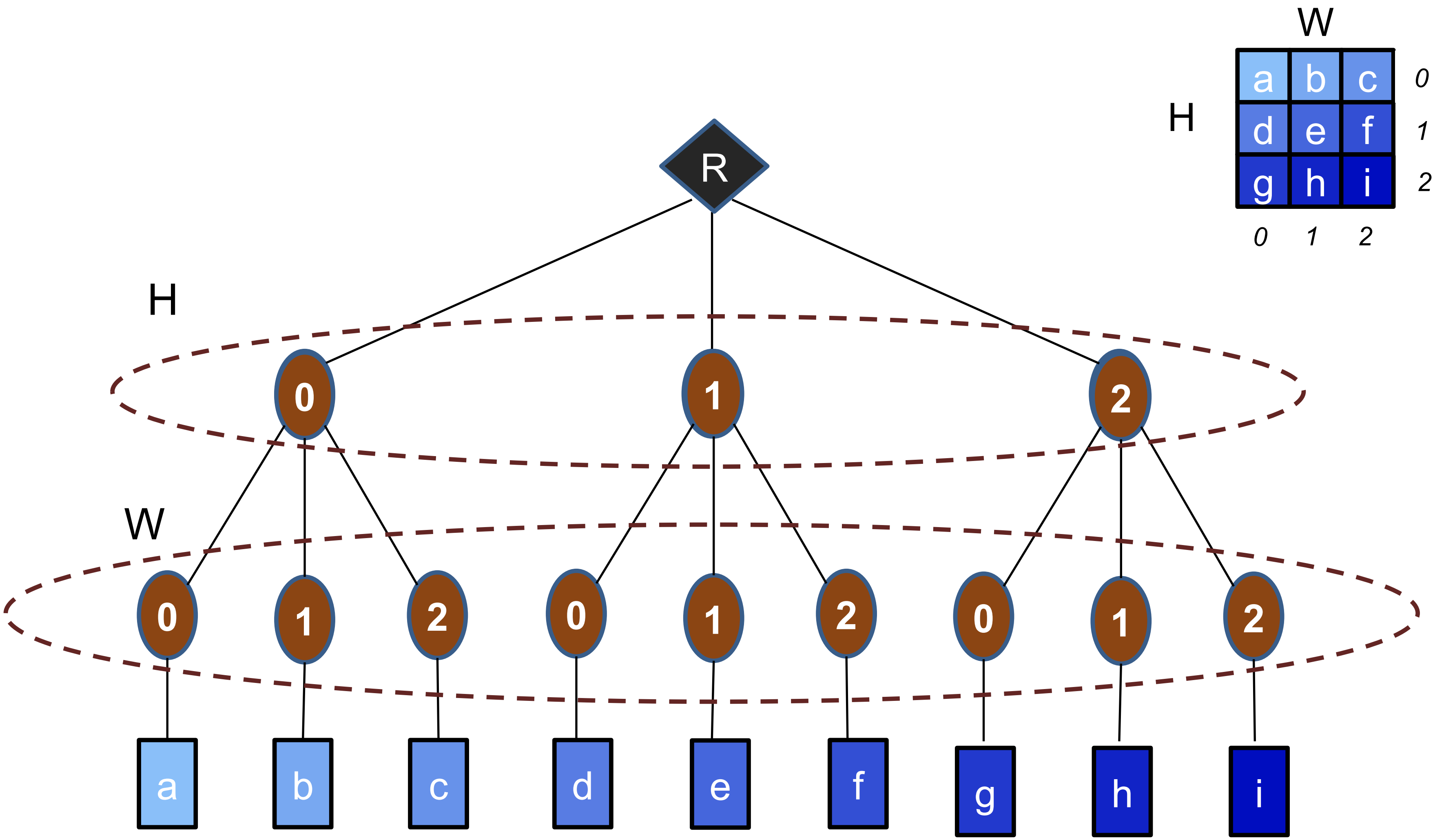
Tensor Data Terminology



- The elements of each “rank” (dimension) are identified by their “coordinates”, e.g., rank H has coordinates 0, 1, 2

- Each element of the tensor is identified by the tuple of coordinates from each of its ranks, i.e., a “point”.
  So (1,2) -> “f”

Tree-based Tensor Abstraction



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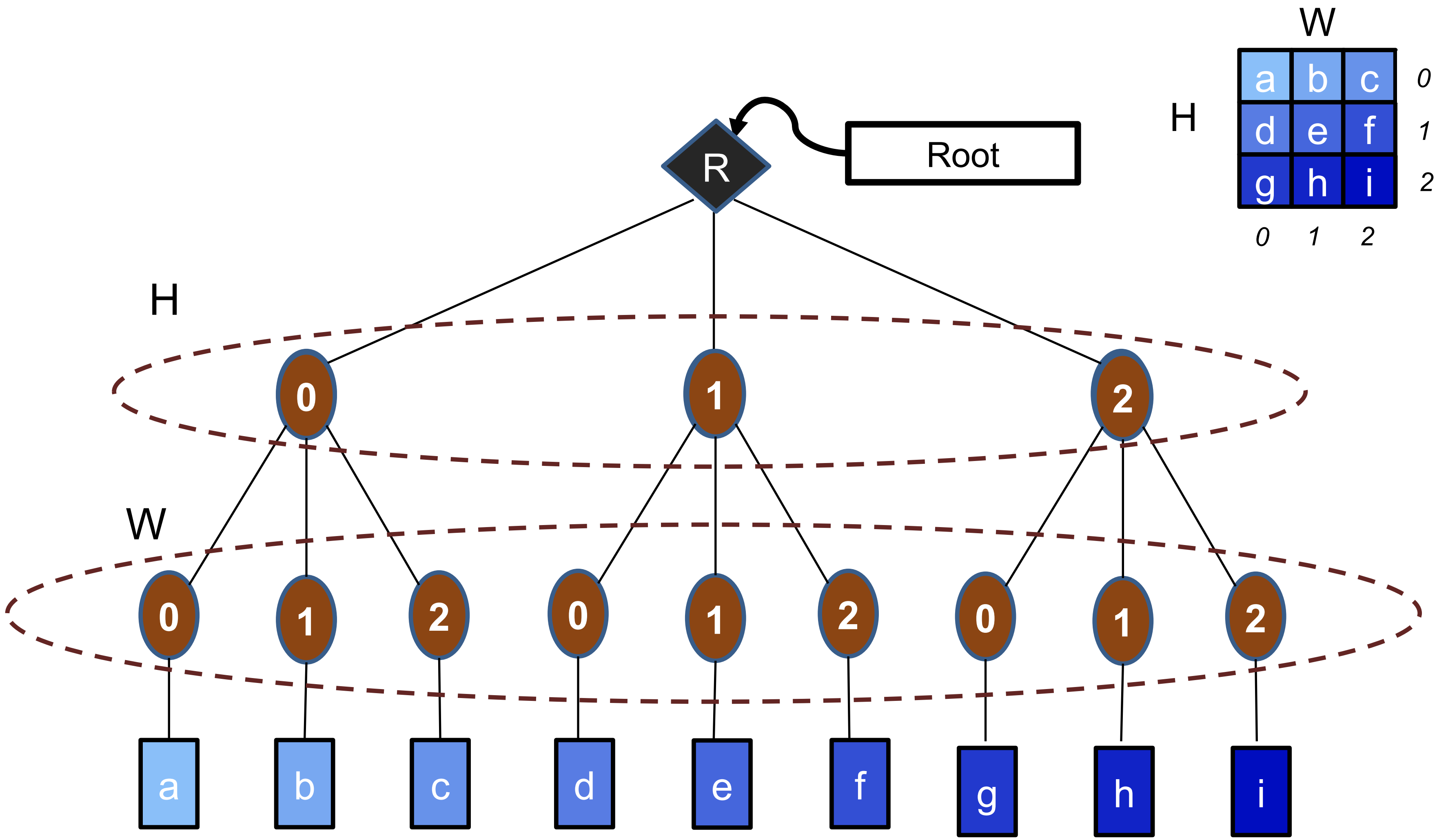

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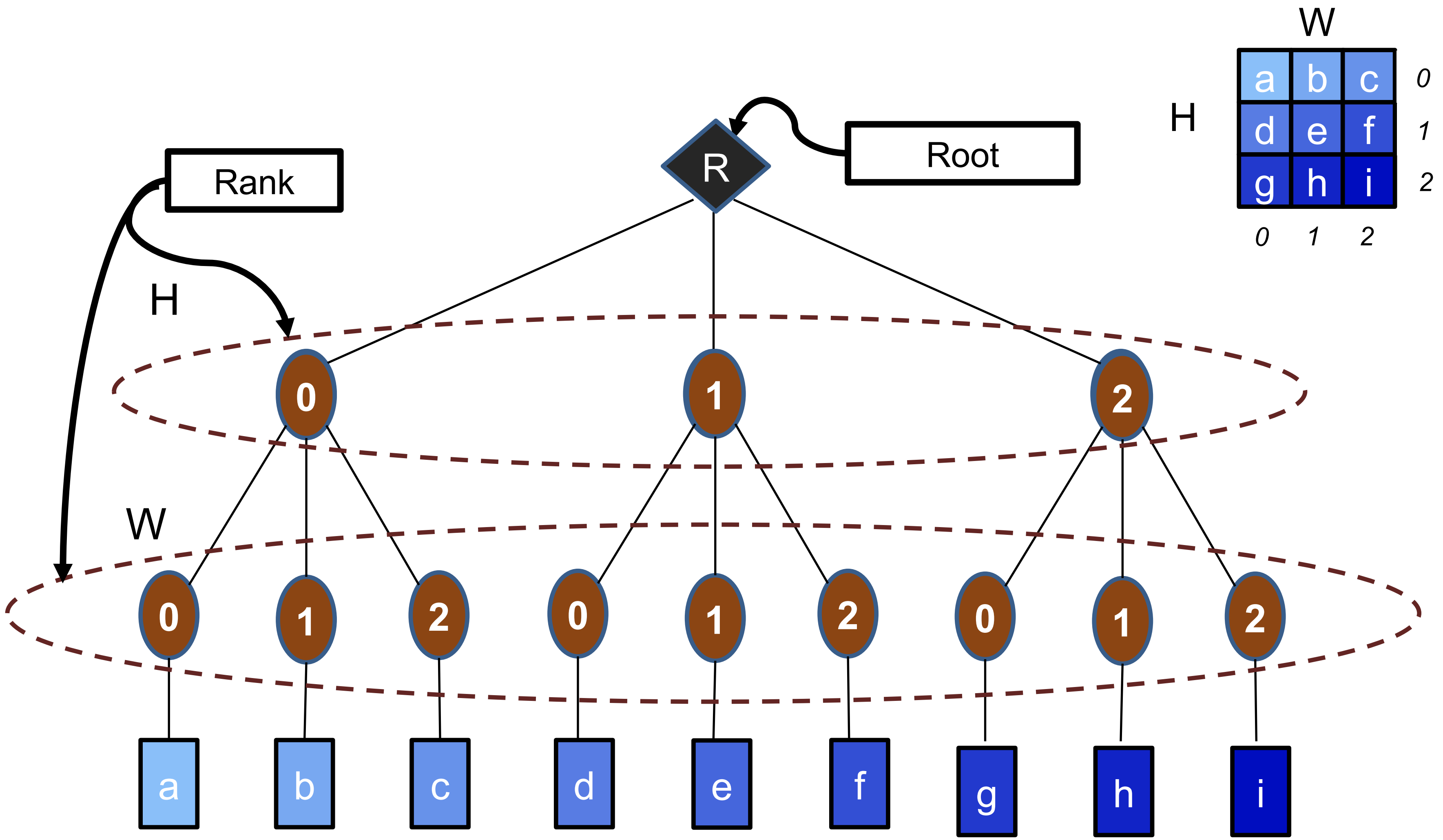

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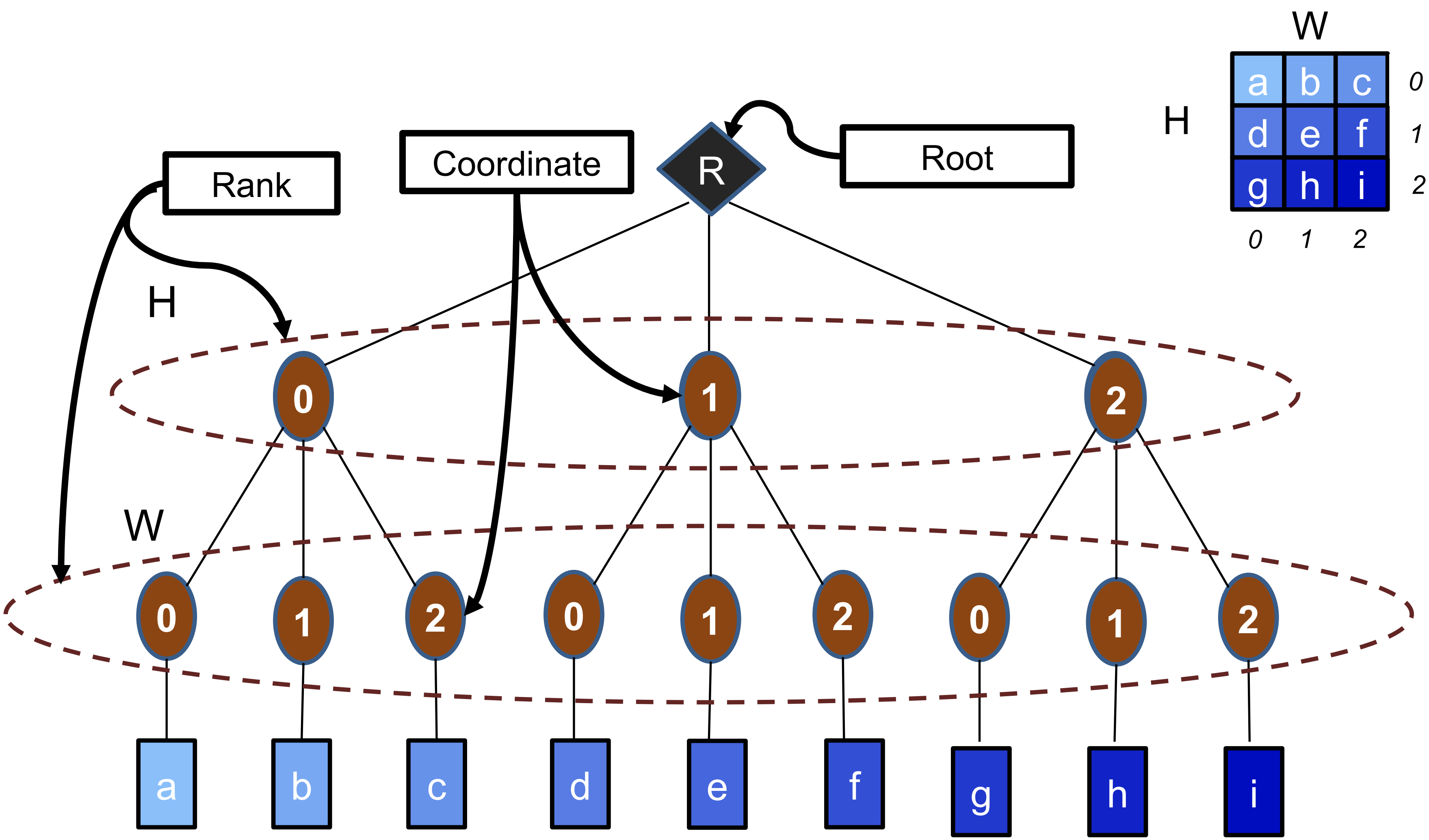

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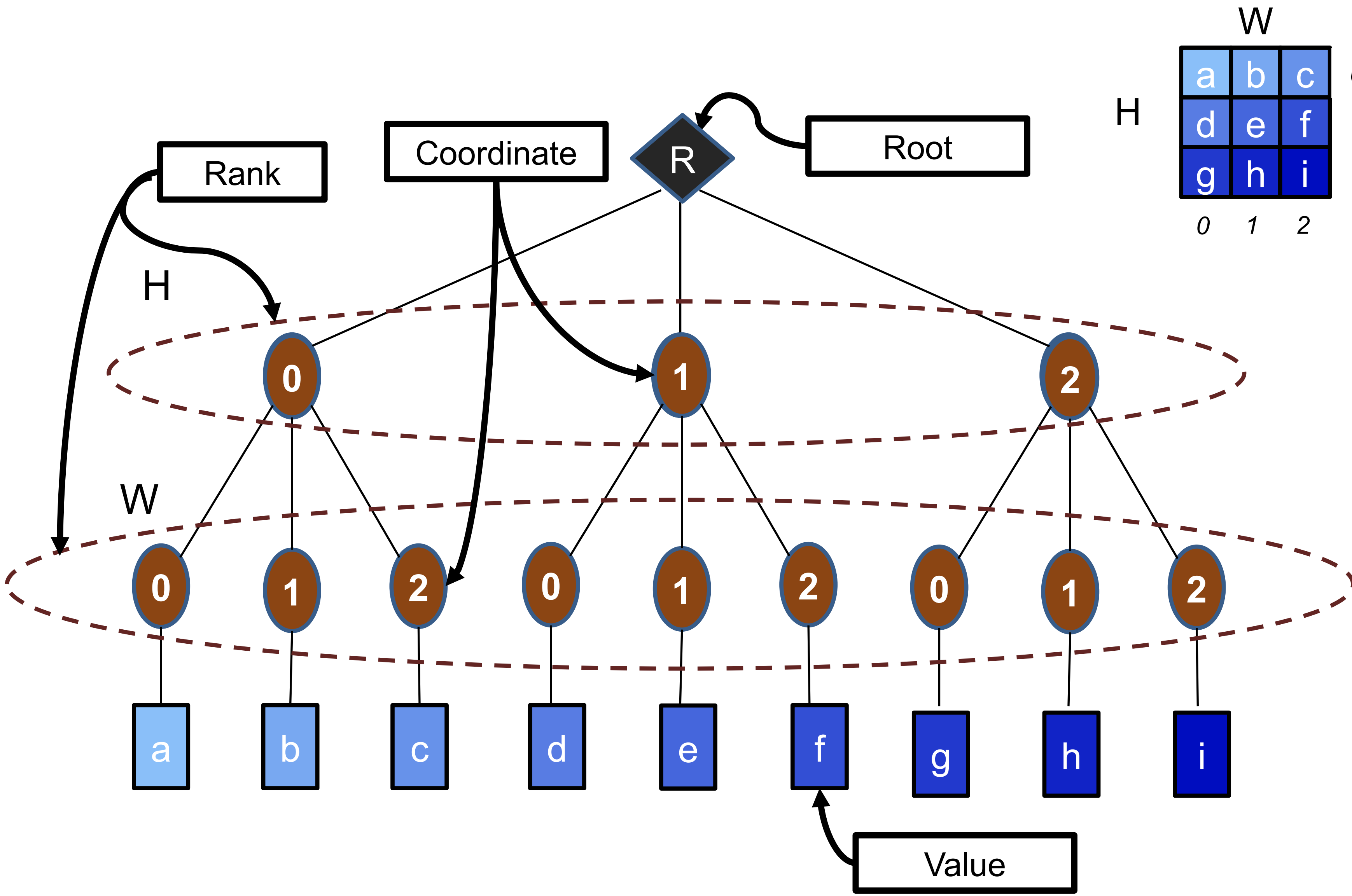
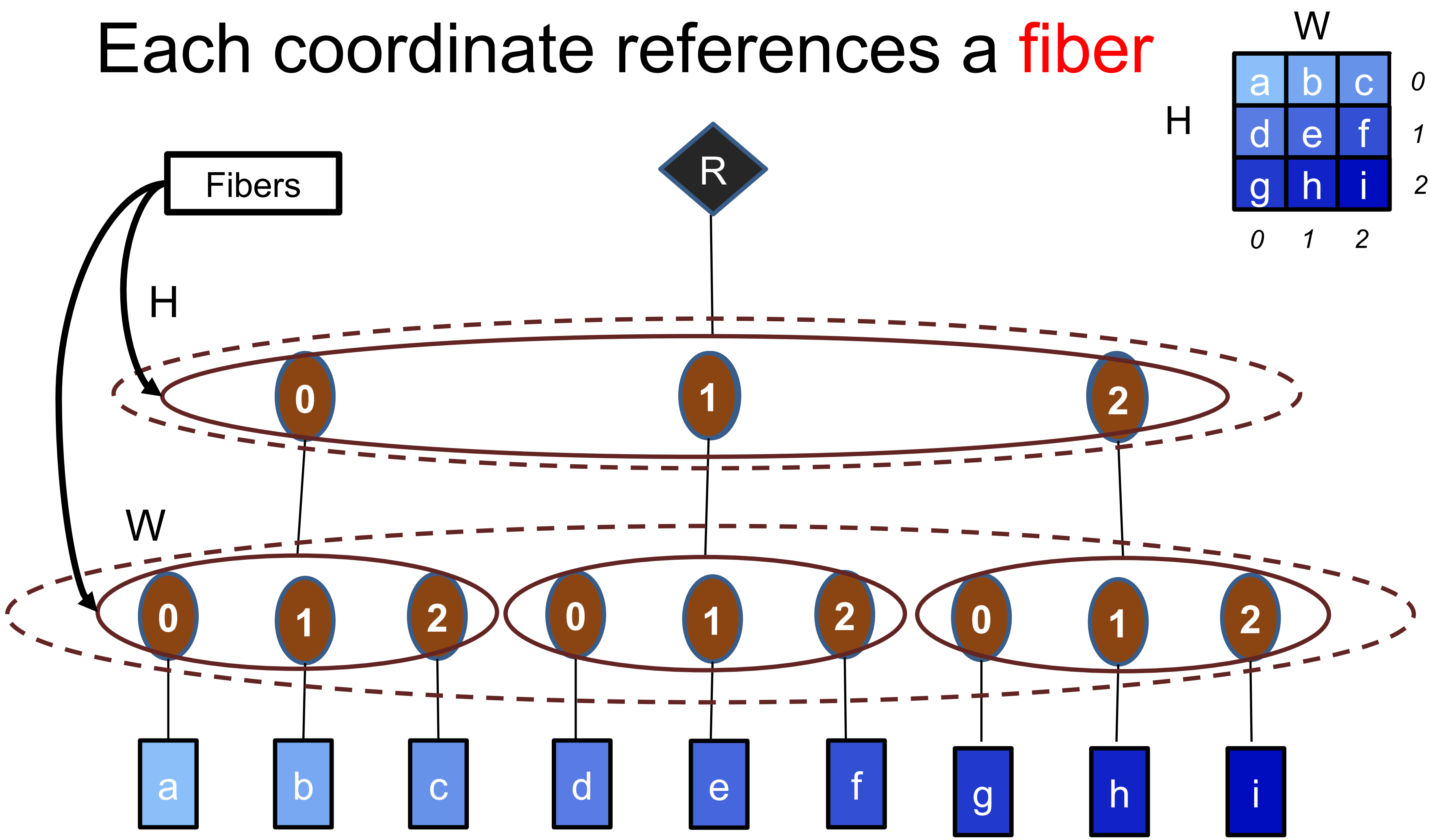


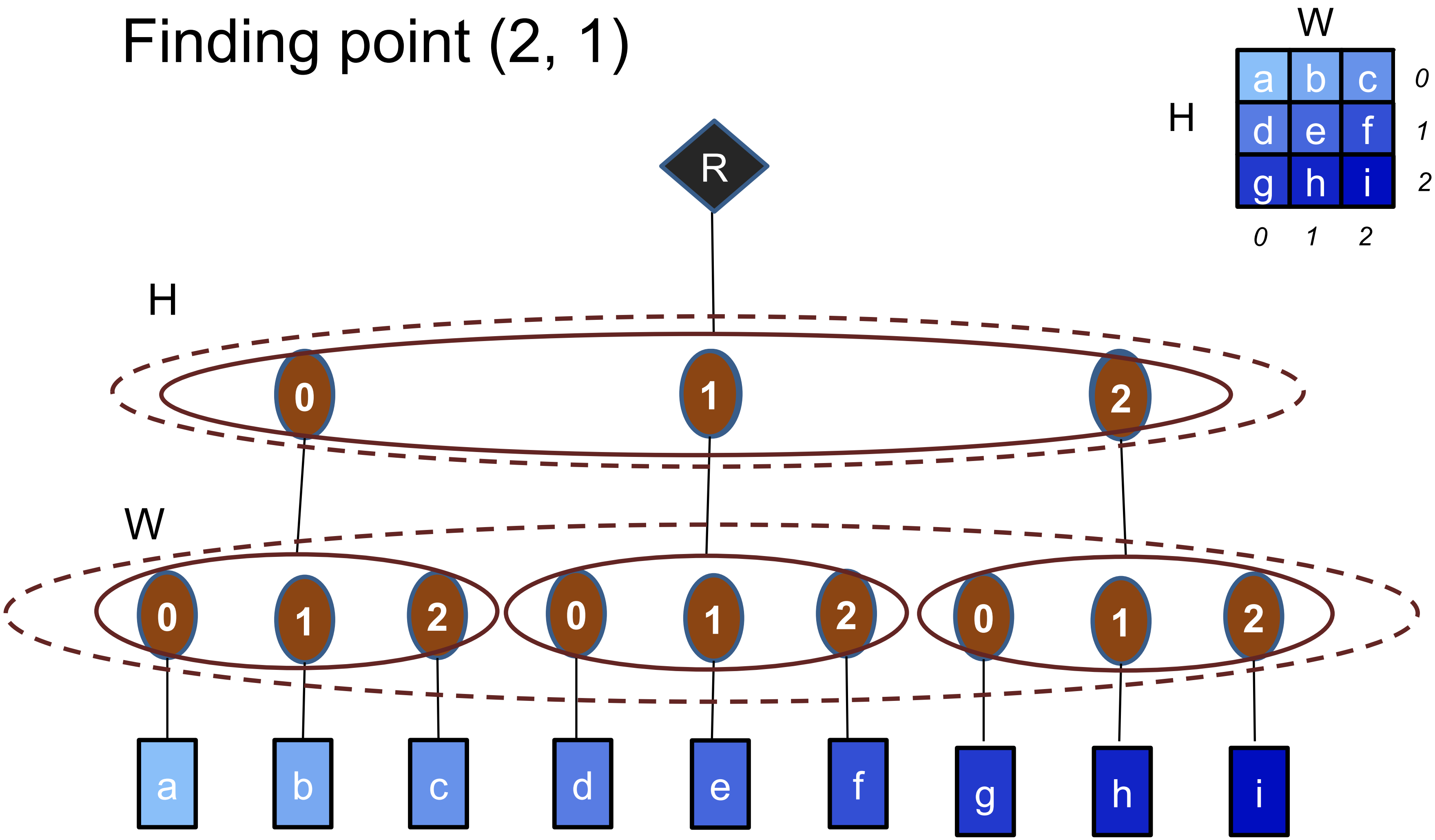
Fibertree Tensor Abstraction

Each coordinate references a fiber



Fibertree Tensor Abstraction

Finding point (2, 1)



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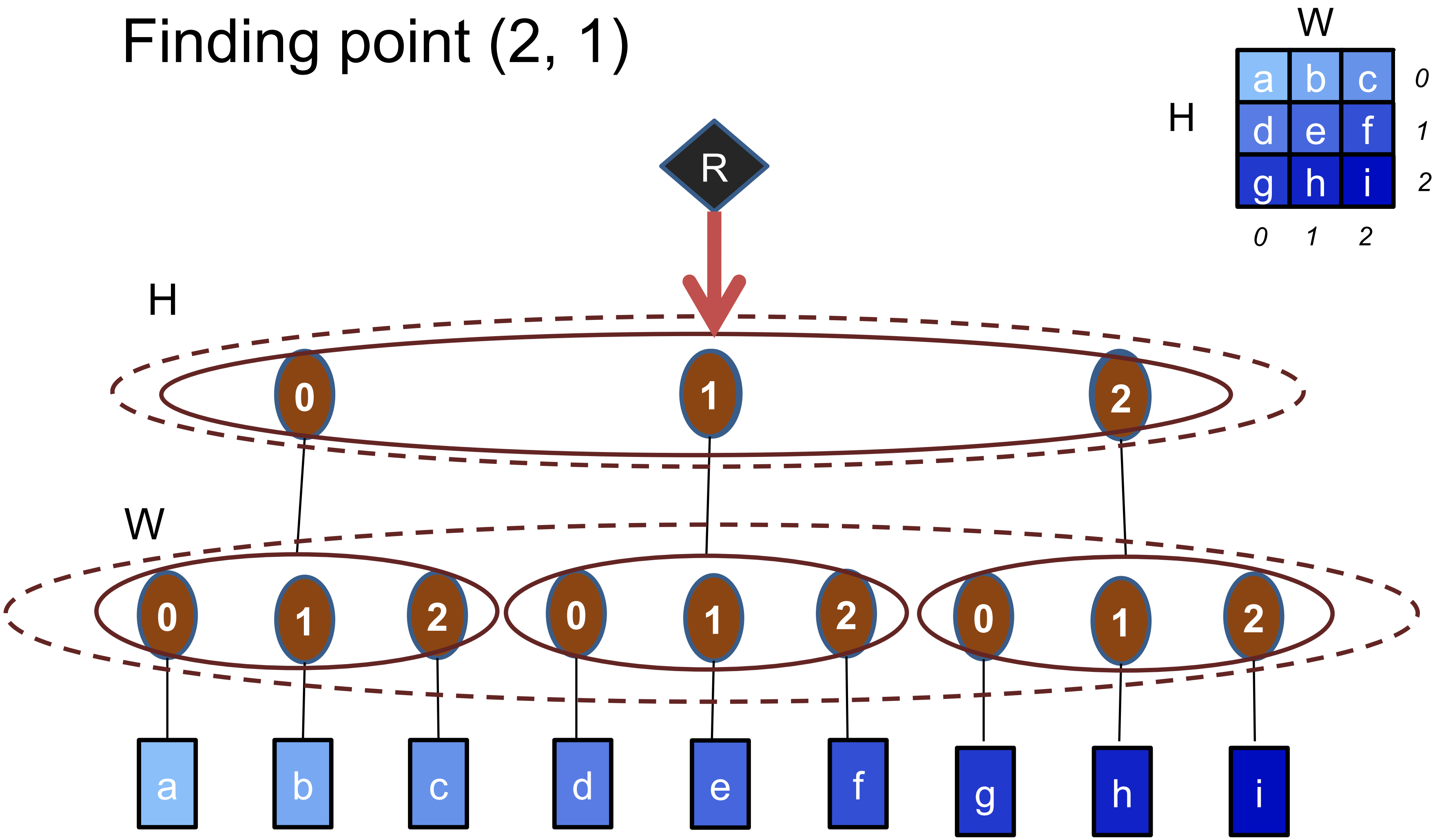

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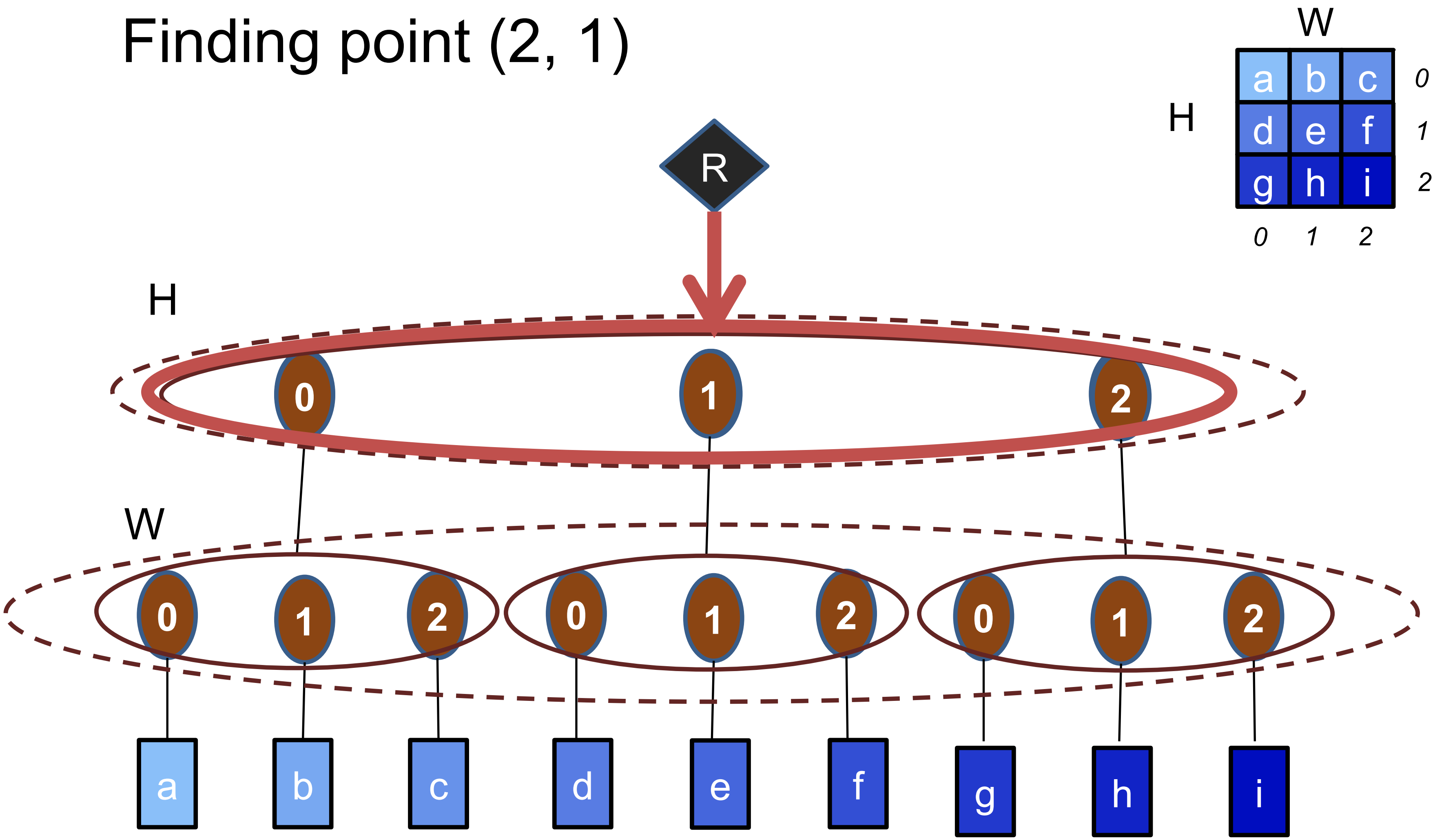

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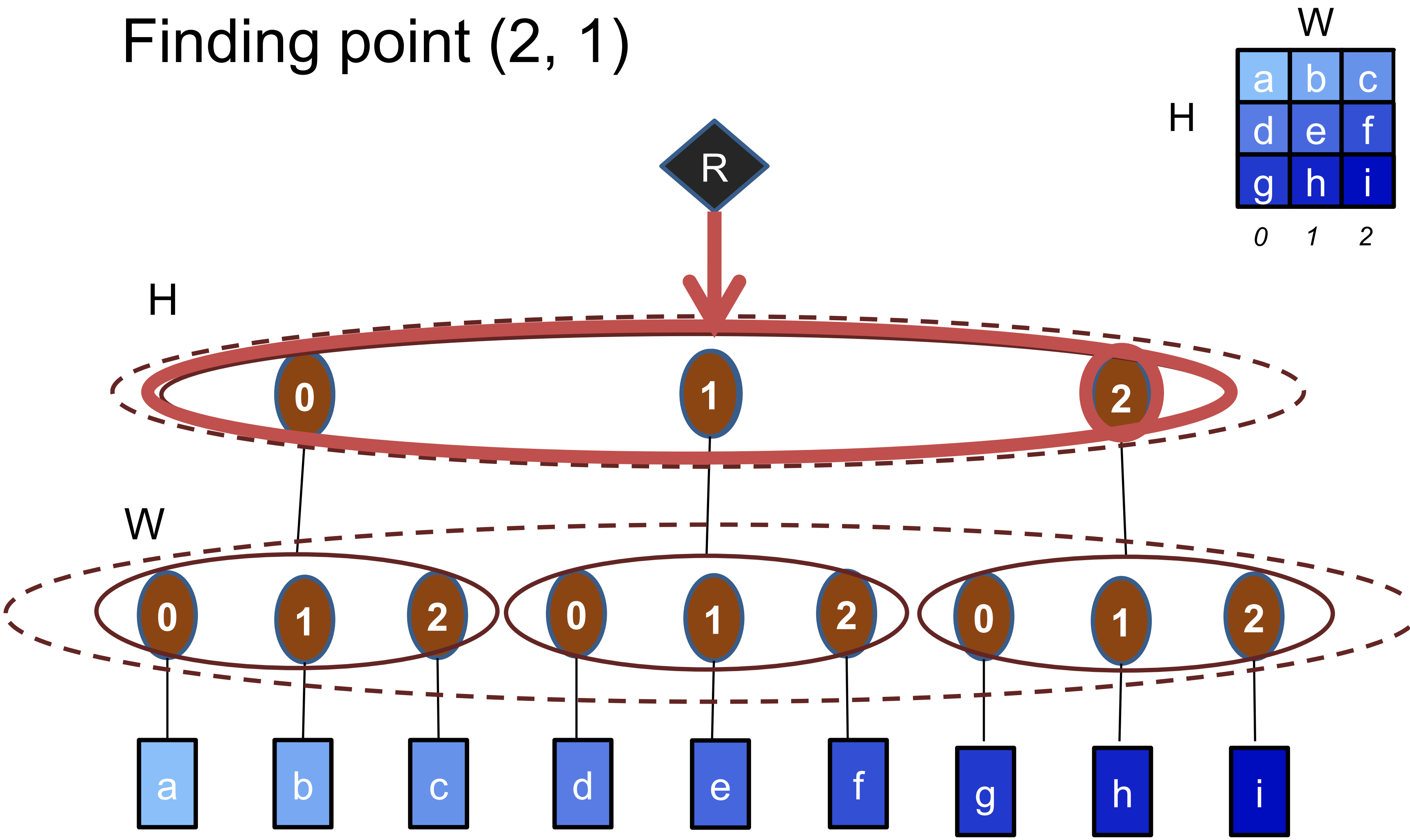

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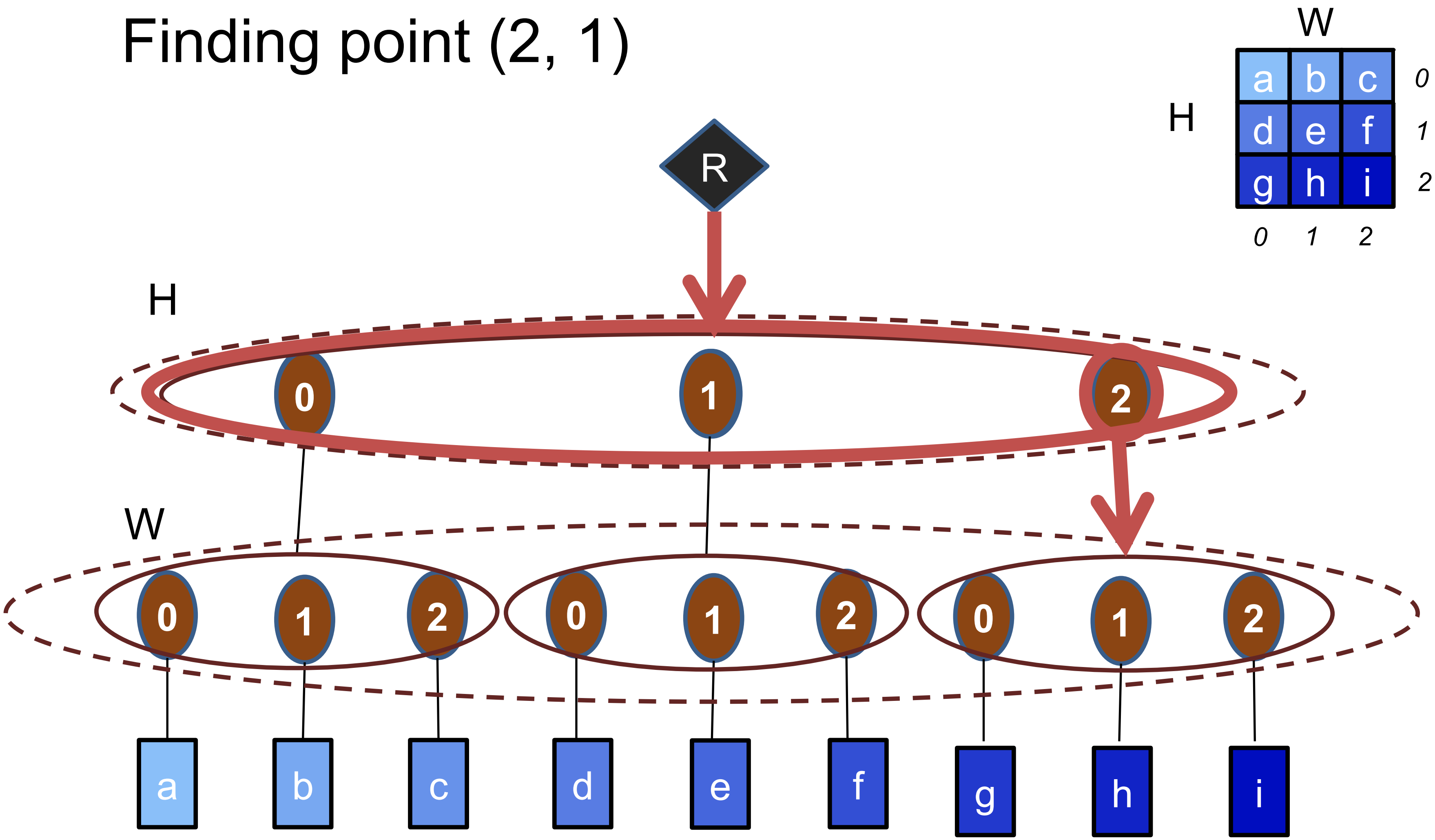

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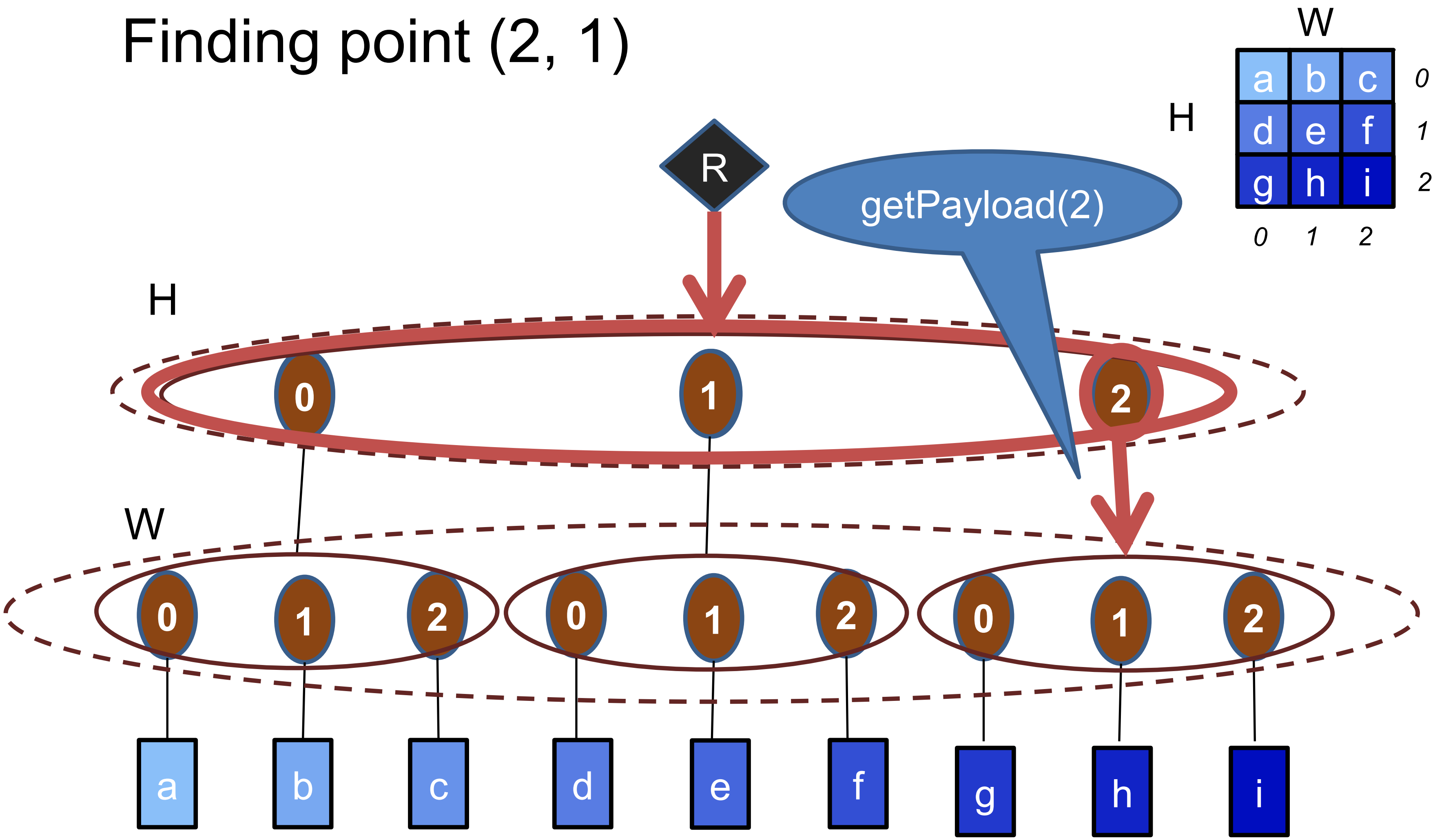

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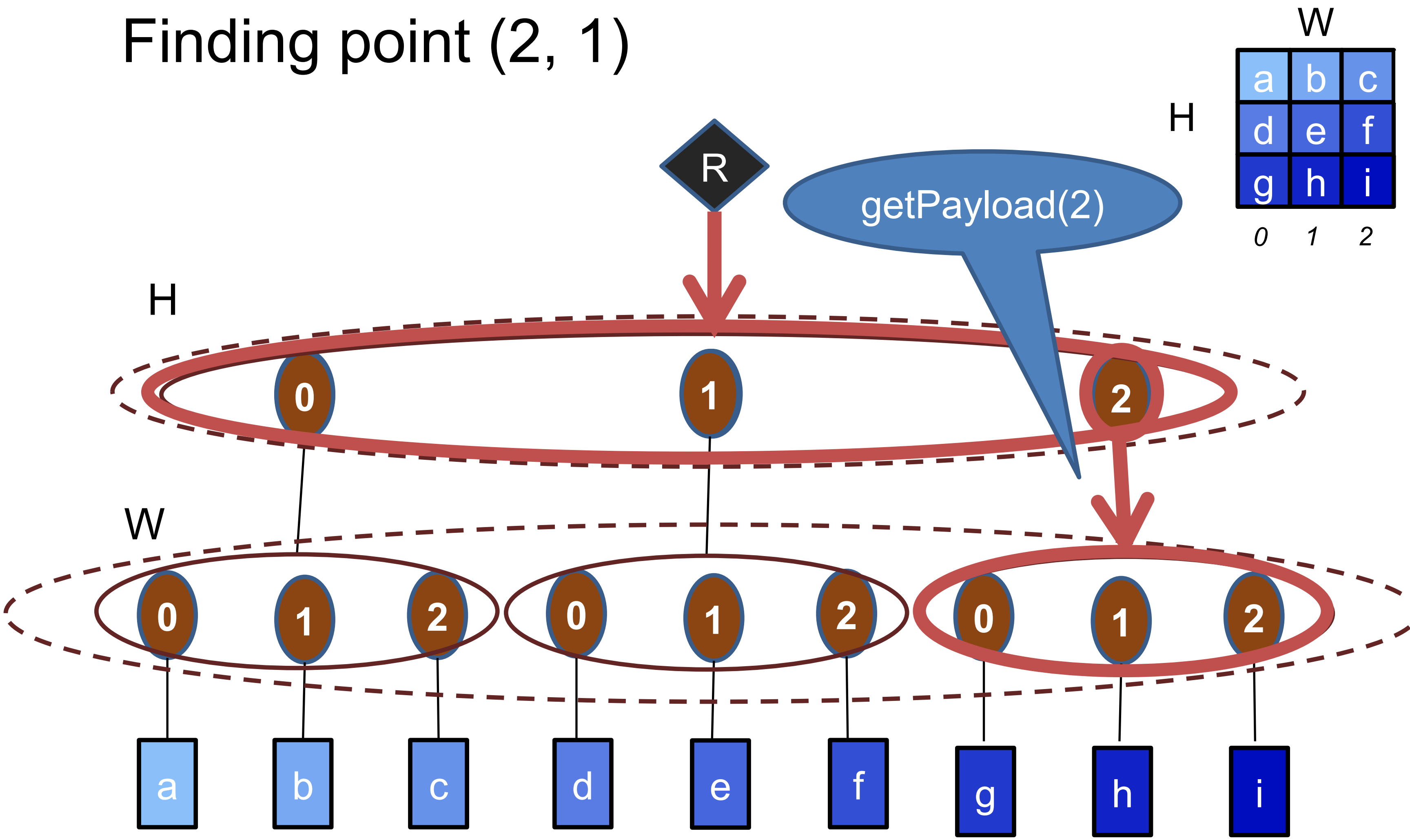

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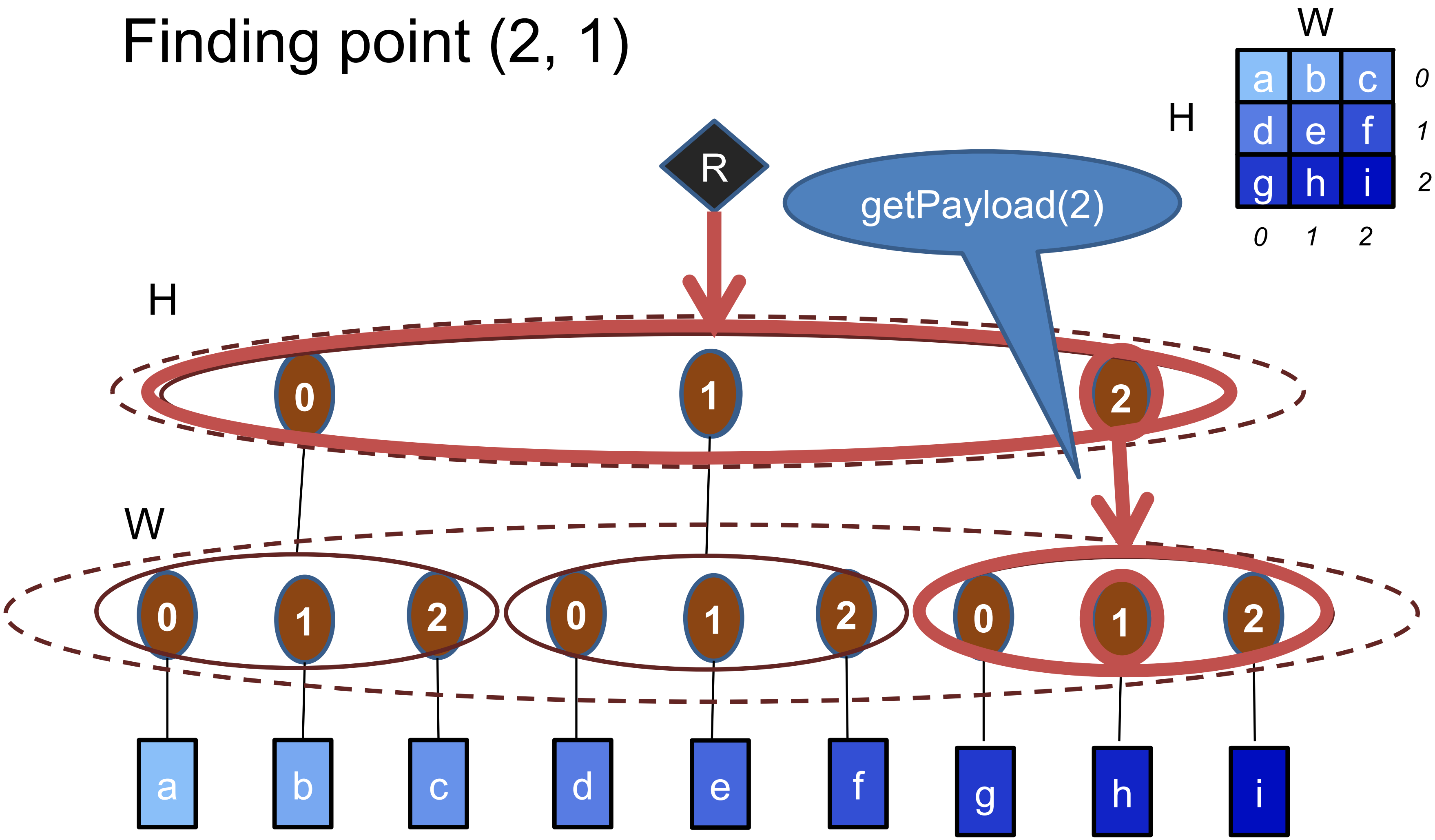

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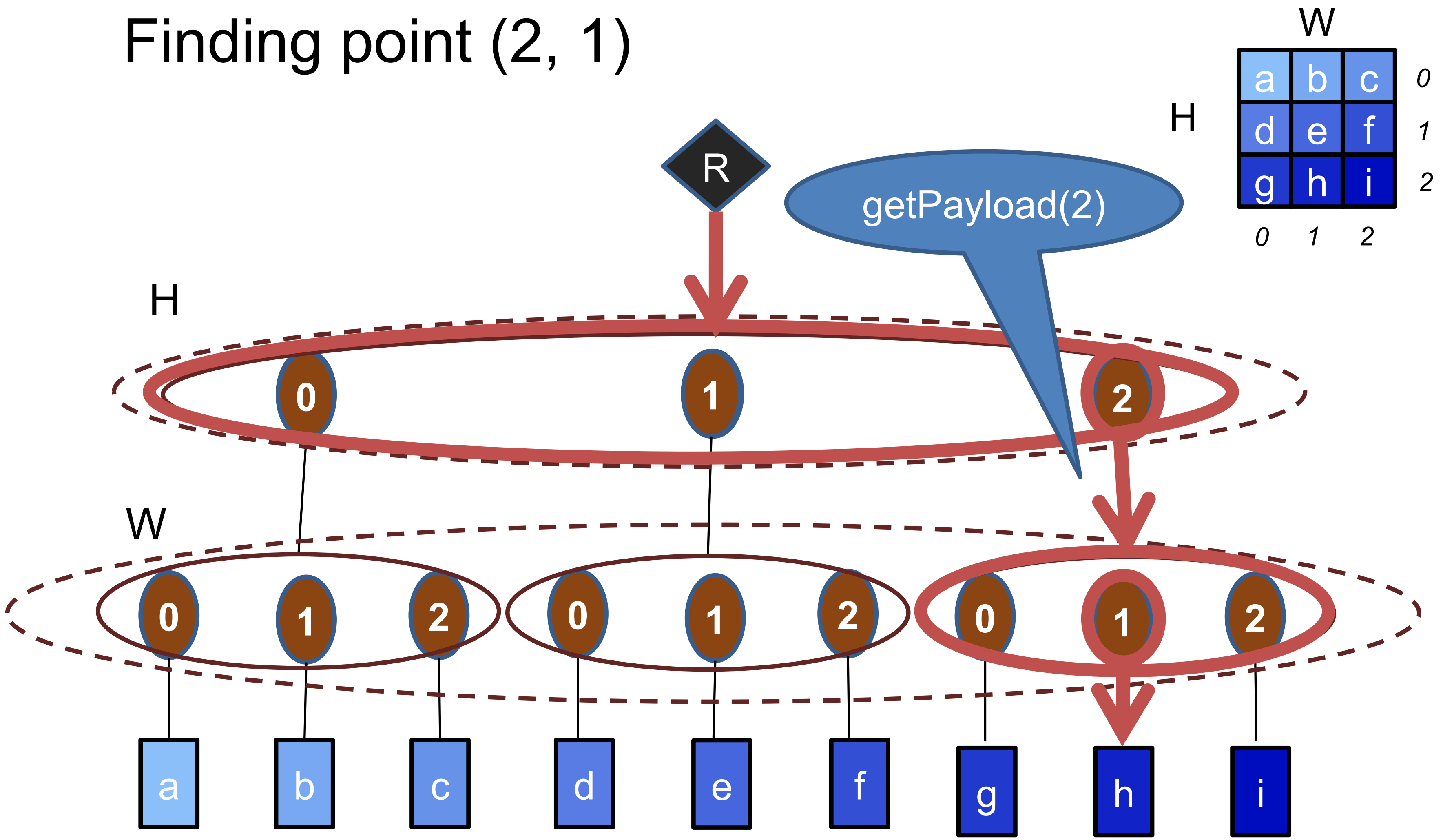

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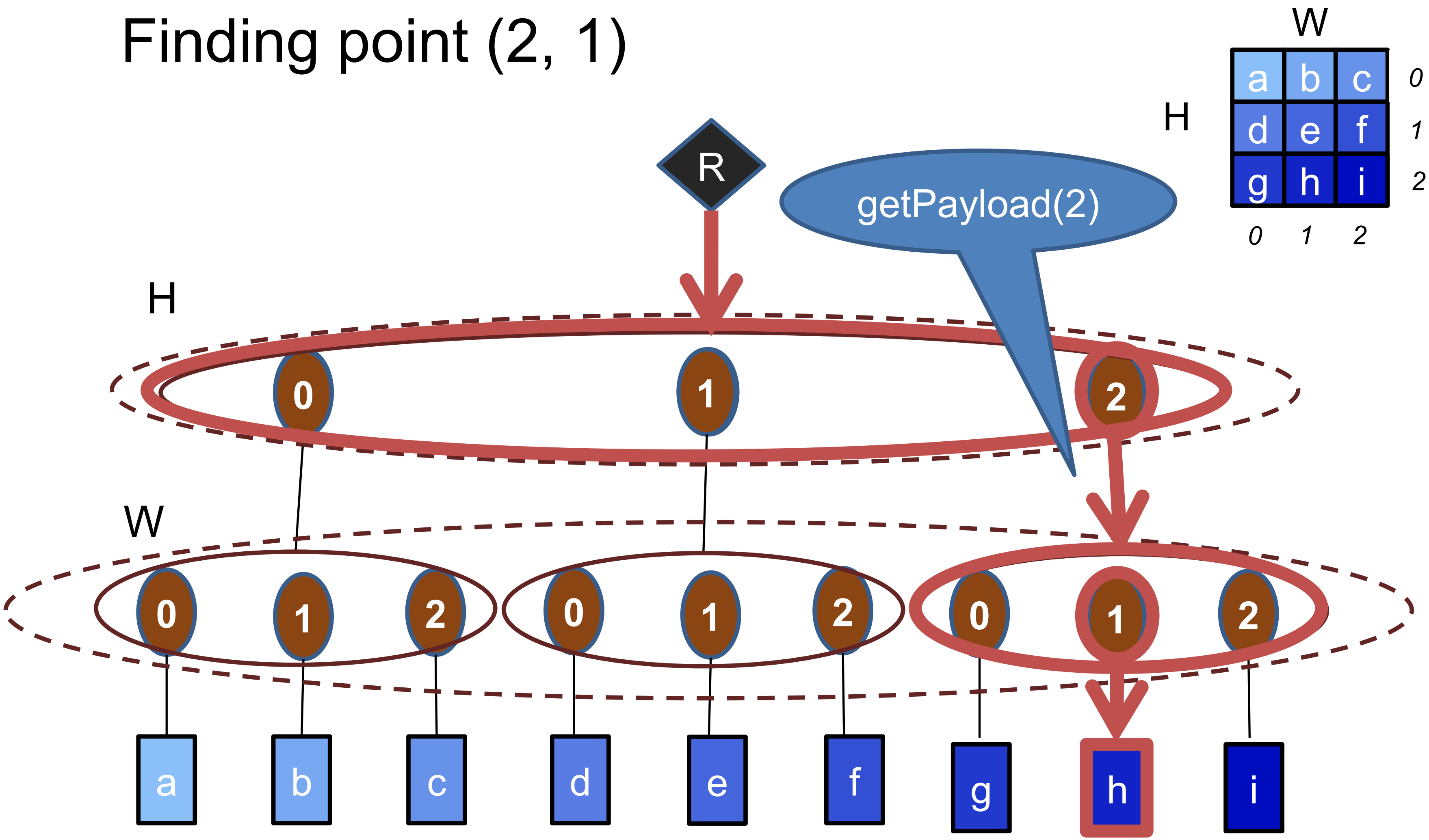

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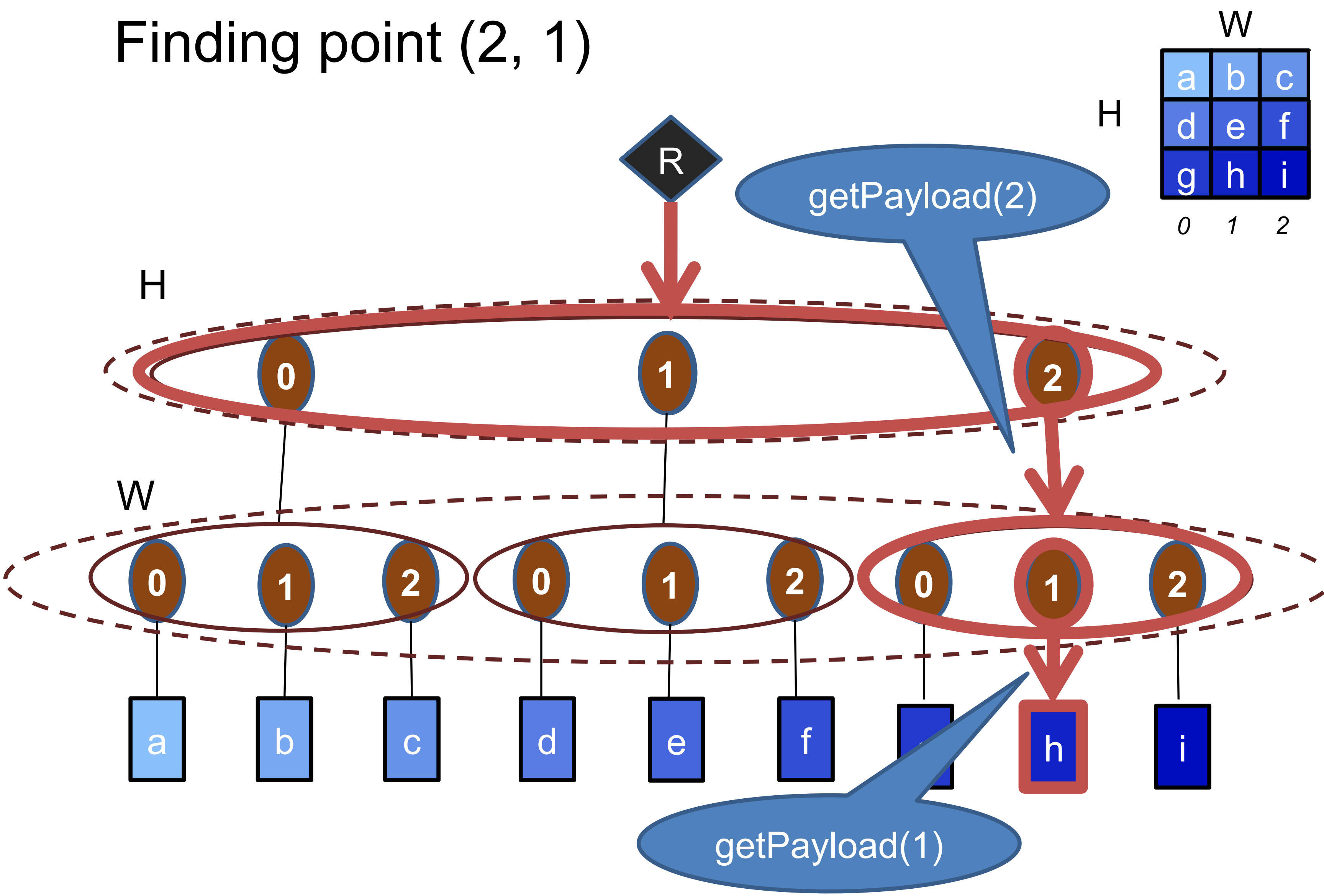

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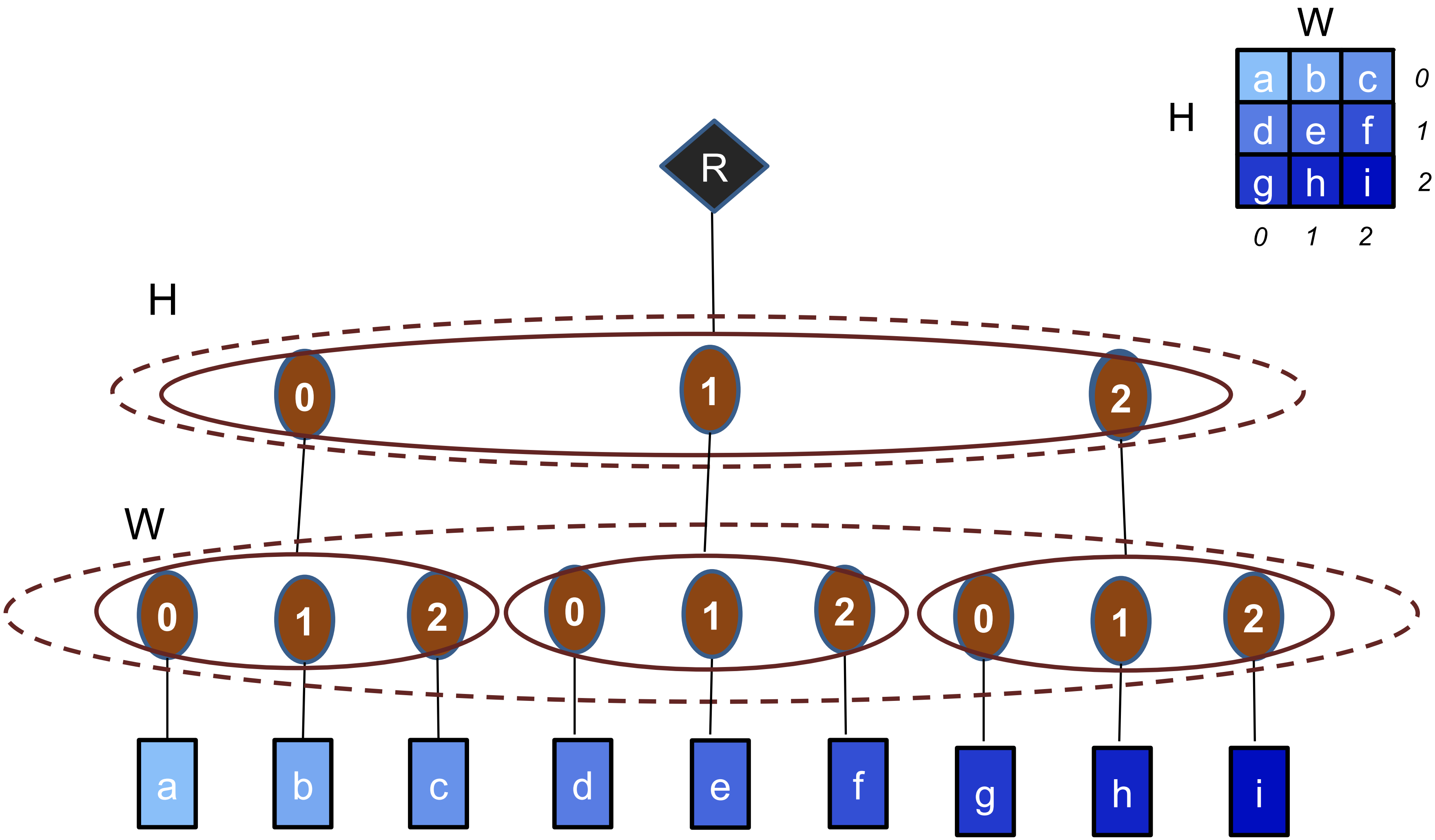
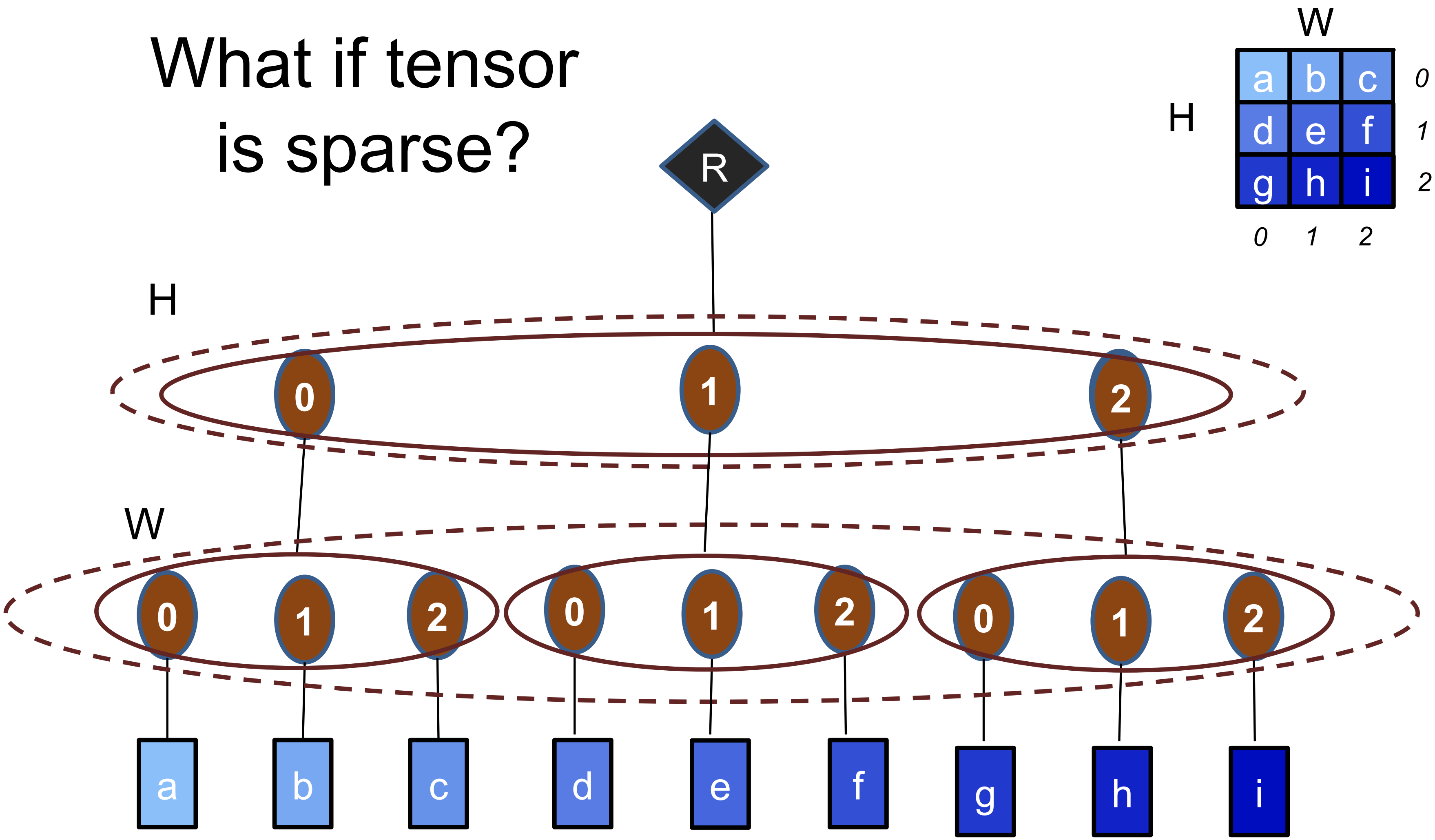


Fibertree Tensor Abstraction

What if tensor is sparse?



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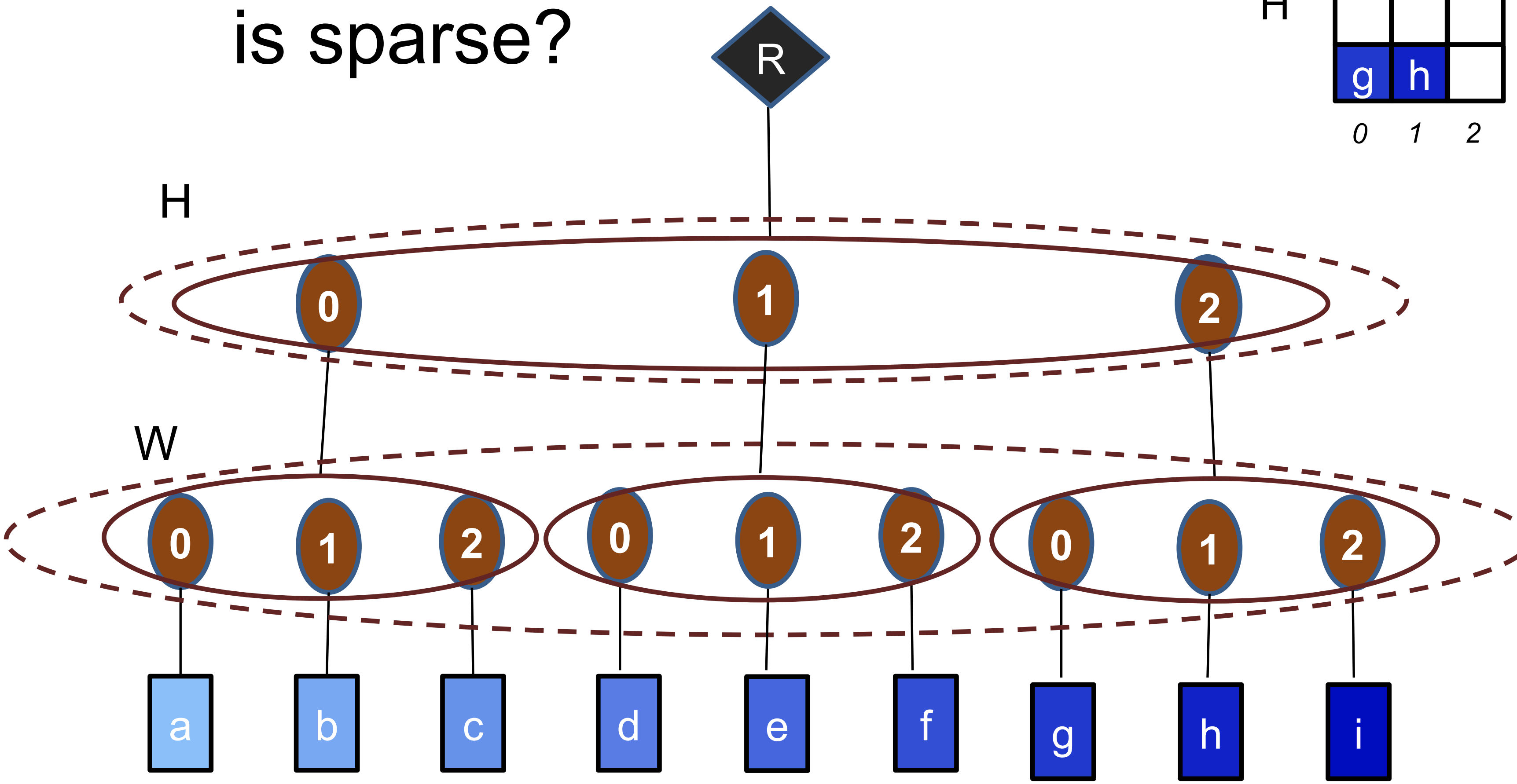

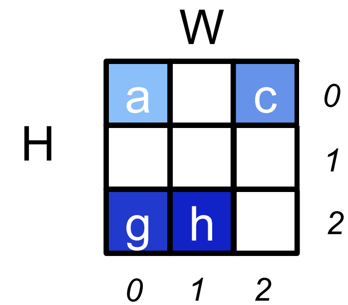

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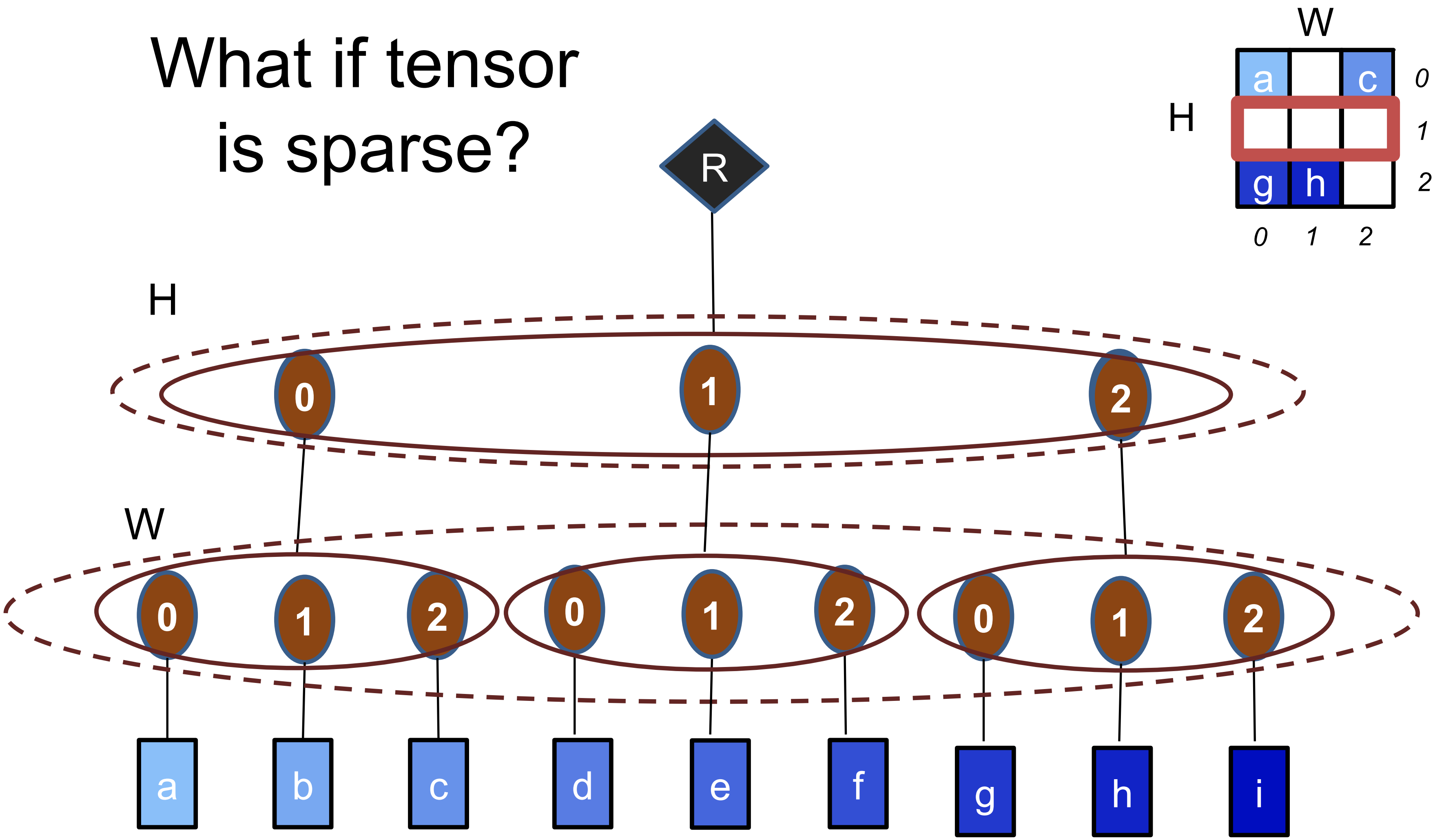

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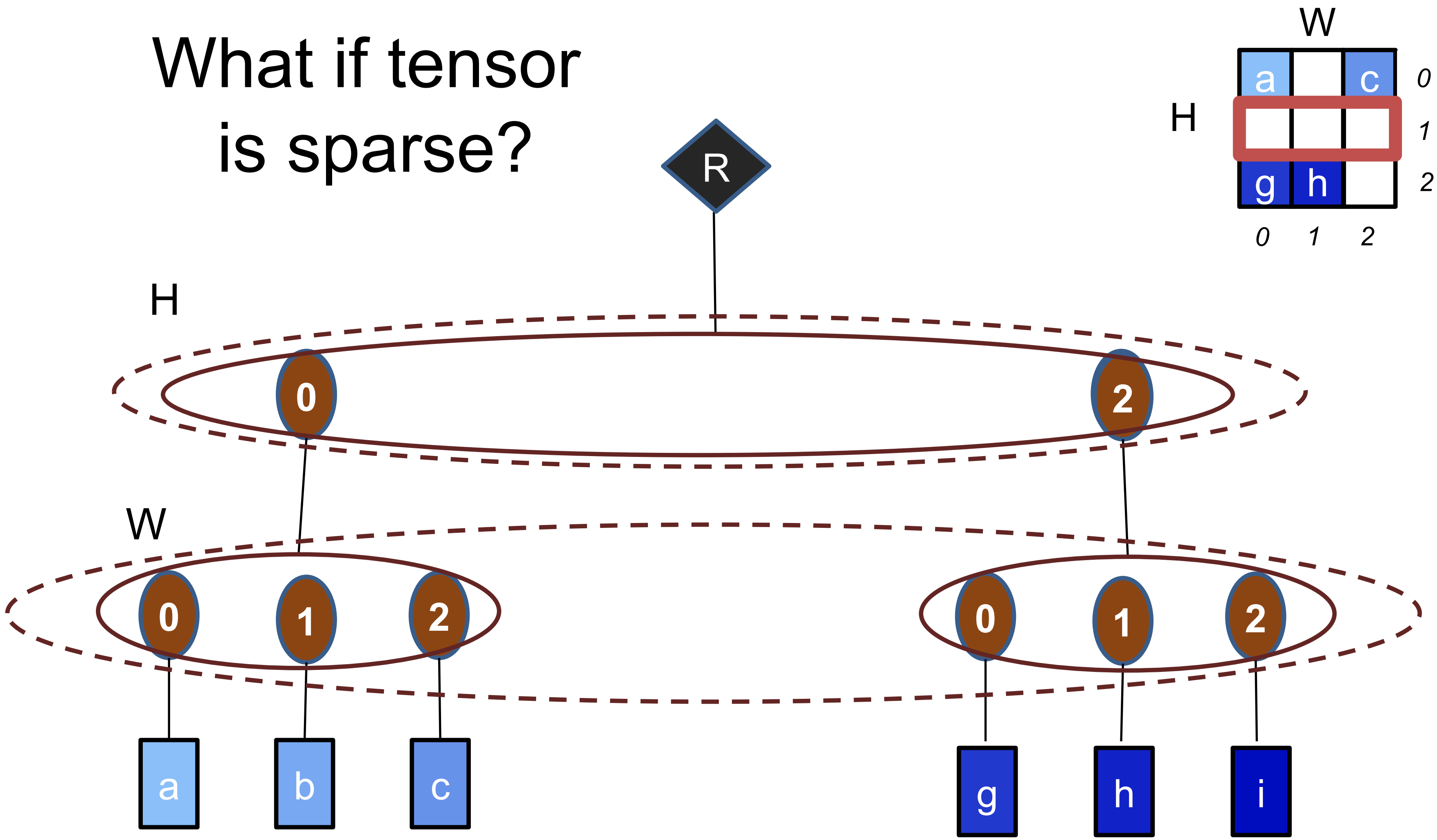

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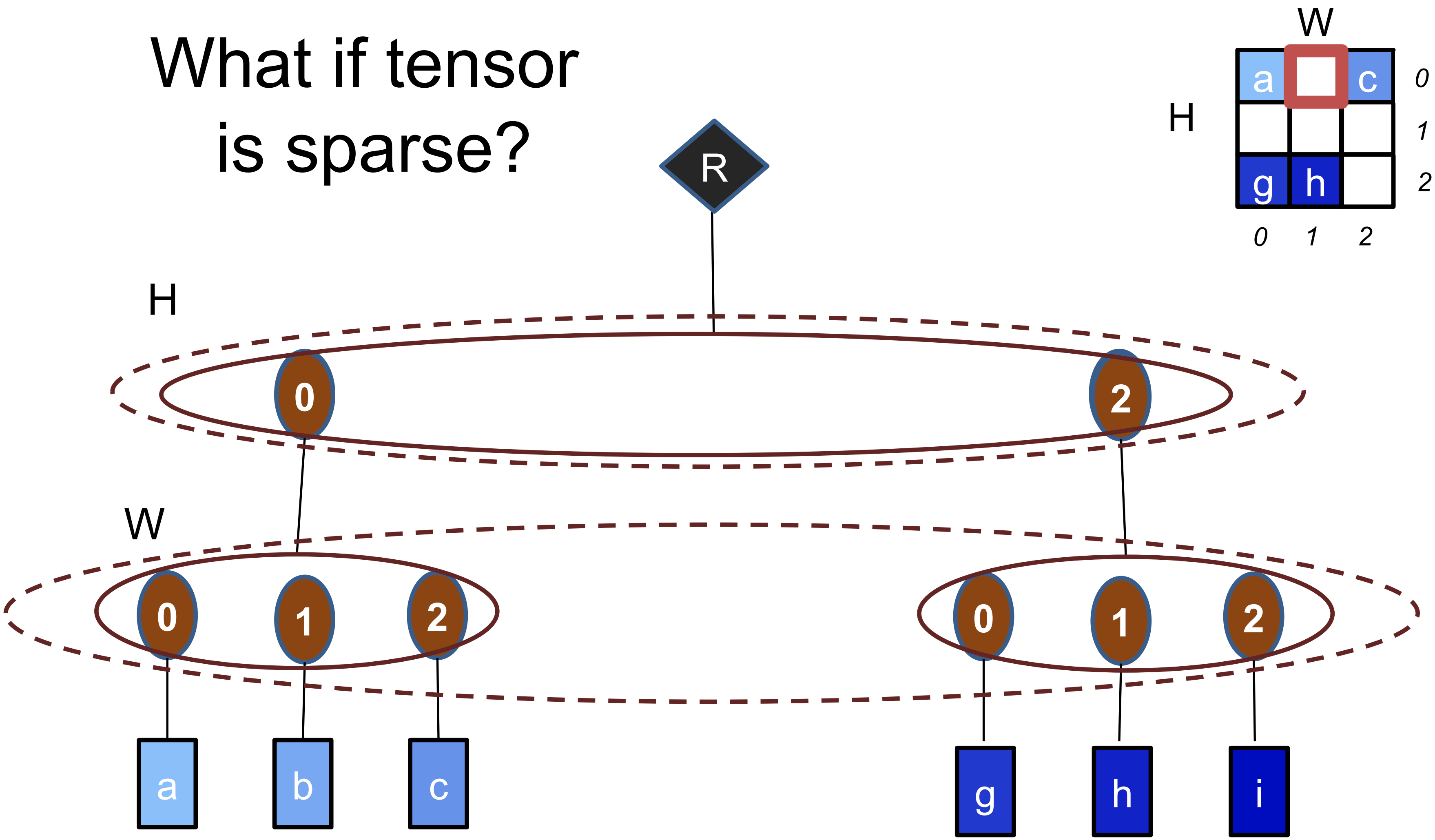

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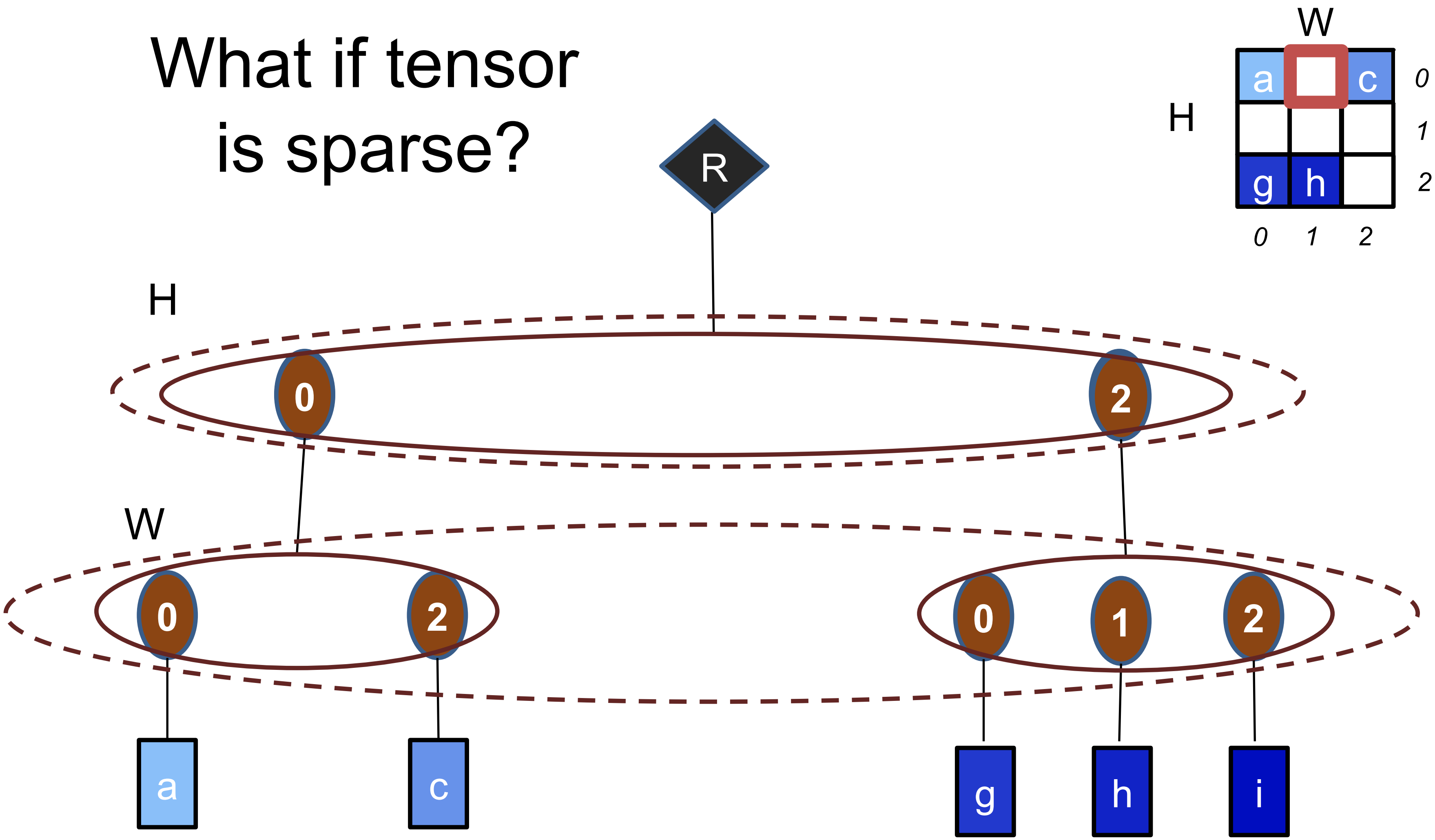

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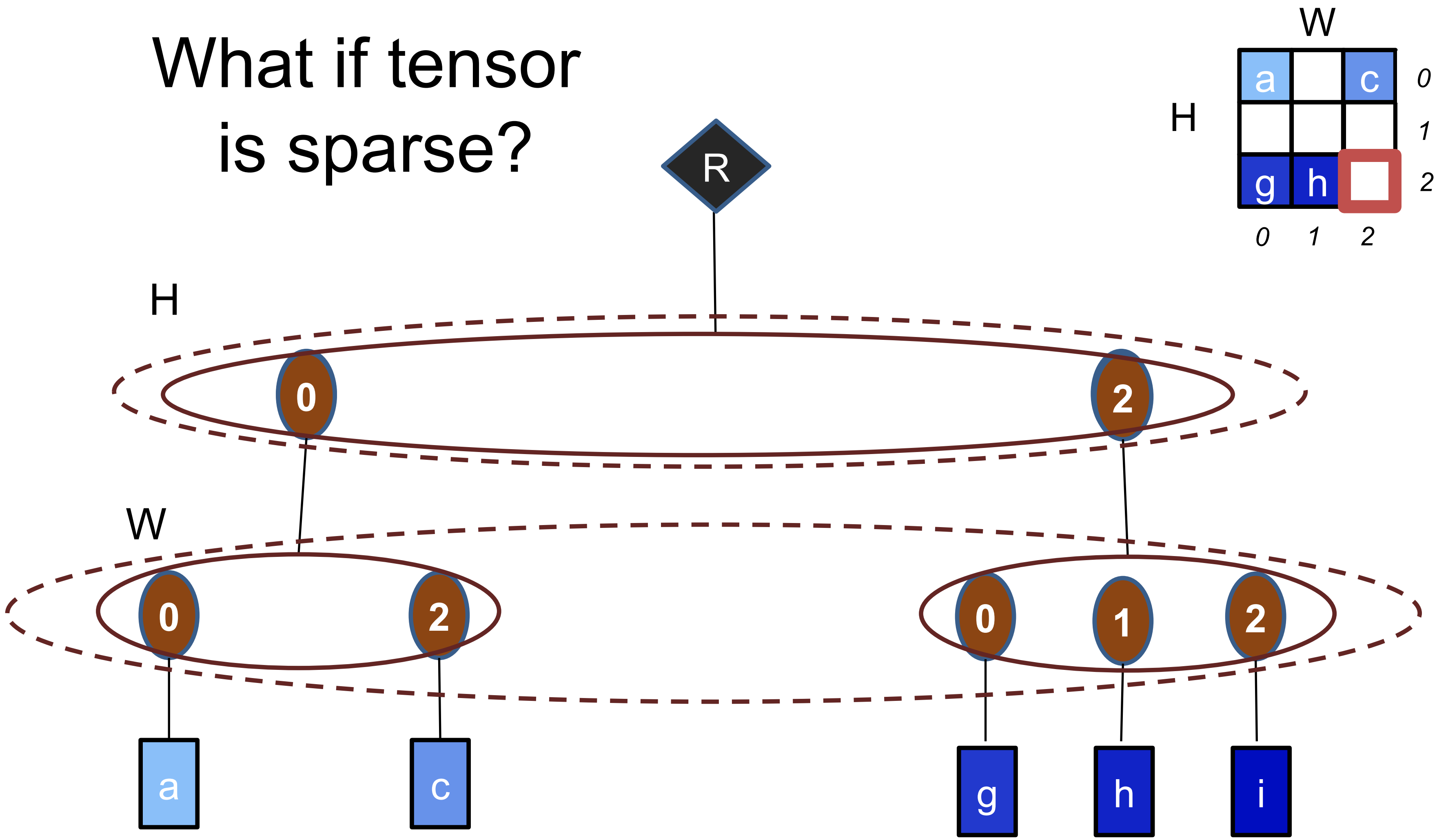

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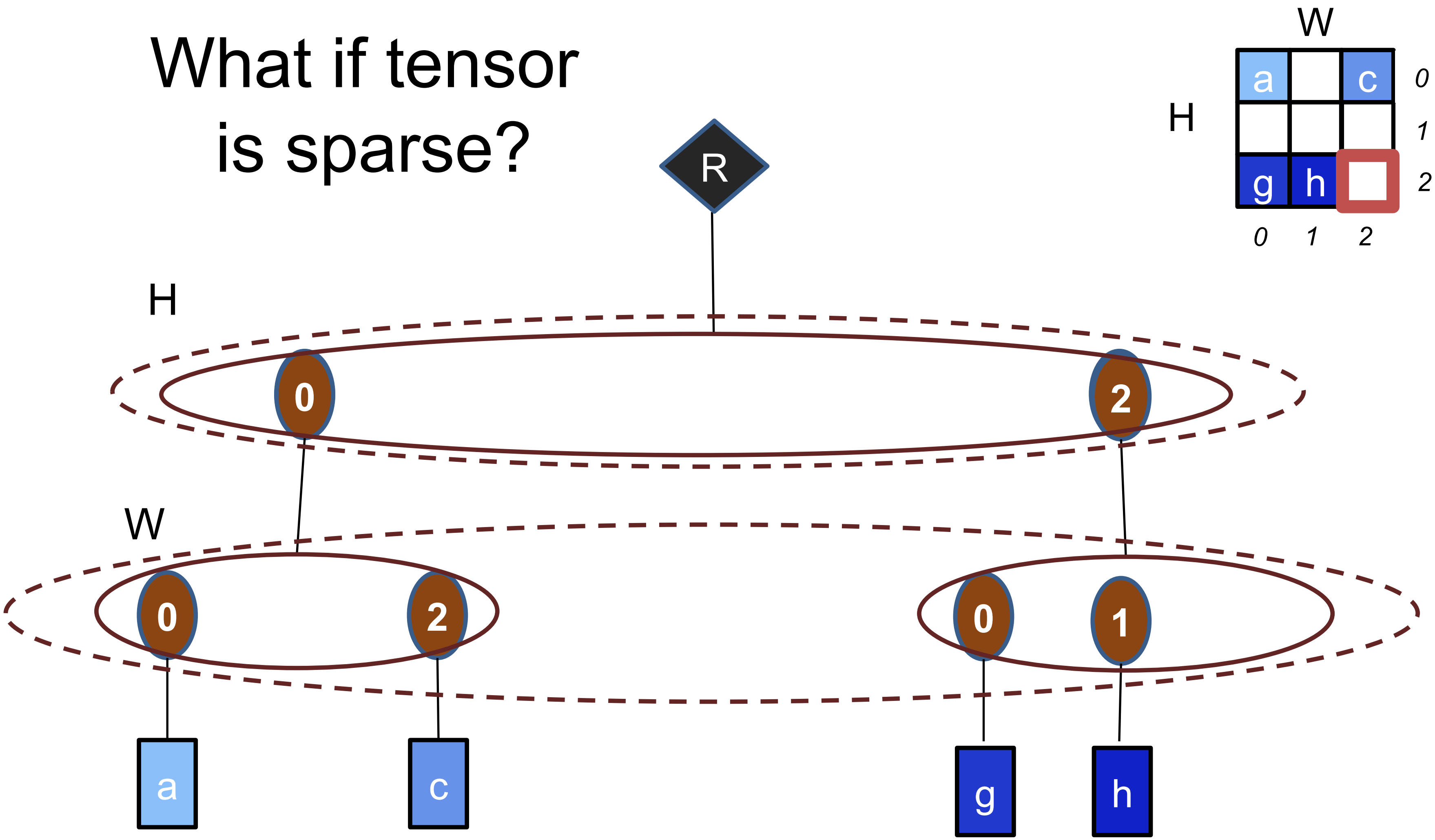

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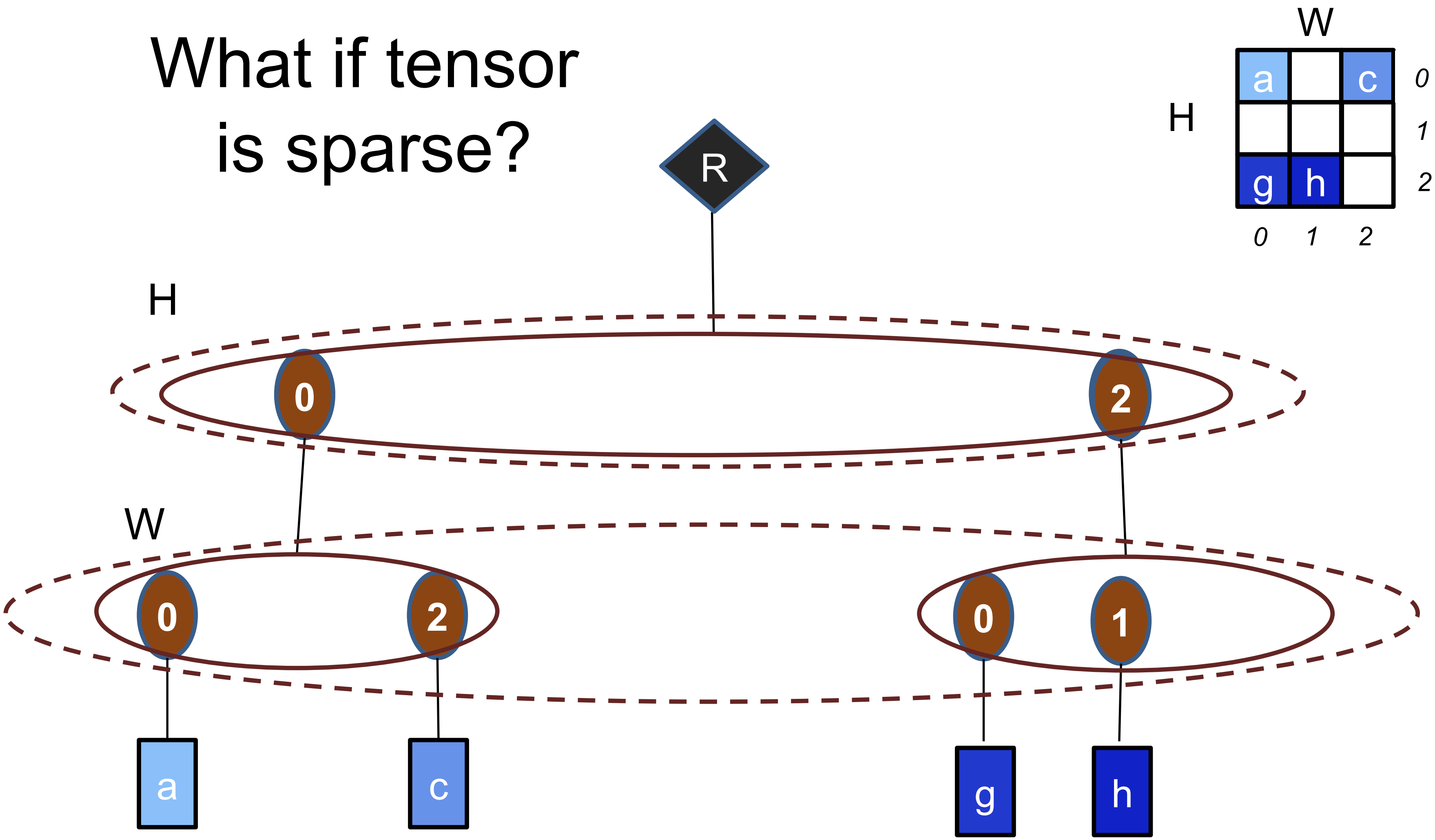
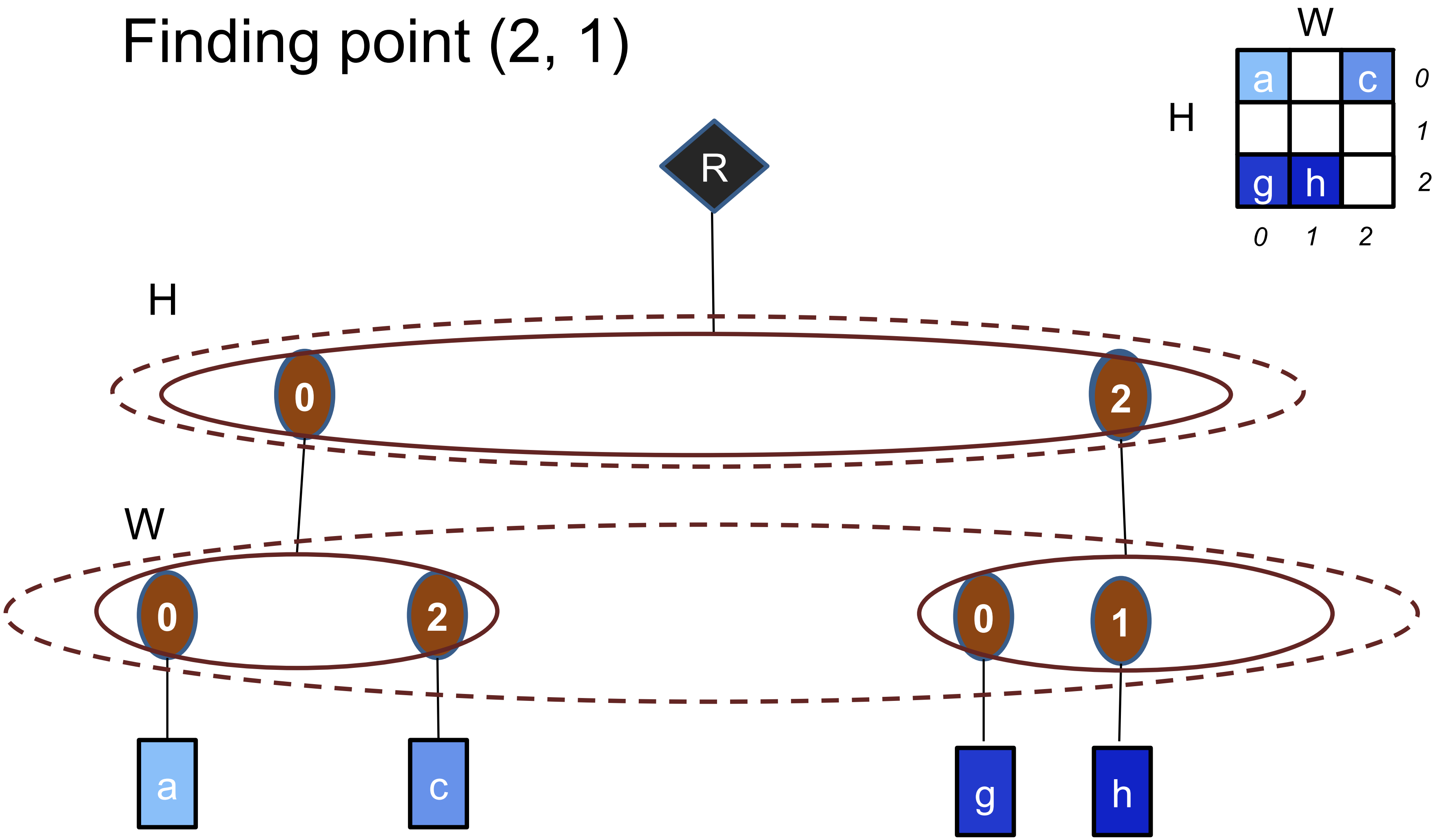


Fibertree Tensor Abstraction

Finding point (2, 1)



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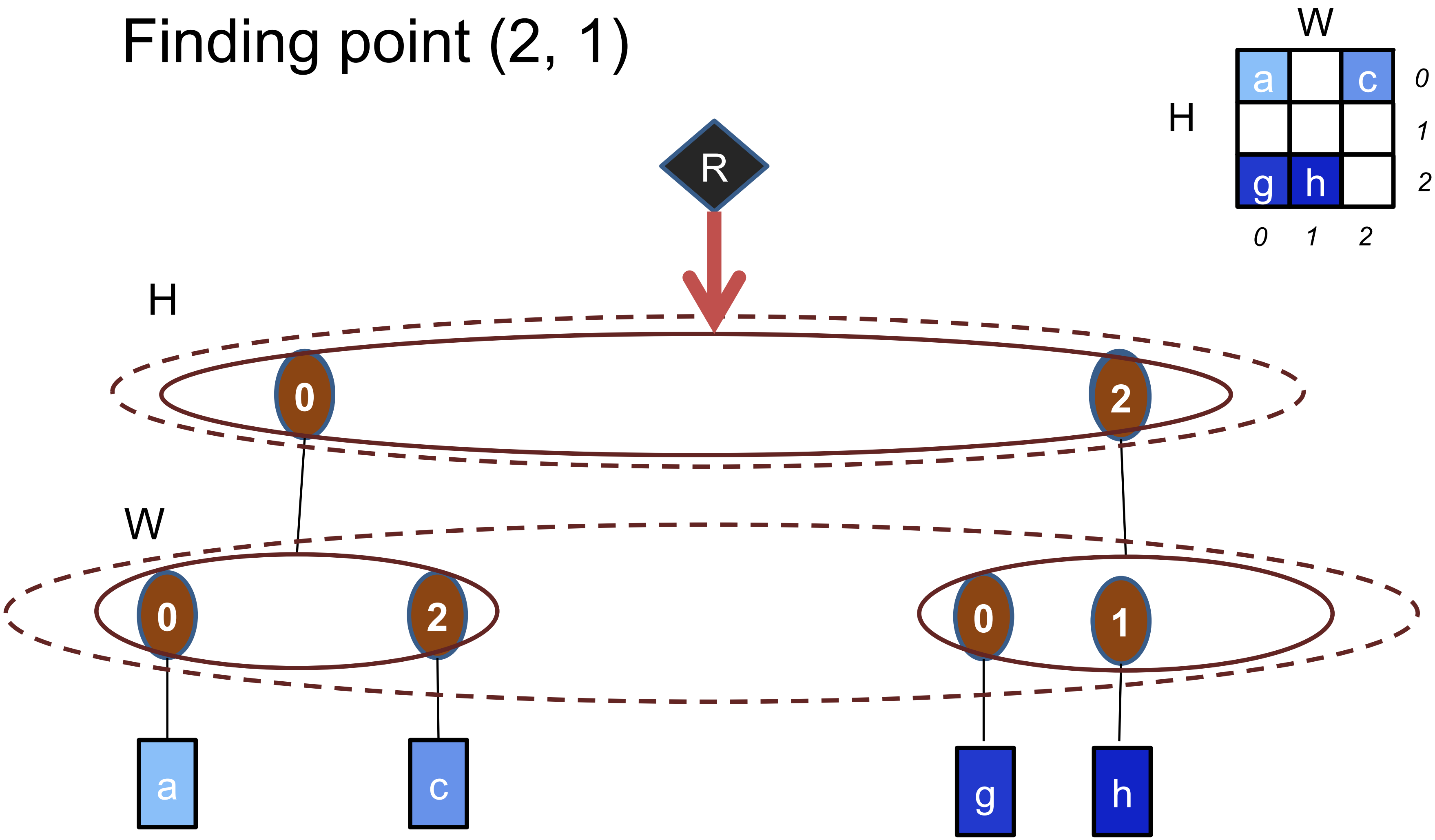

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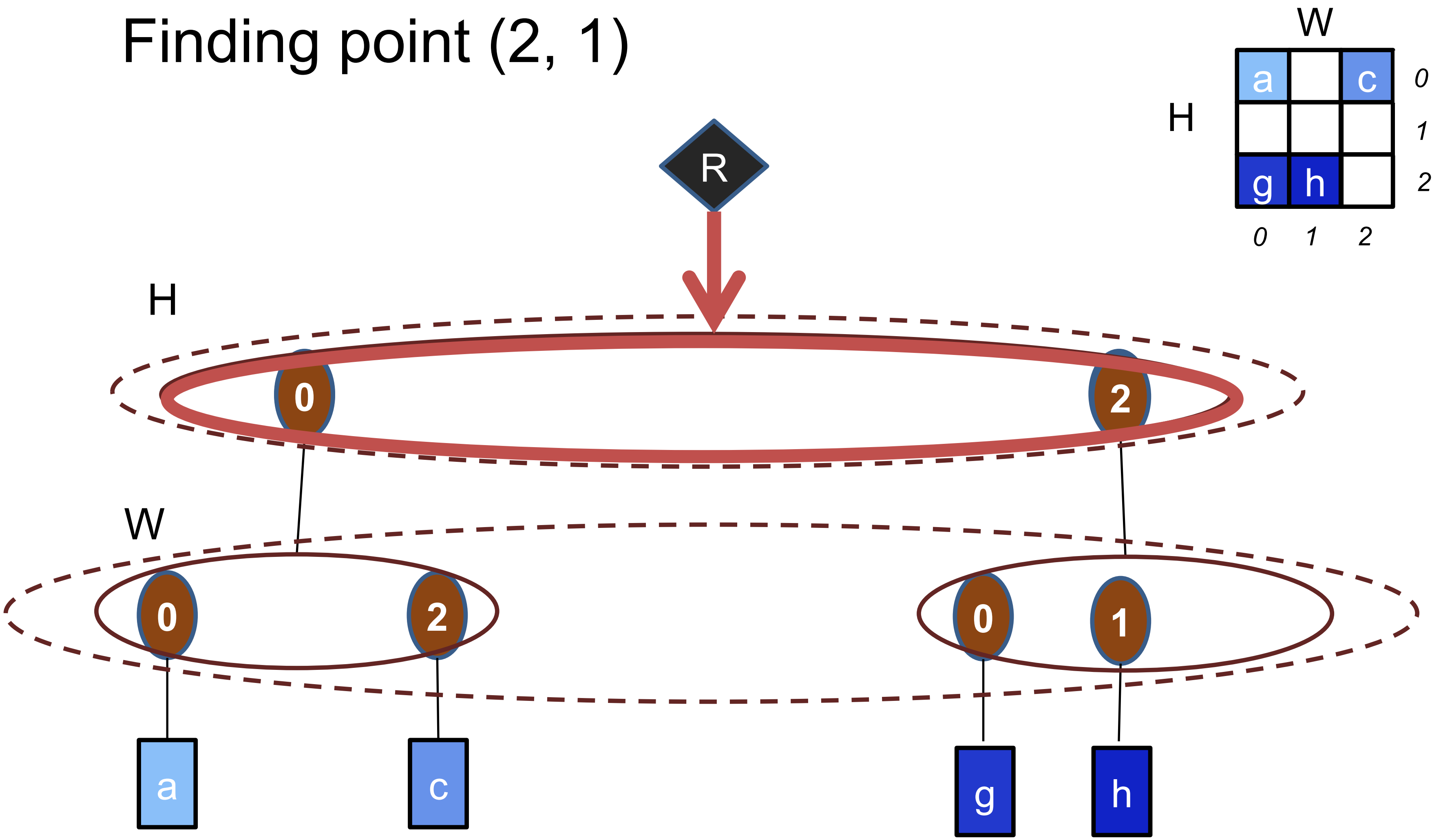

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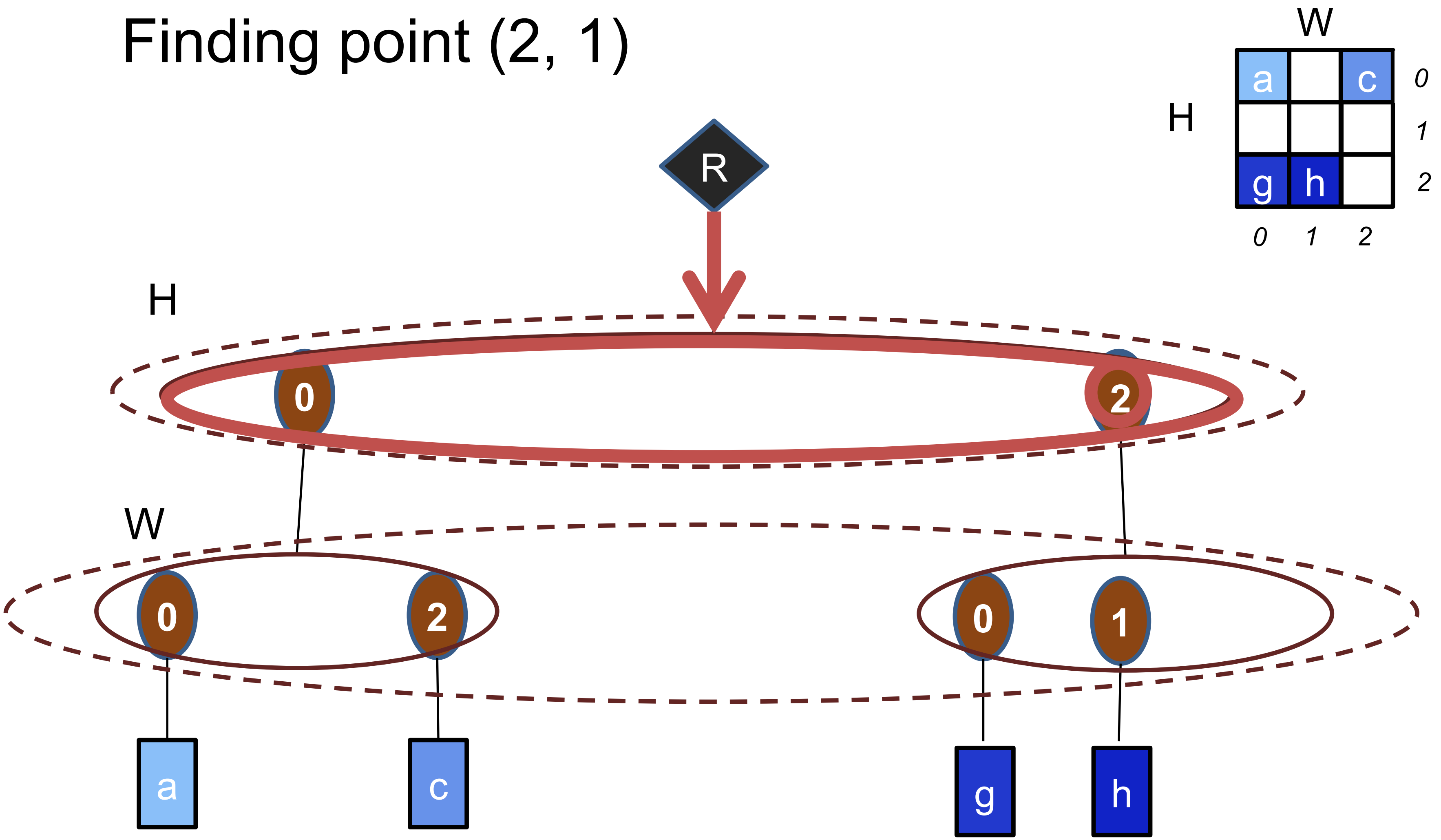

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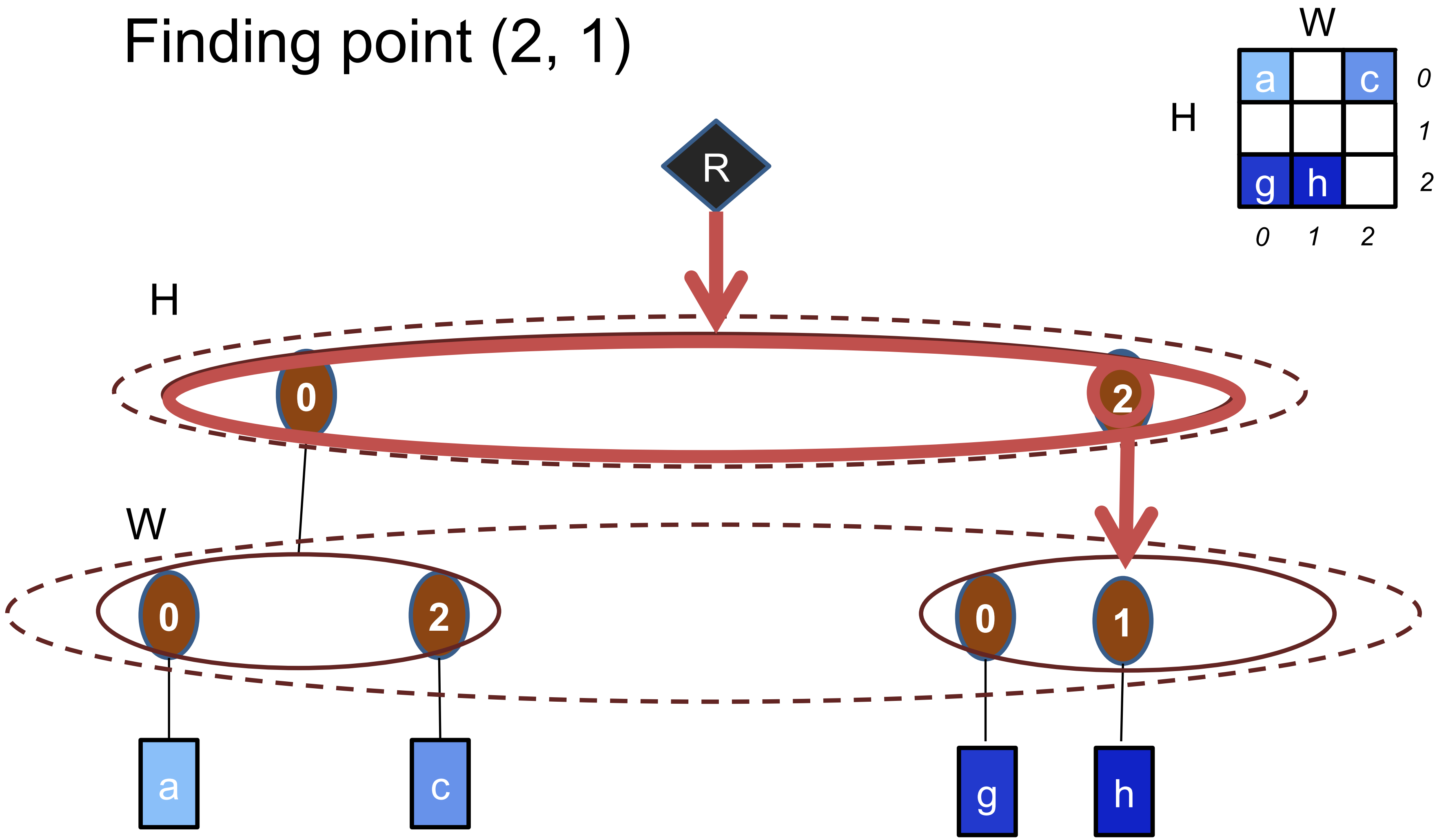

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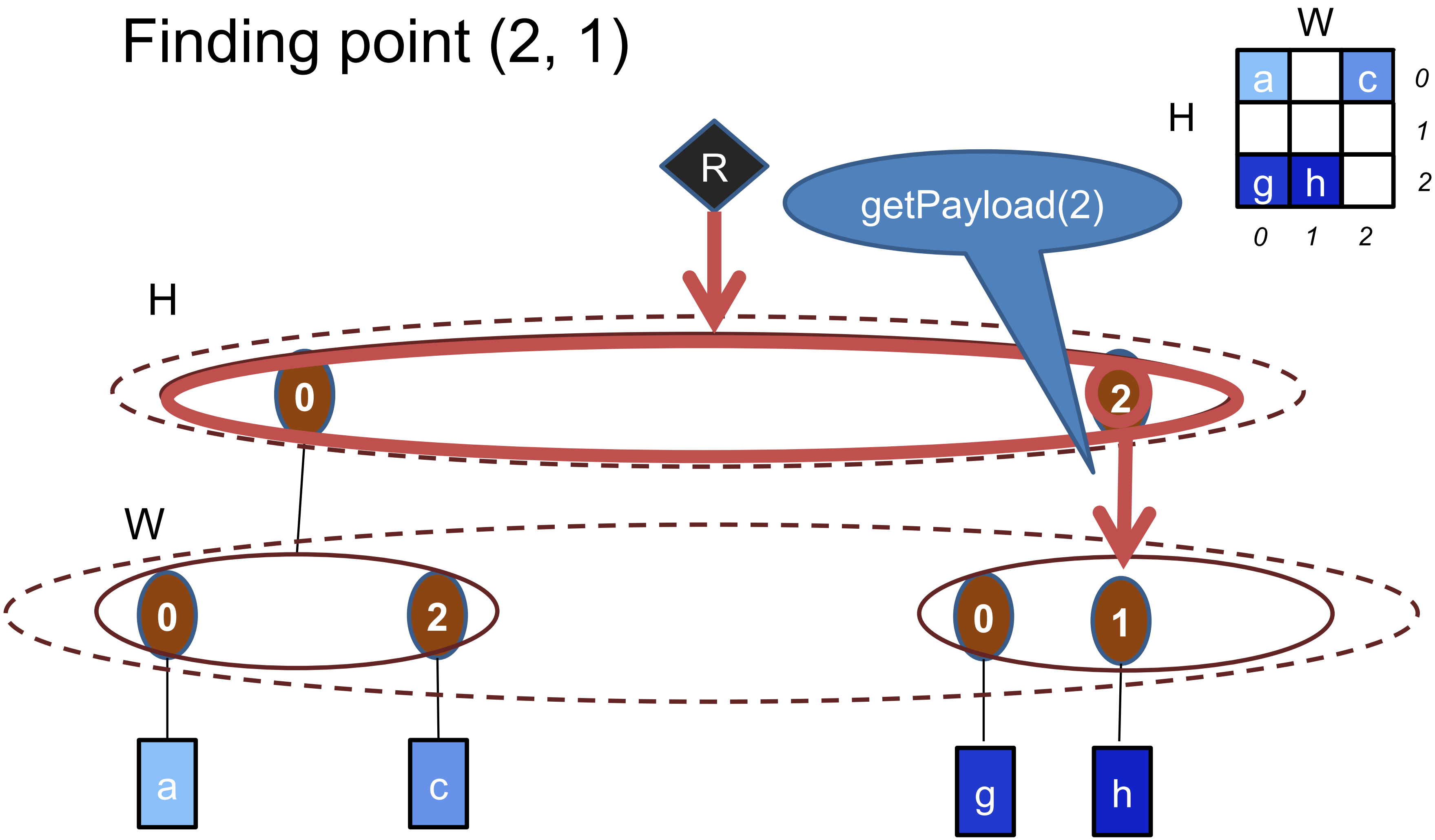

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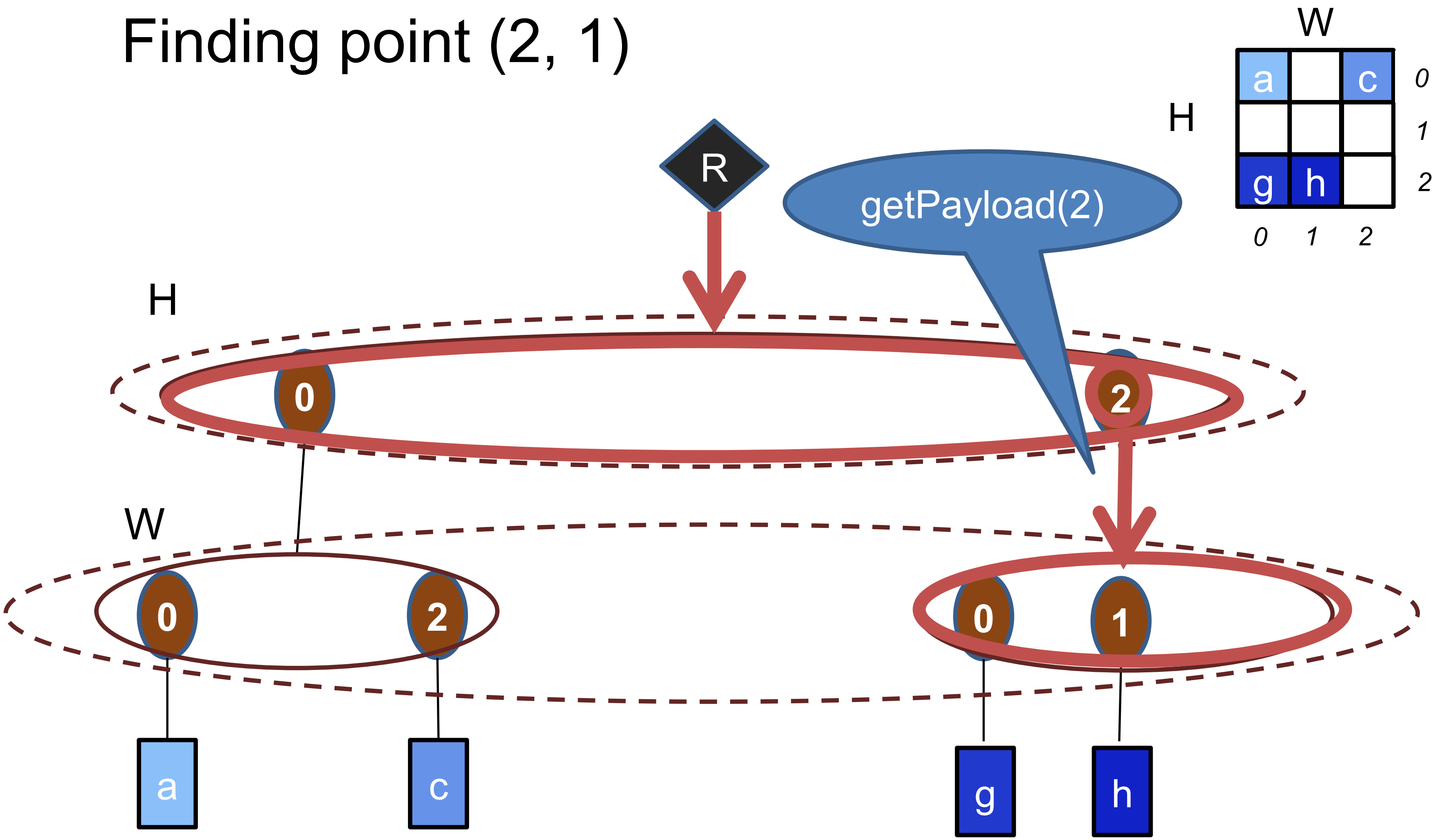

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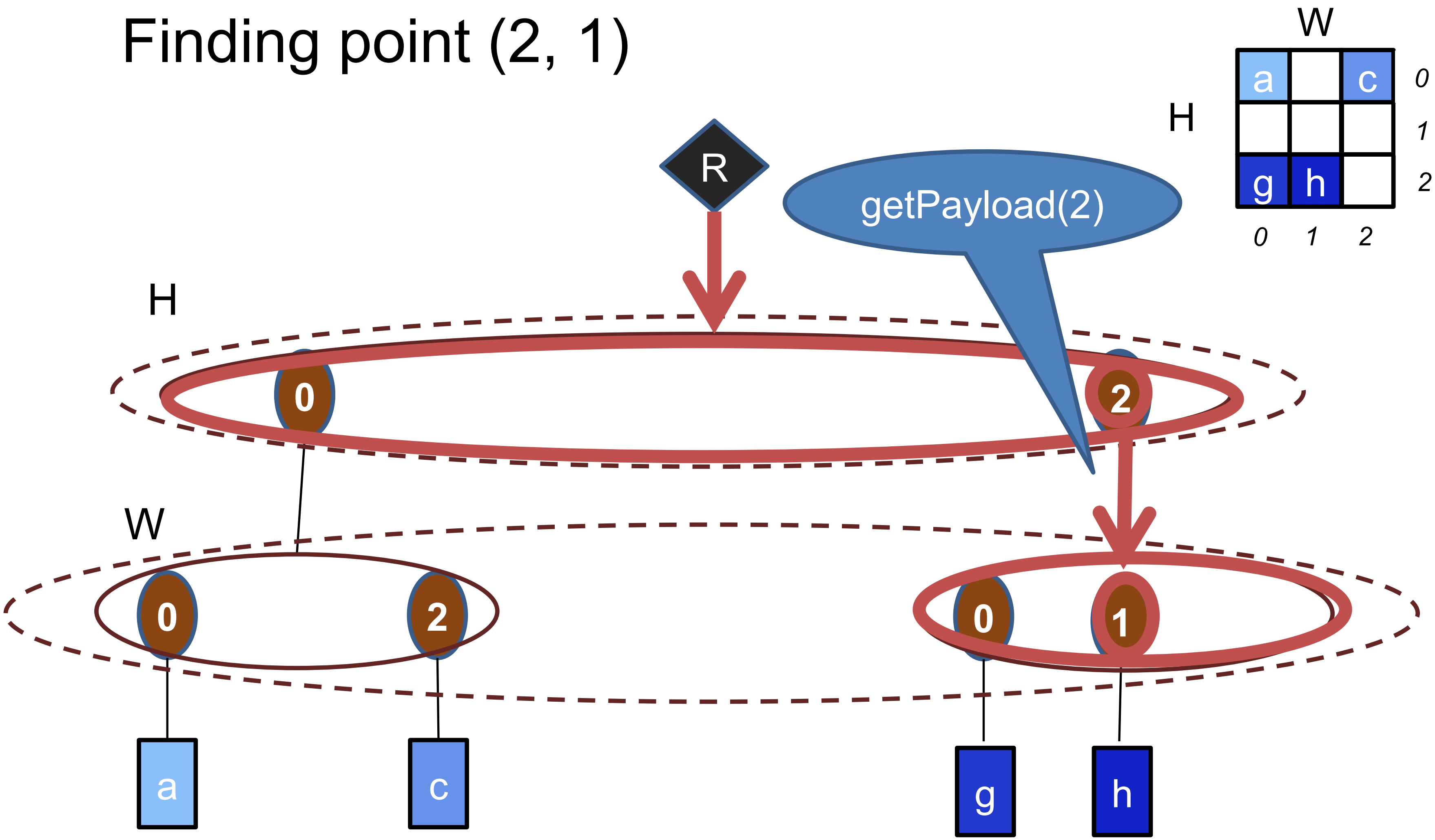

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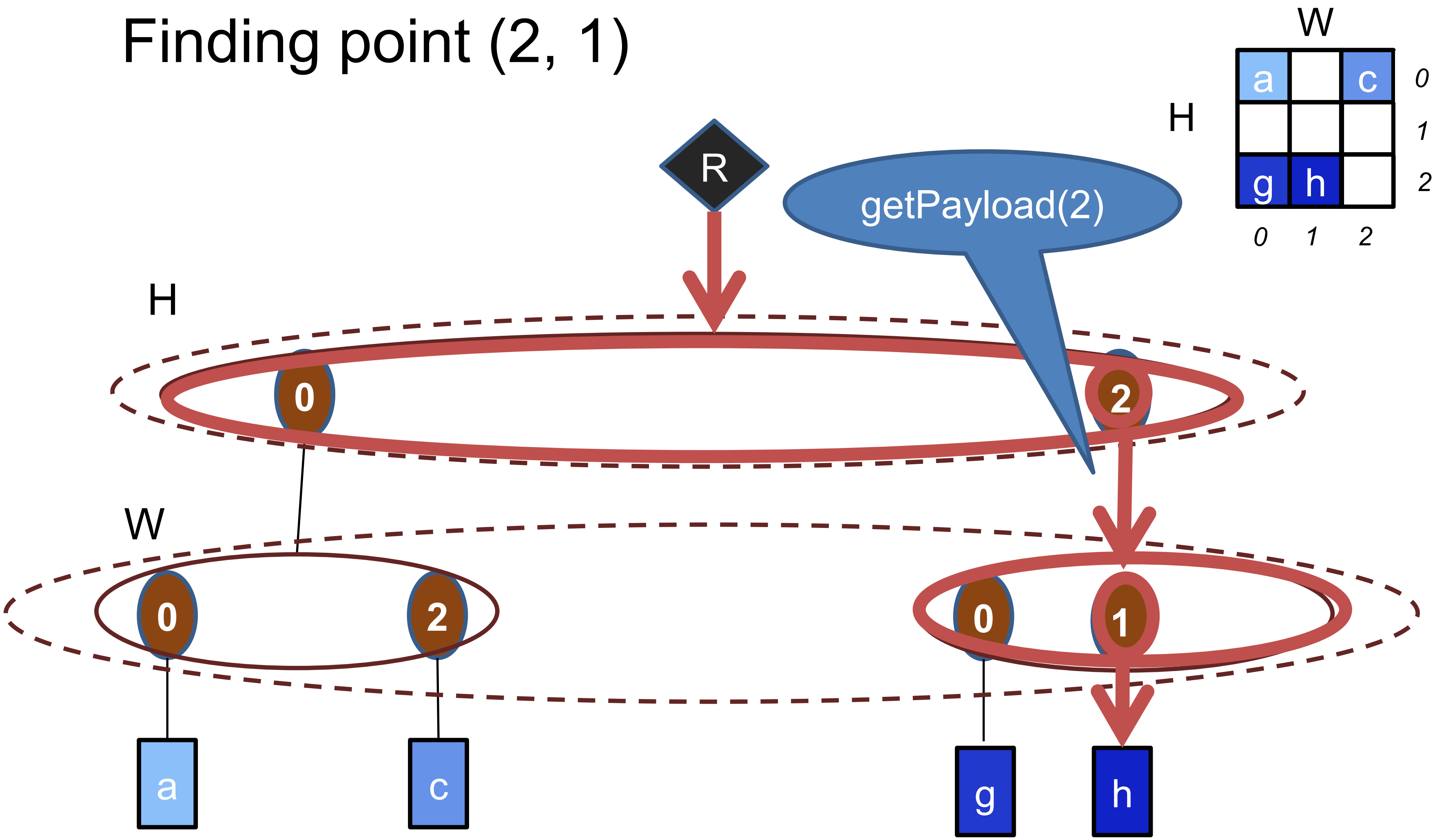

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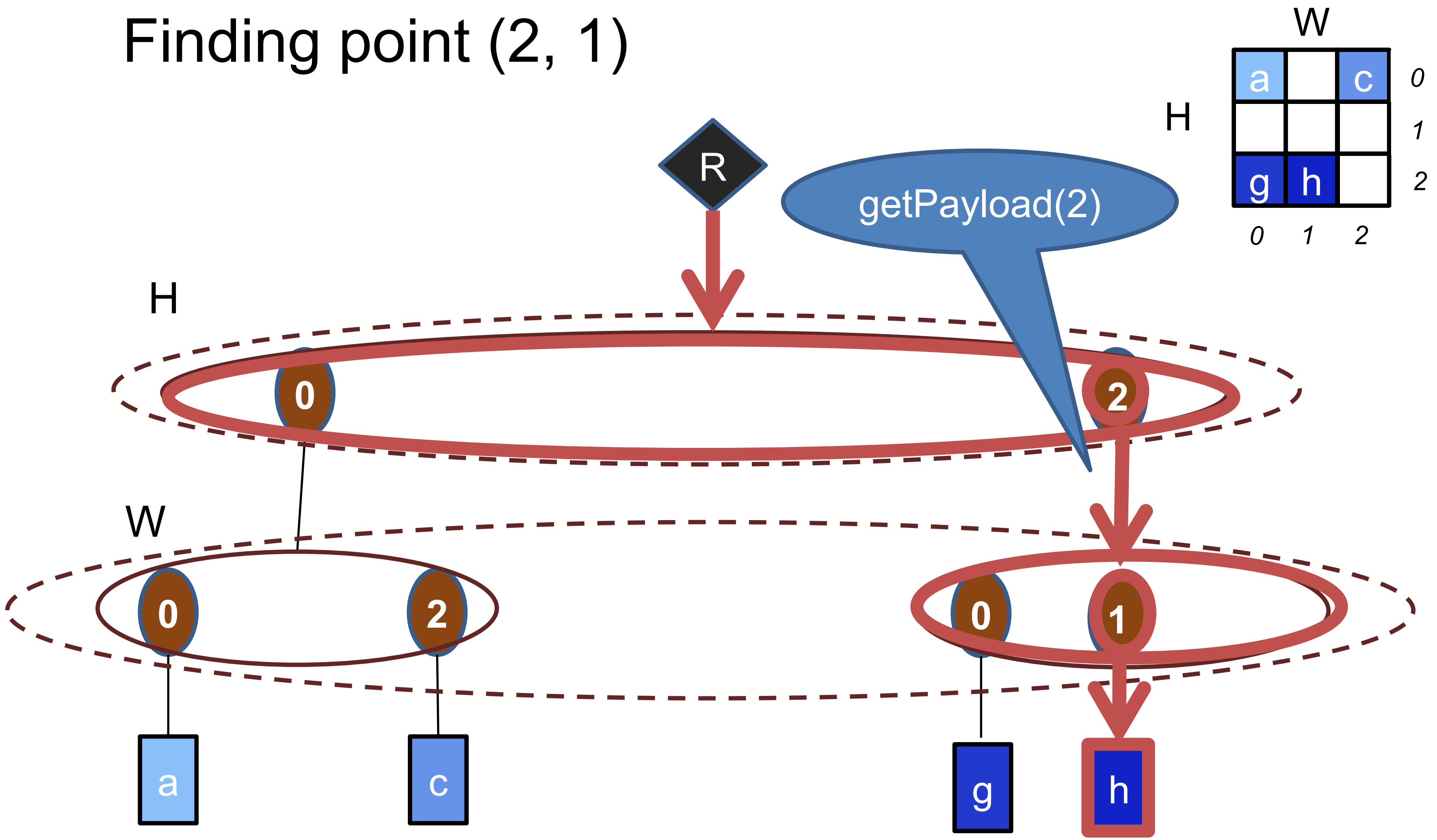

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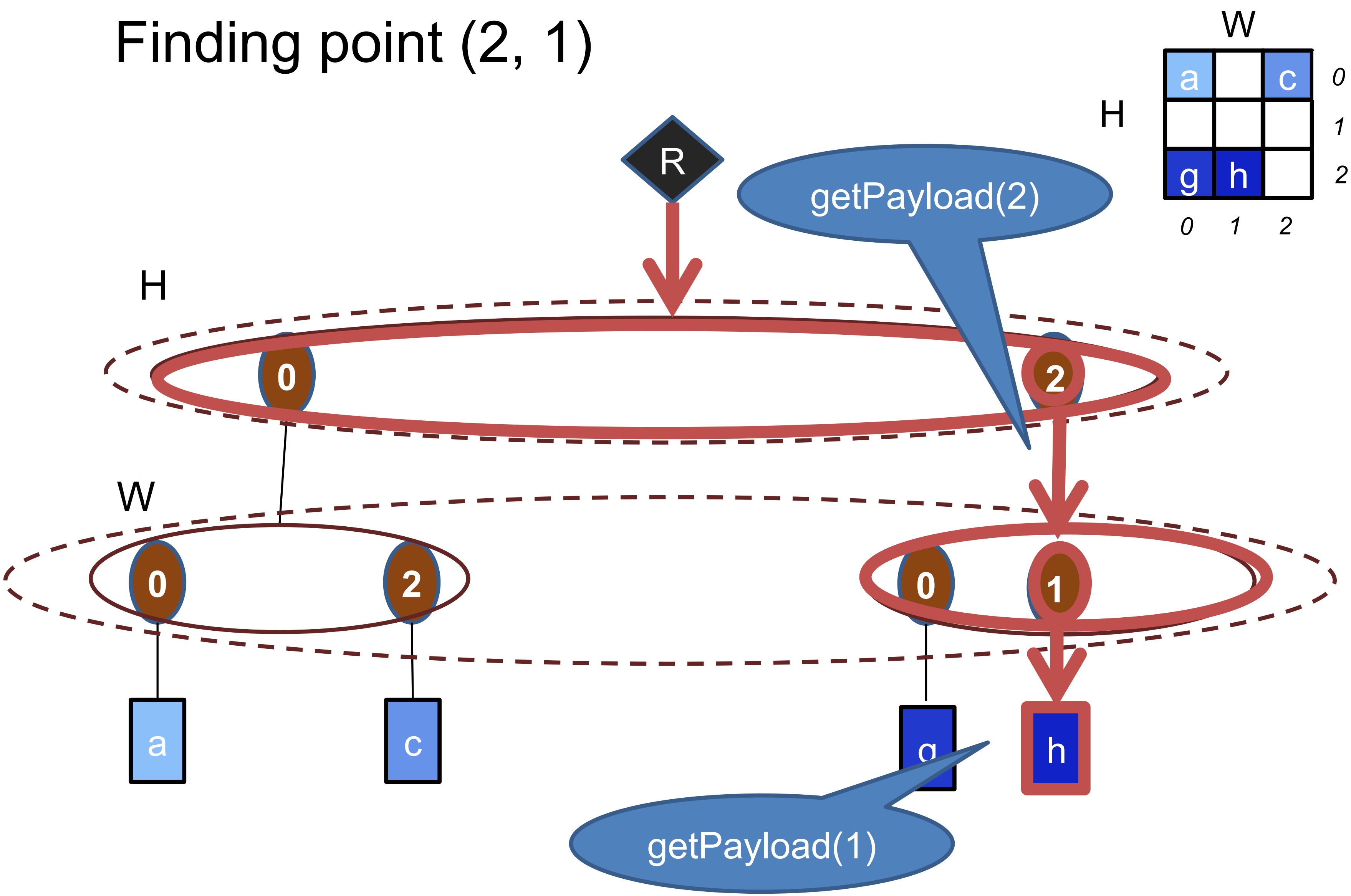

Tensor Traversal (2-D)

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# 2-D Tensor Traversal

t = Tensor(H,W)

sum = 0
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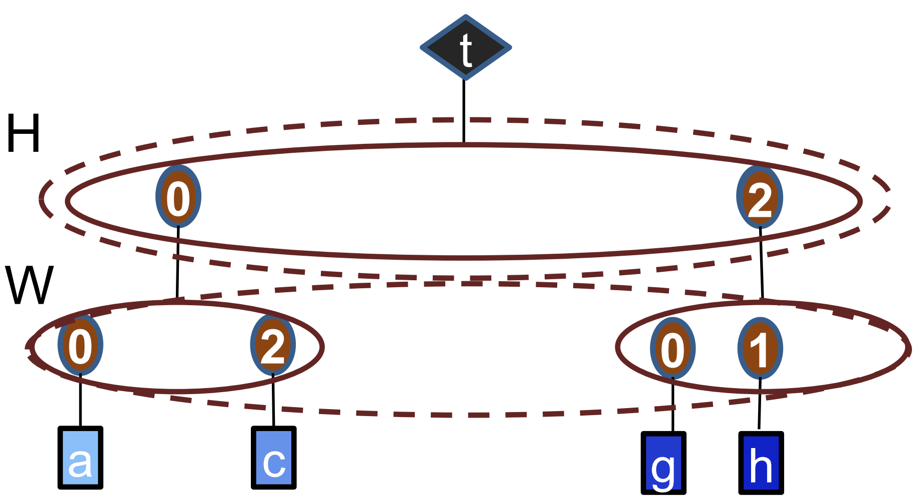

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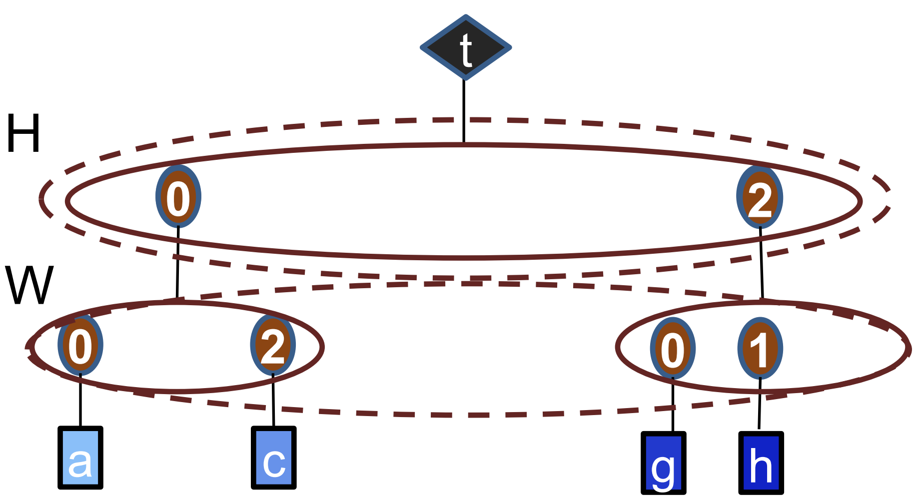

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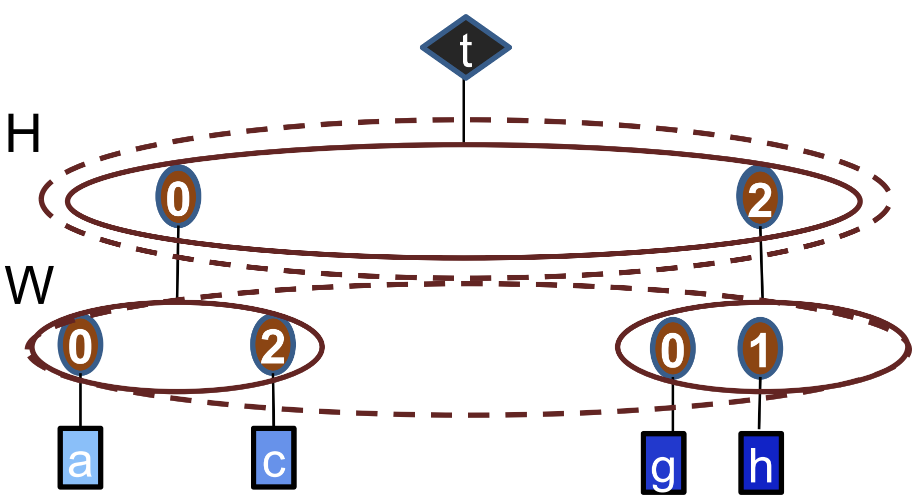

t_pos	h	t_h_pos	w	t_val
0	0	?	?	?
0	0	0	0	a
0	0	1	2	c
1	2	?	?	?
…	…	…	…	…

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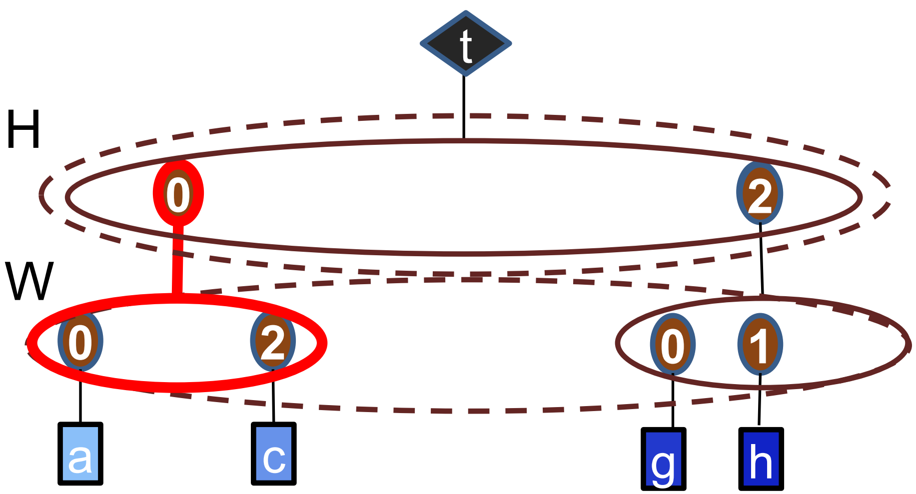

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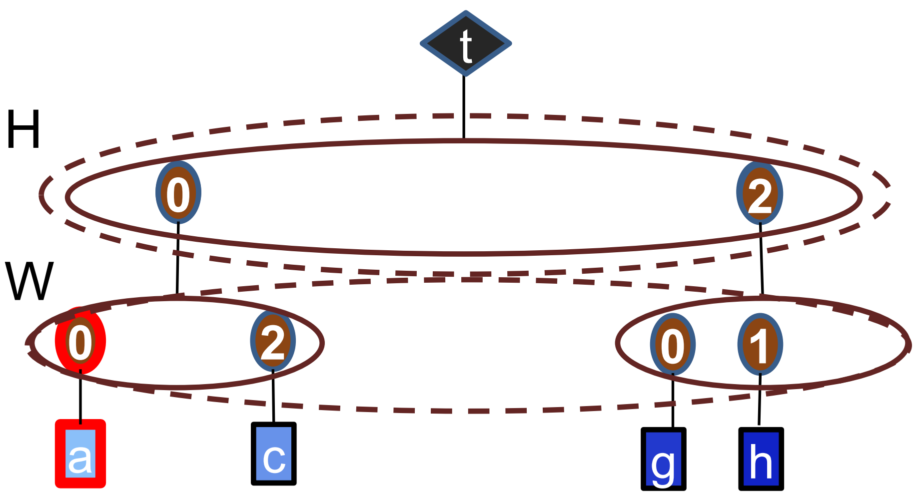

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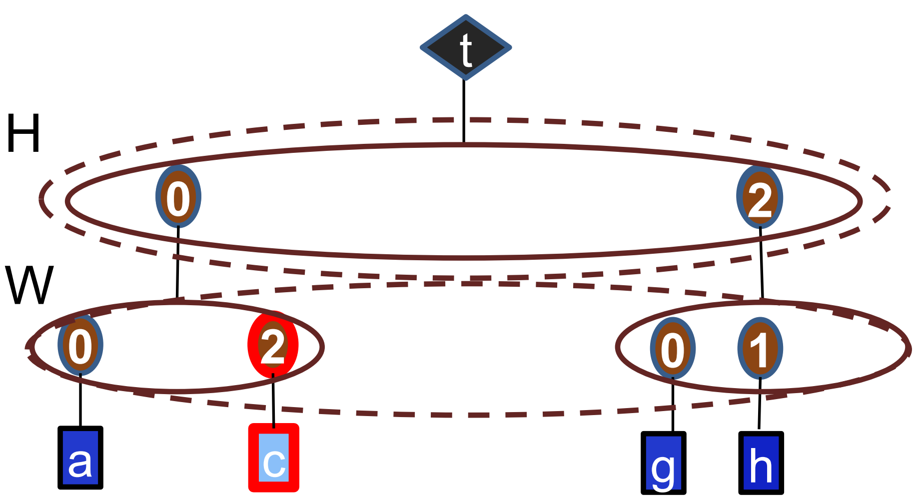

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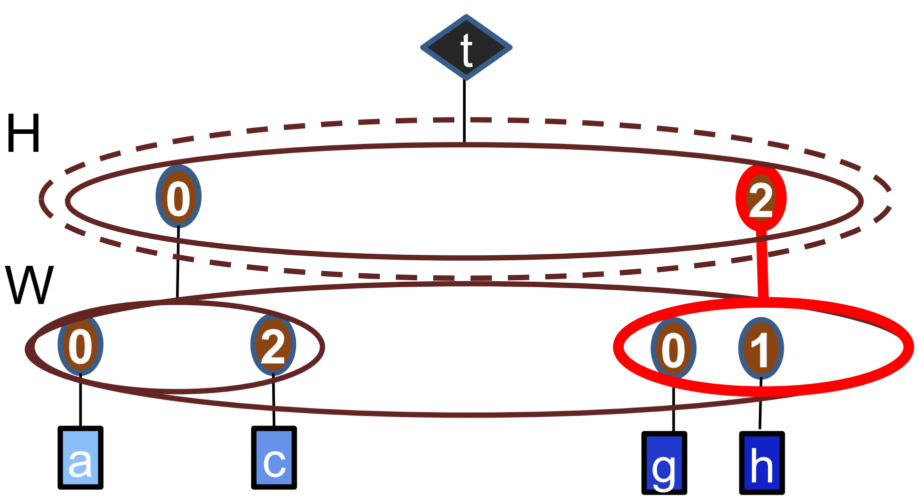

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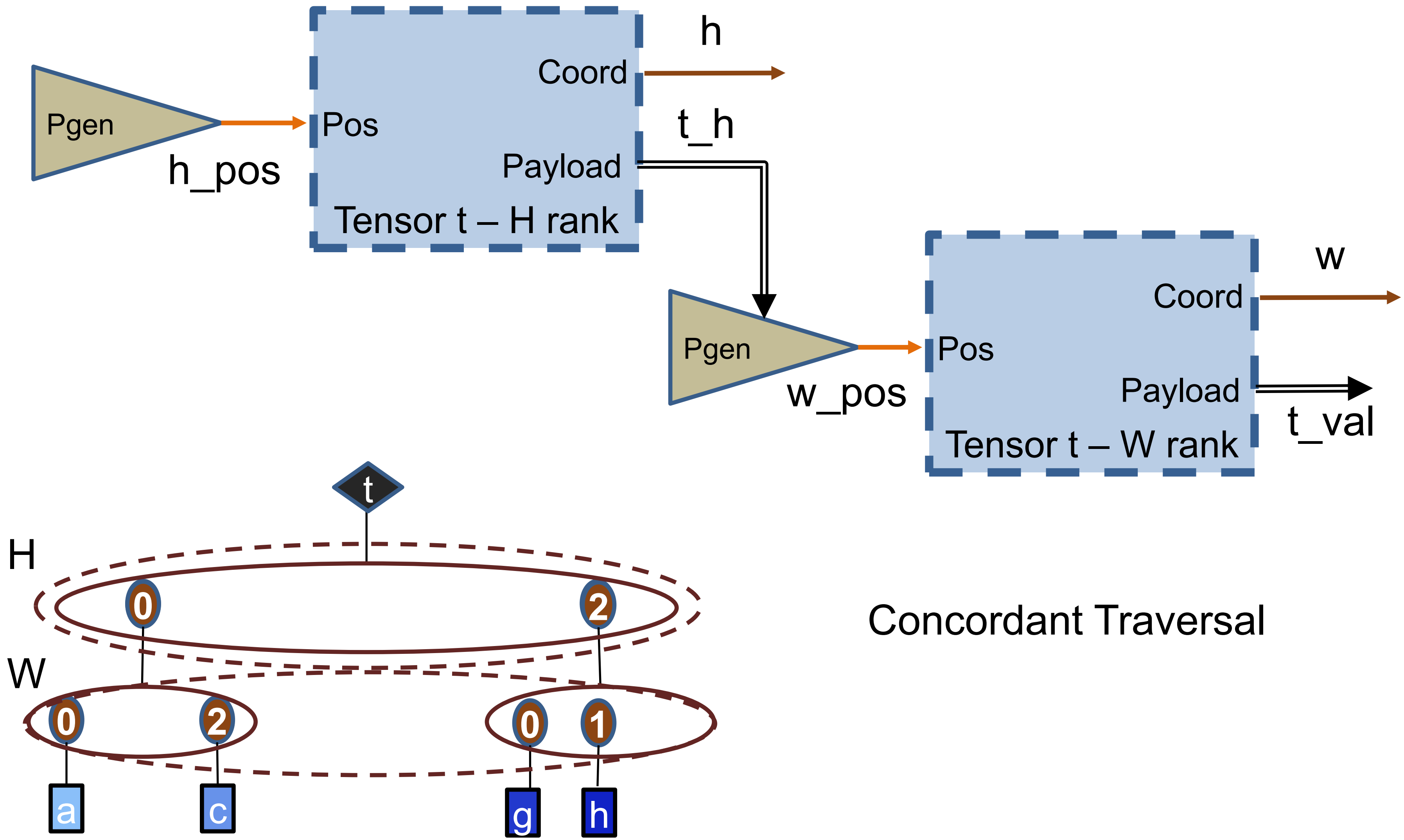
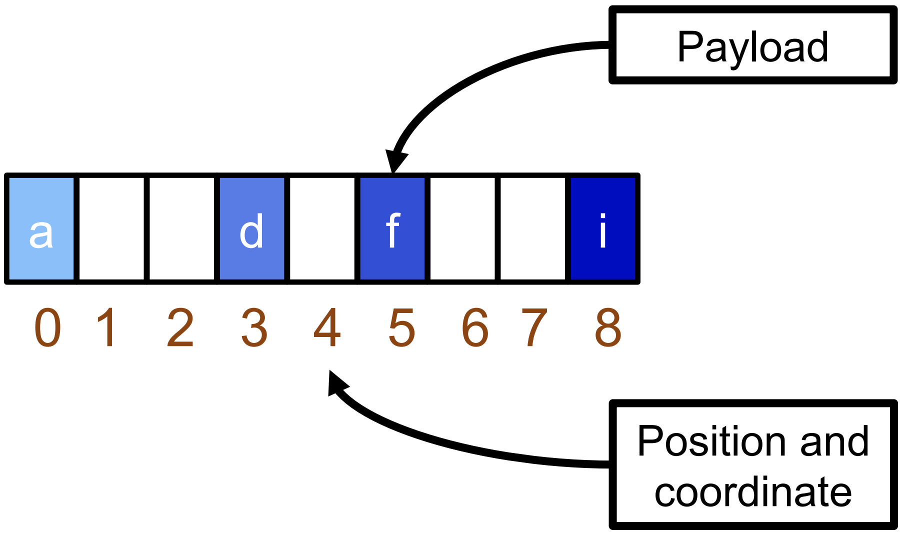


Concordant Traversal

Example Fiber Representations

Each fiber has a set of (coordinate, "payload") tuples

Array



Data in a fiber is accessed by its position or offset in memory

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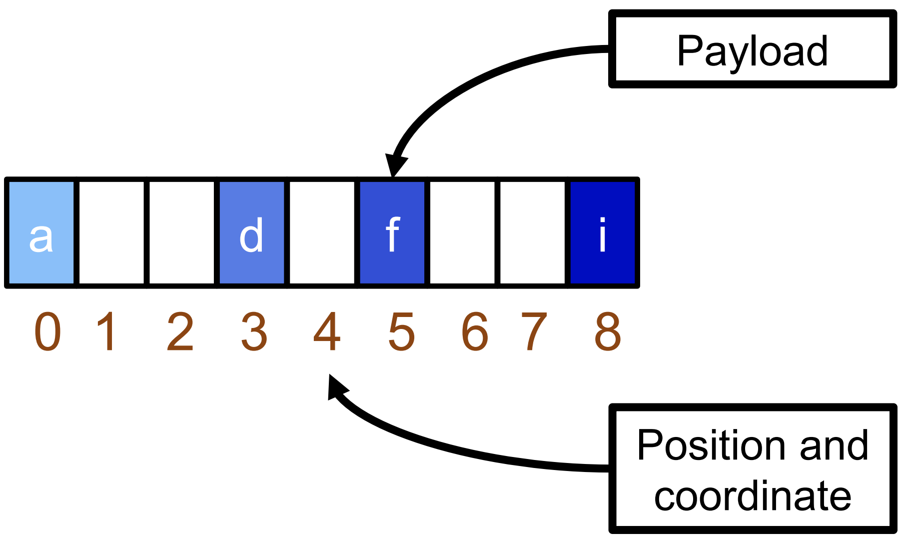
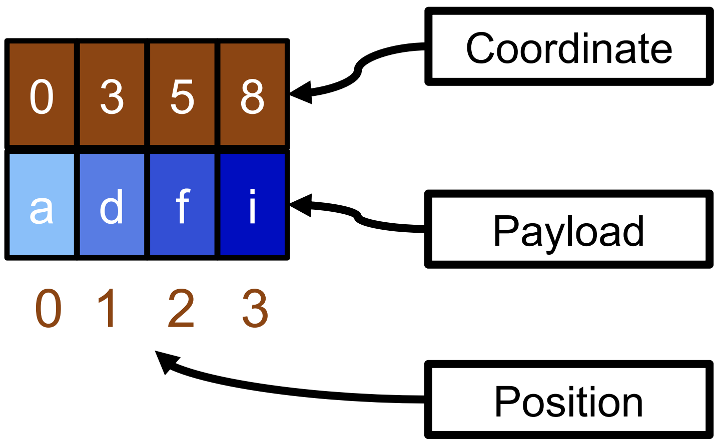


Coordinate/Payload List



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Fiber Representation Choices

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  - Uncompressed (no metadata required)
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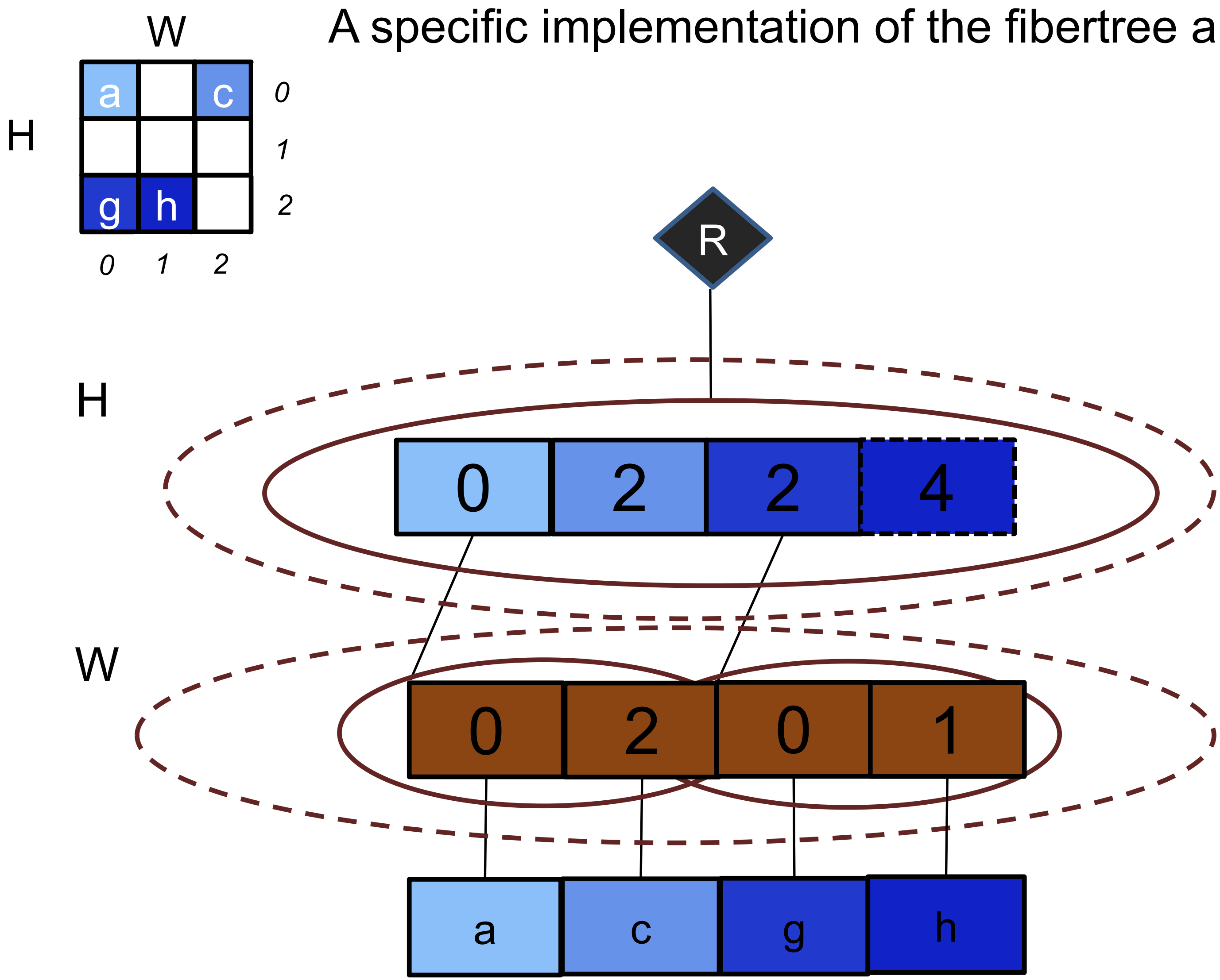
*Note: sparsity/density is an attribute of the data.

Uncompressed/Compressed Representation

A specific implementation of the fibertree abstract type



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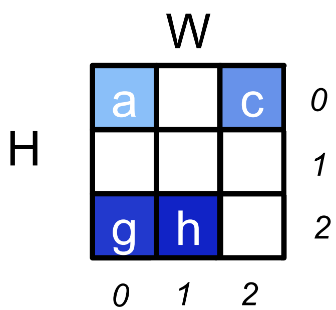

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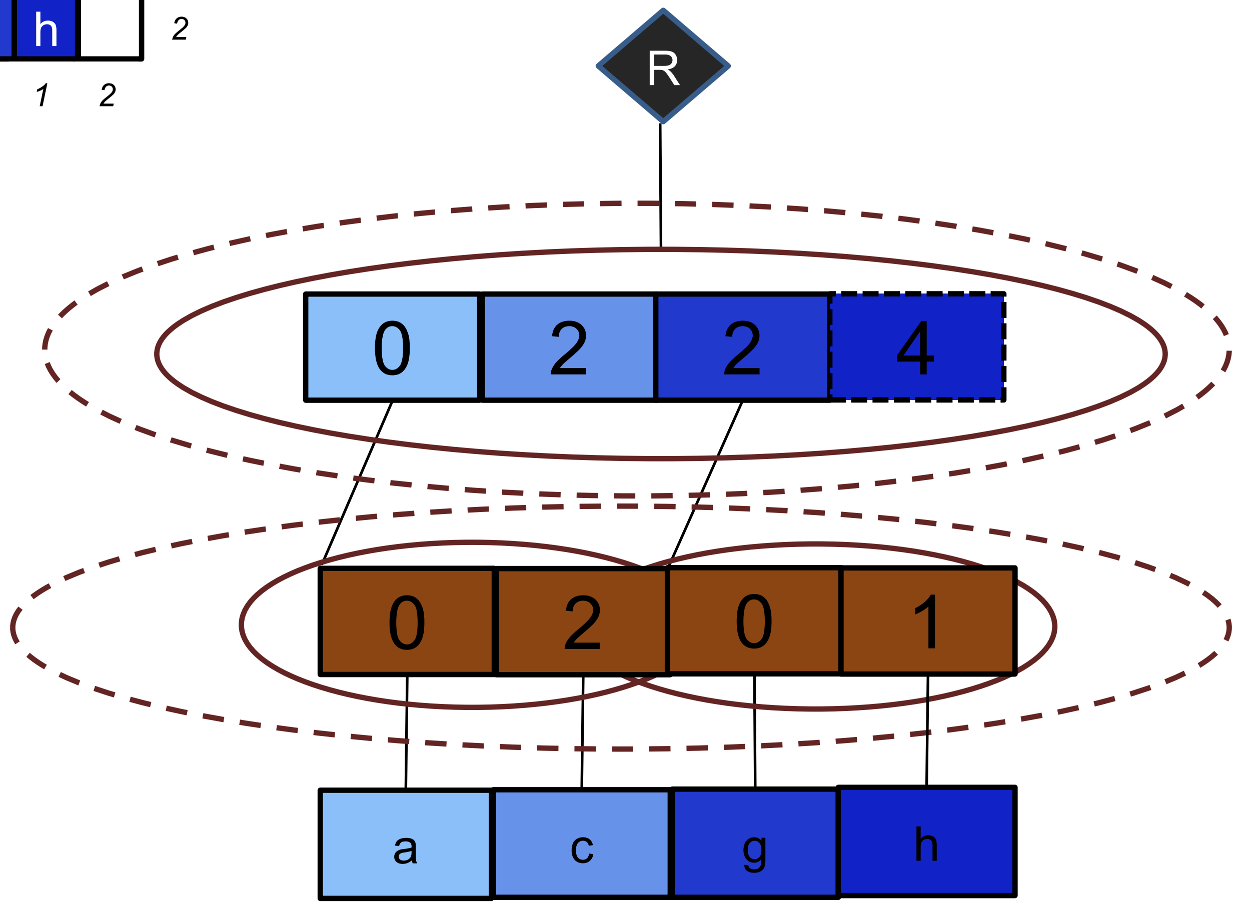
Compressed Sparse Row (CSR)



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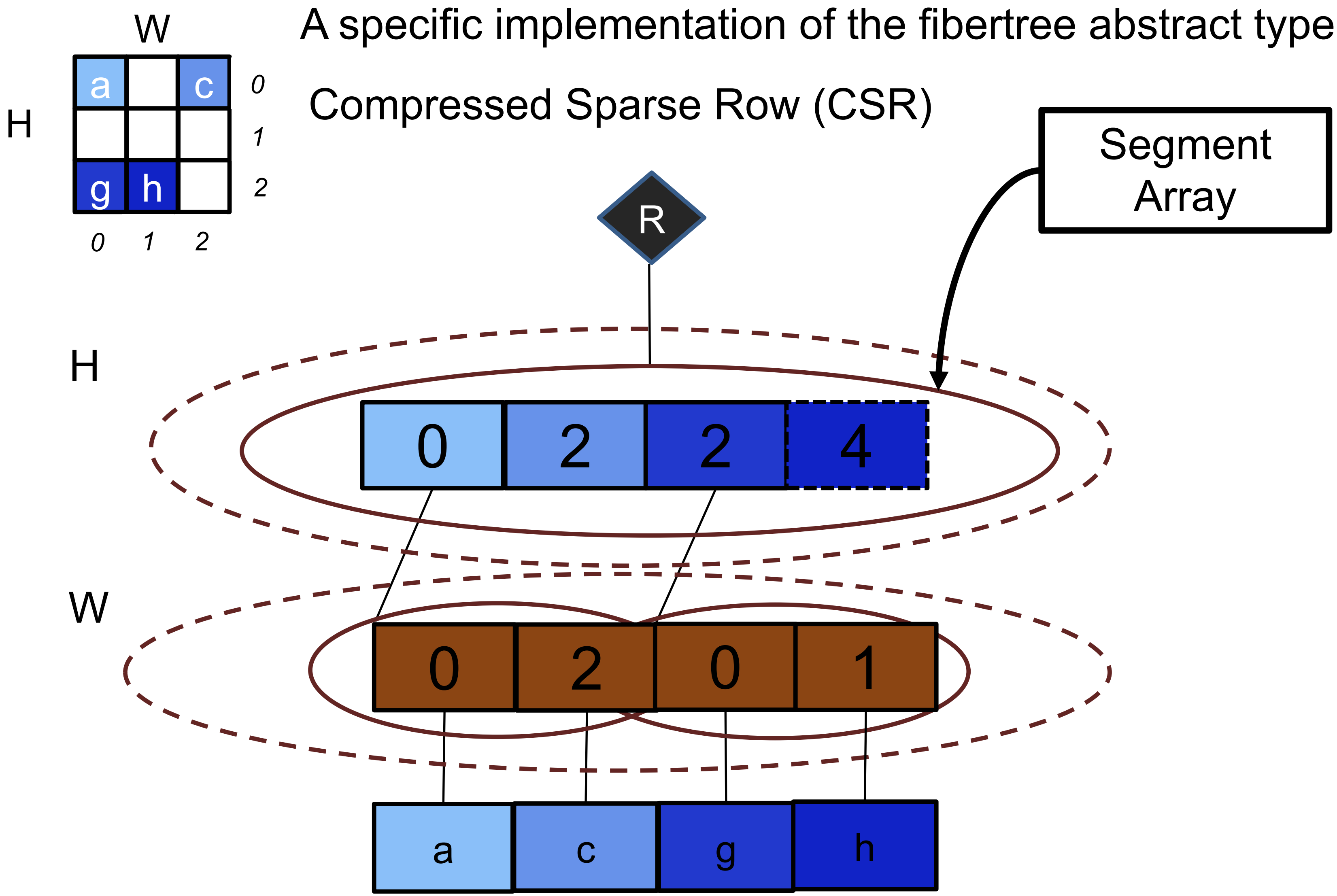

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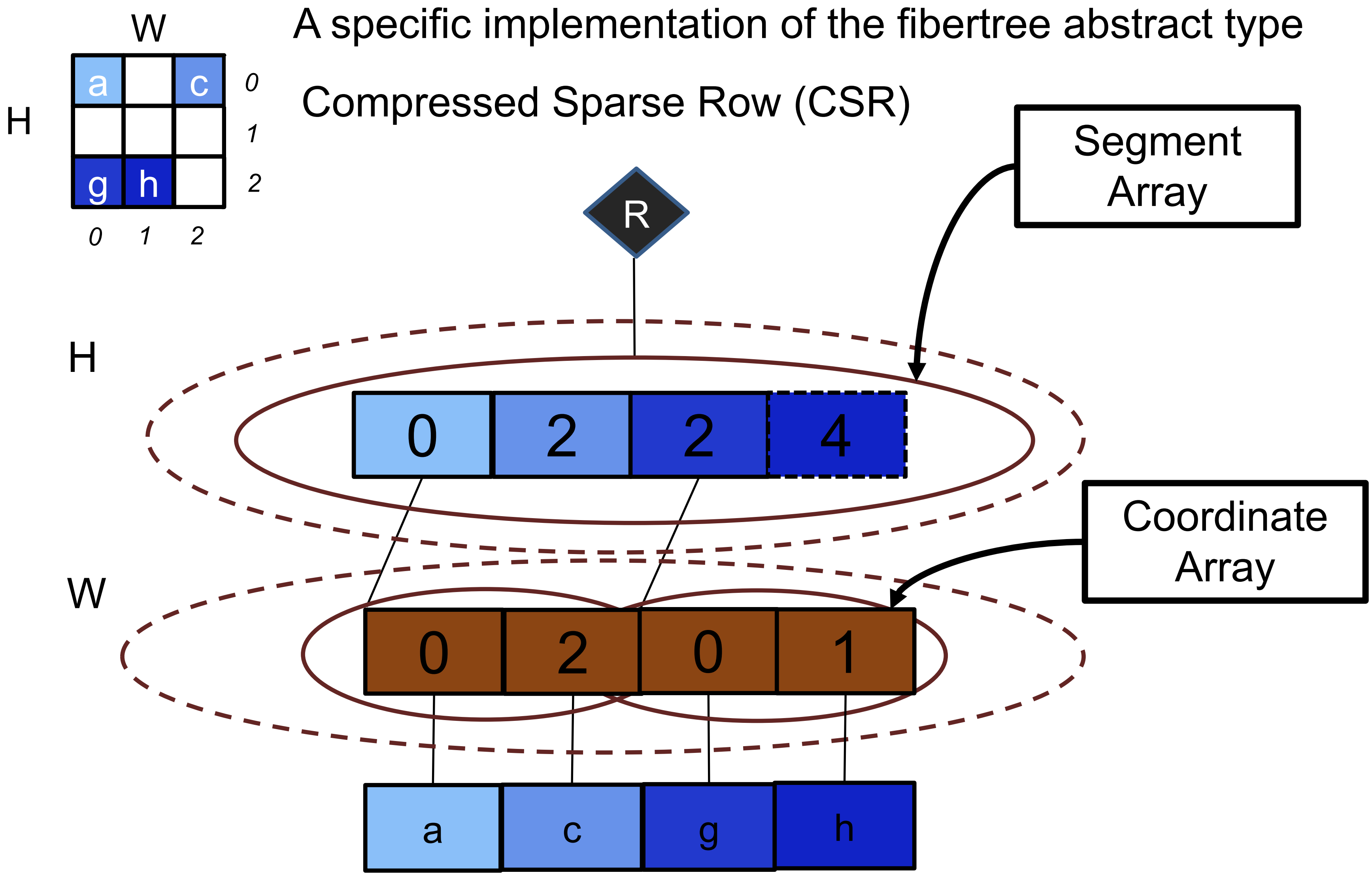

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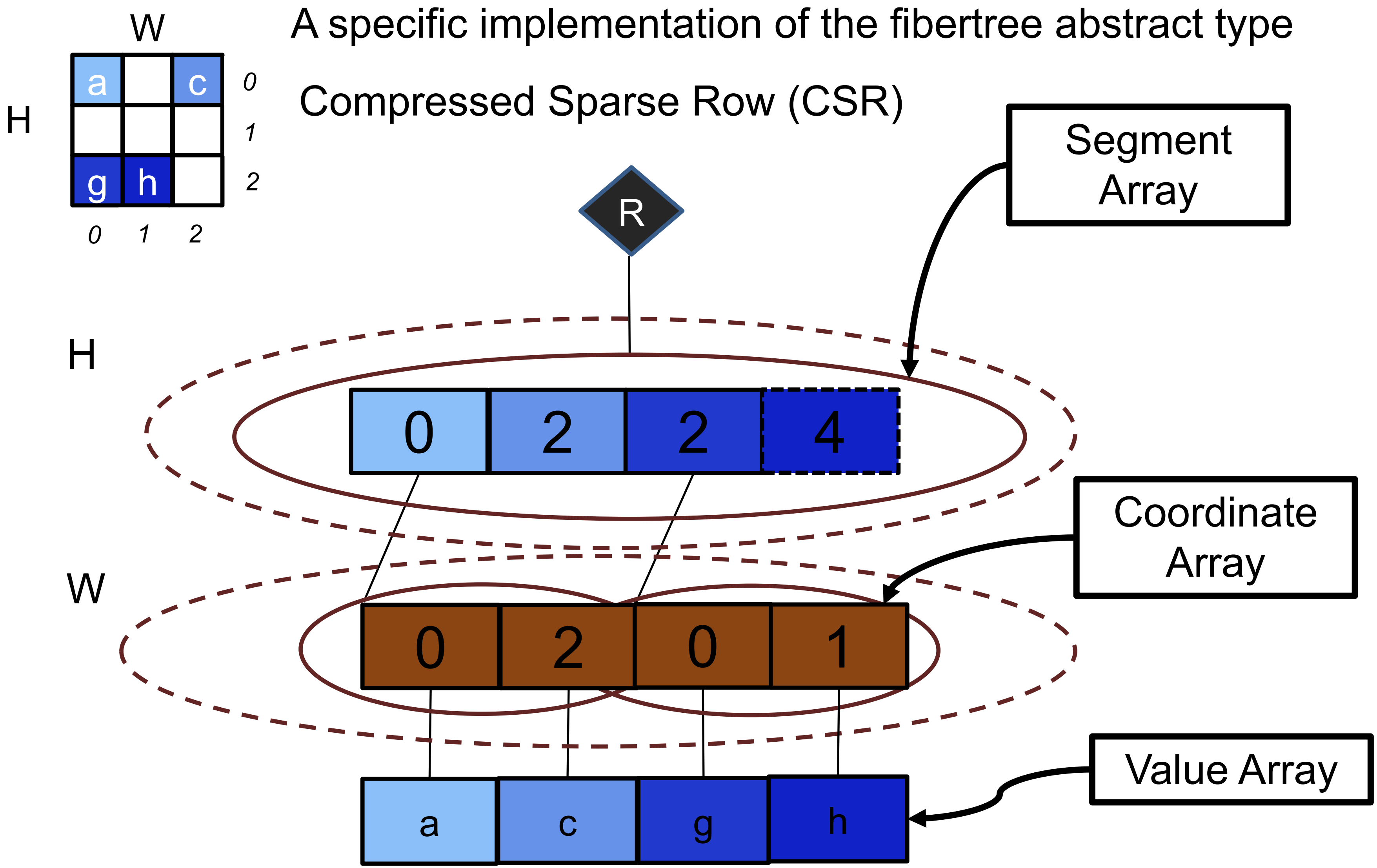

Tensor Traversal (CSR Style)

```
# 2-D Tensor Traversal (CSR)

t_segs = Array(H)
t_coords = Array(W)
t_vals = Array(W)

sum = 0
for t_h_pos in [0,H):
  h = t_h_pos
  t_w_start = t_segs[t_h_pos]
  t_w_len = t_segs[t_h_pos+1]-t_w_start
  for t_w_pos in [t_w_start, t_w_len):
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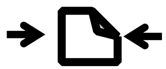
For uncompressed rank coordinate equals position

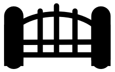
Coordinates not actually used in this example

CONV: Exploiting Sparse Weights

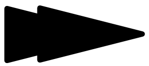
Hardware Sparse Acceleration Features



Format:
Choose tensor representations to save necessary storage spaces and energy associated zero accesses



Gating:
Explicitly eliminate ineffectual storage accesses and computes by letting the hardware unit staying idle for the cycle to save energy



Skipping:
Explicitly eliminate ineffectual storage accesses and computes by skipping the cycle to save energy and time

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i = Array(W)          # Input activations
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o = Array(Q)          # Output activations

for (s, f_val) in f:
  for q in [0, Q):
    w = q + s
    o[q] += i[w] * f_val
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Weight Stationary - Sparse Weights

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Concordant traversal

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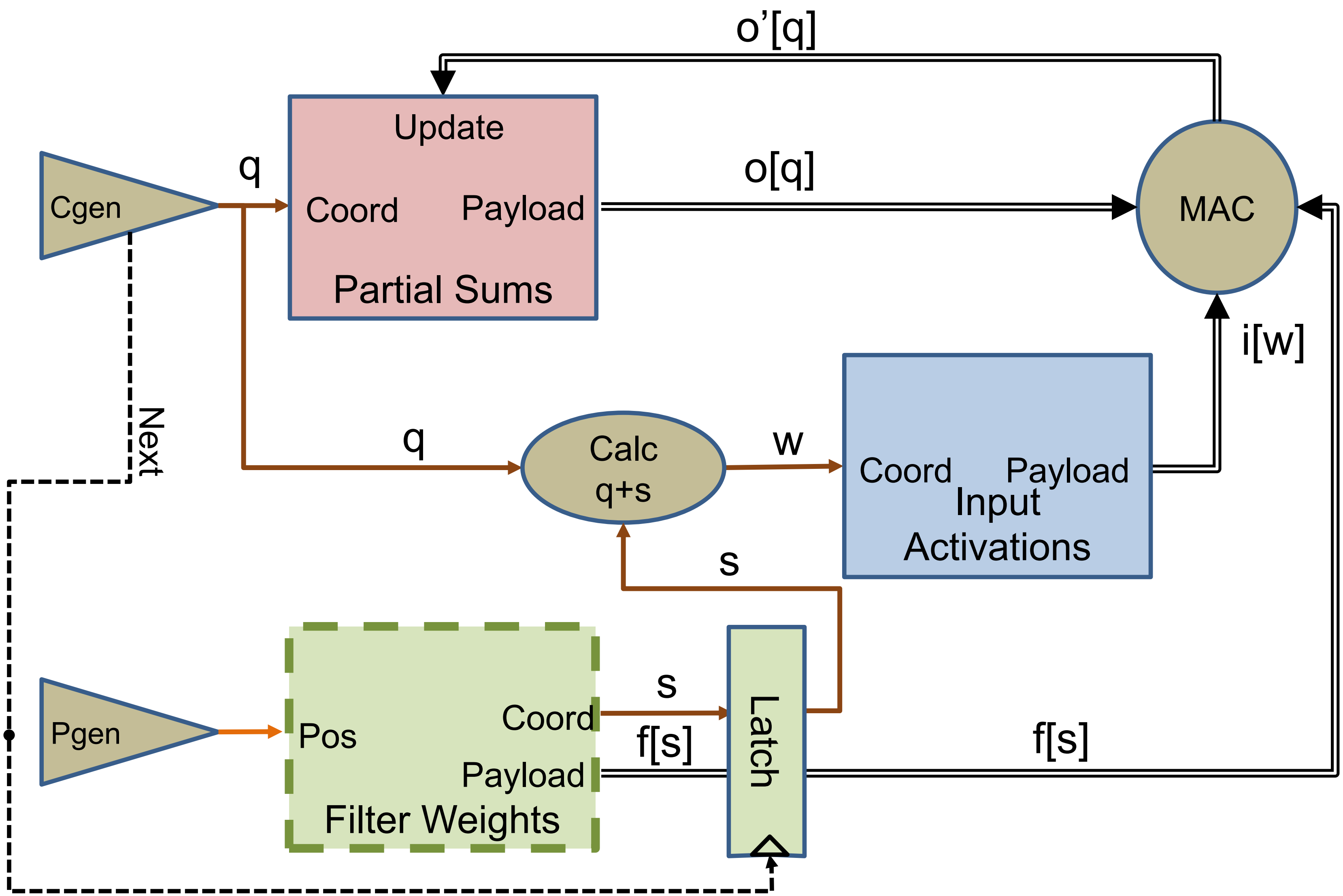
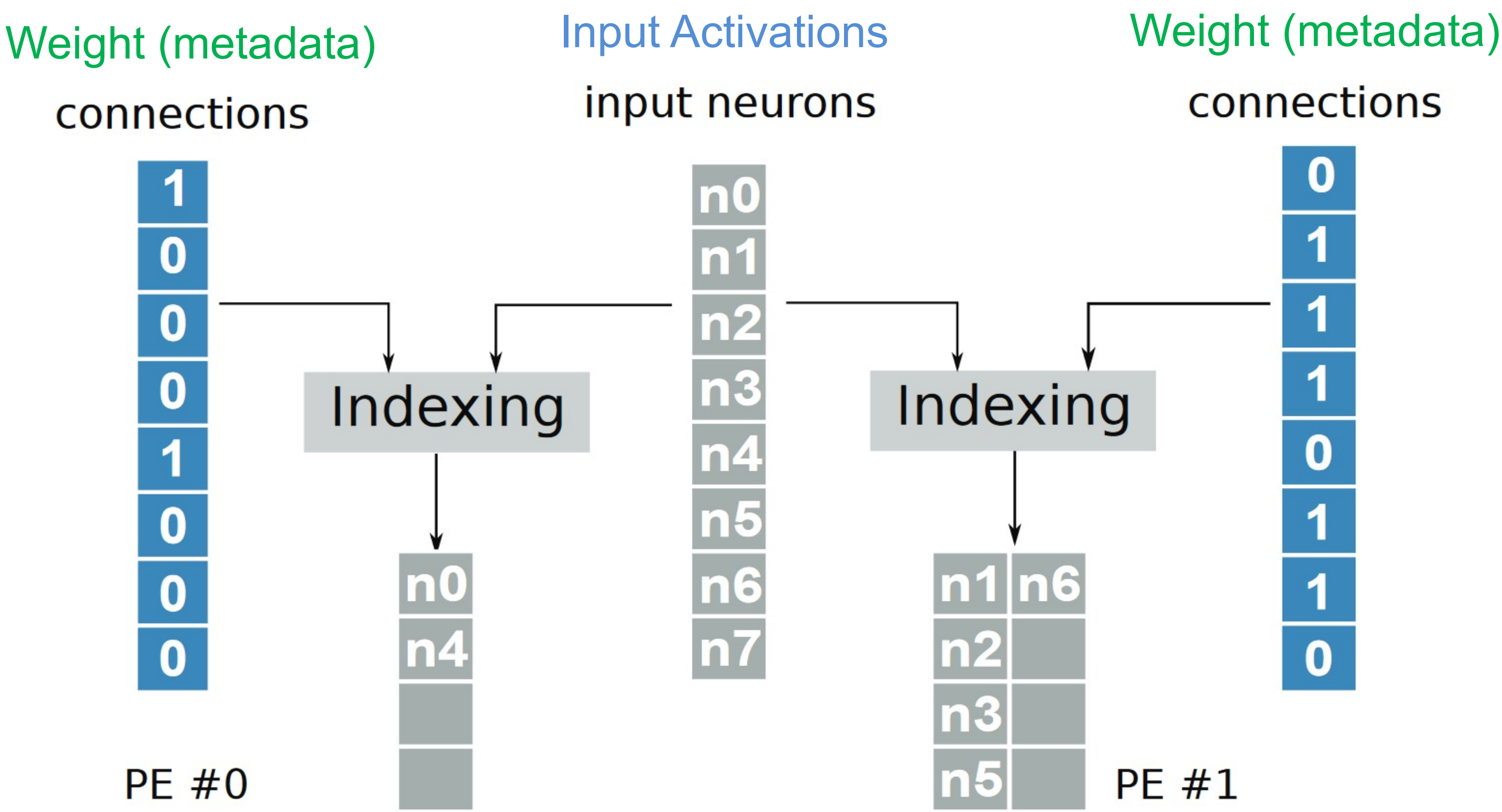


Cambricon-X – Activation Access



Cambricon-X – Zhang et.al., Micro 2016

To Extend to Other Dimensions of DNN

- **Need to add loop nests for:**
  - 2-D input activations and filters
  - Multiple input channels
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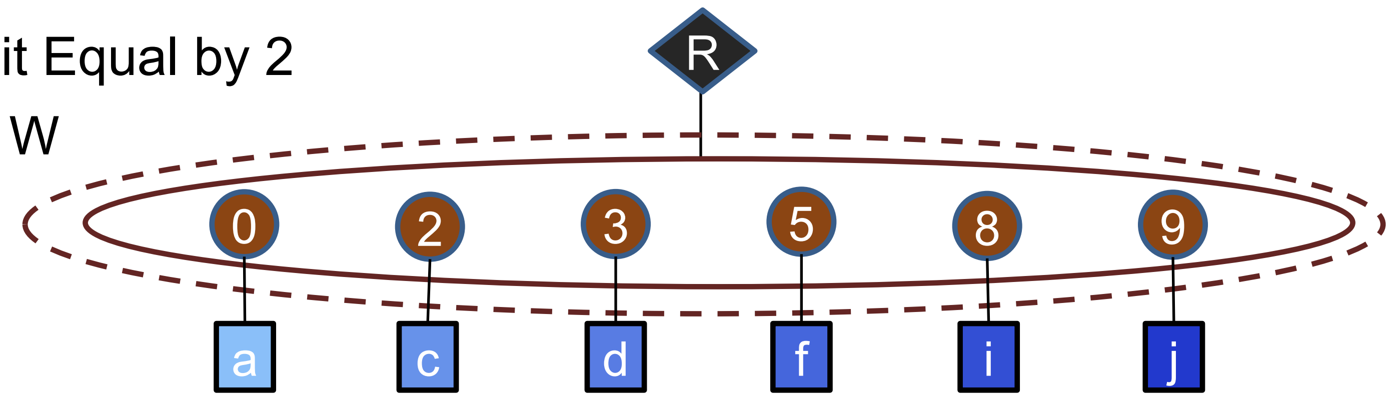
- **Add parallelism…**

Fiber Splitting Equally in Position Space

Before Split Equal by 2



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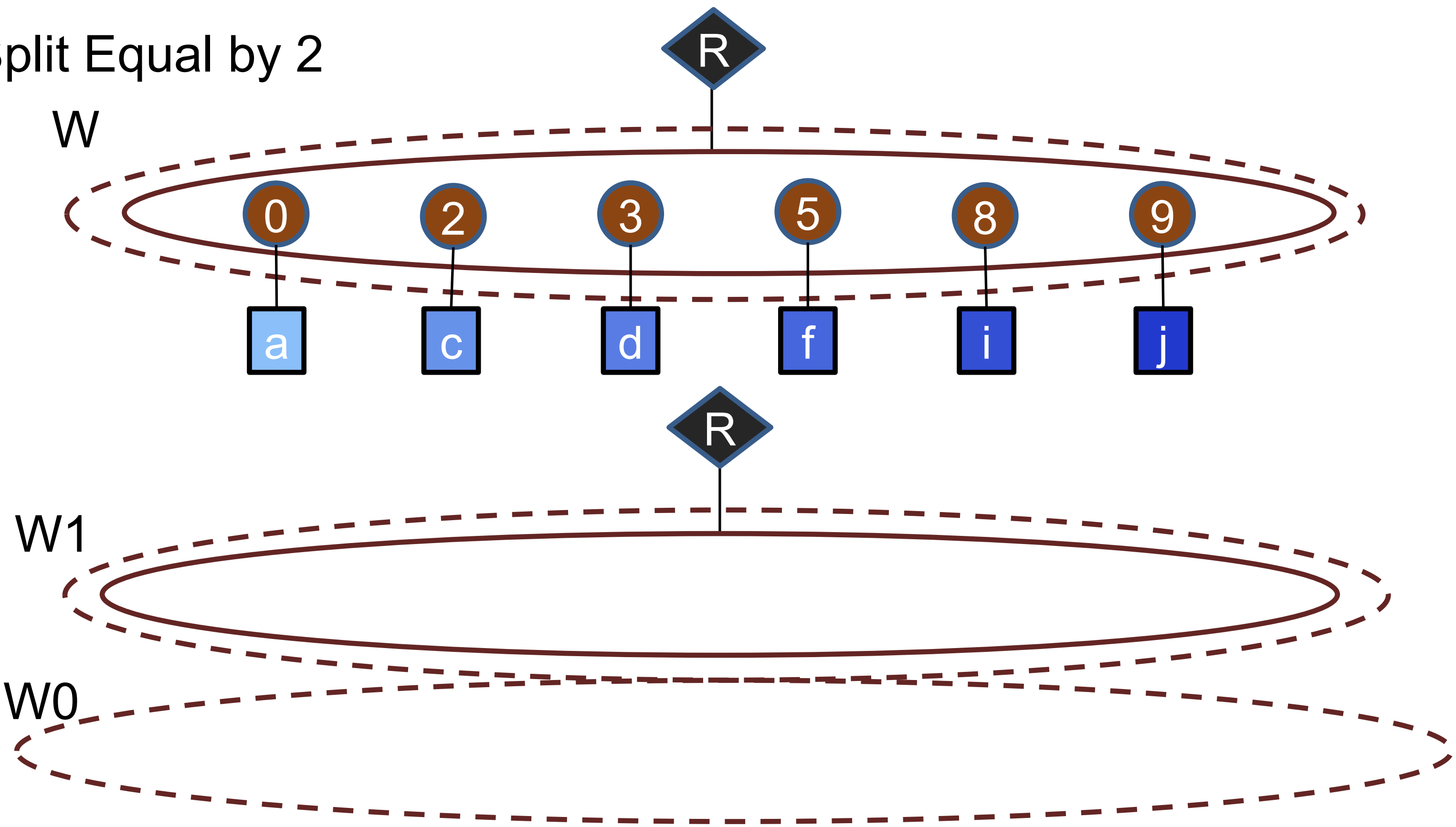

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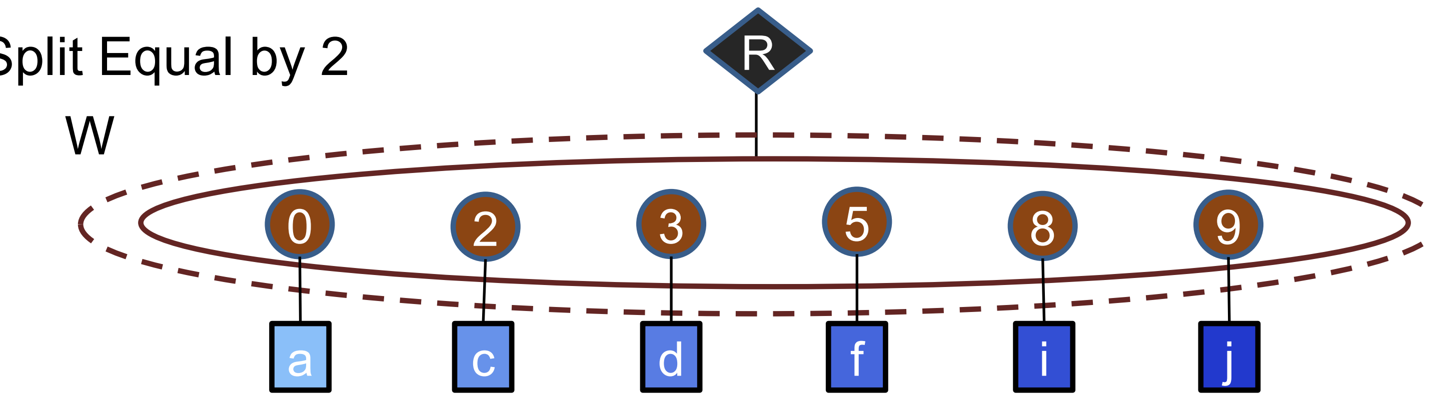
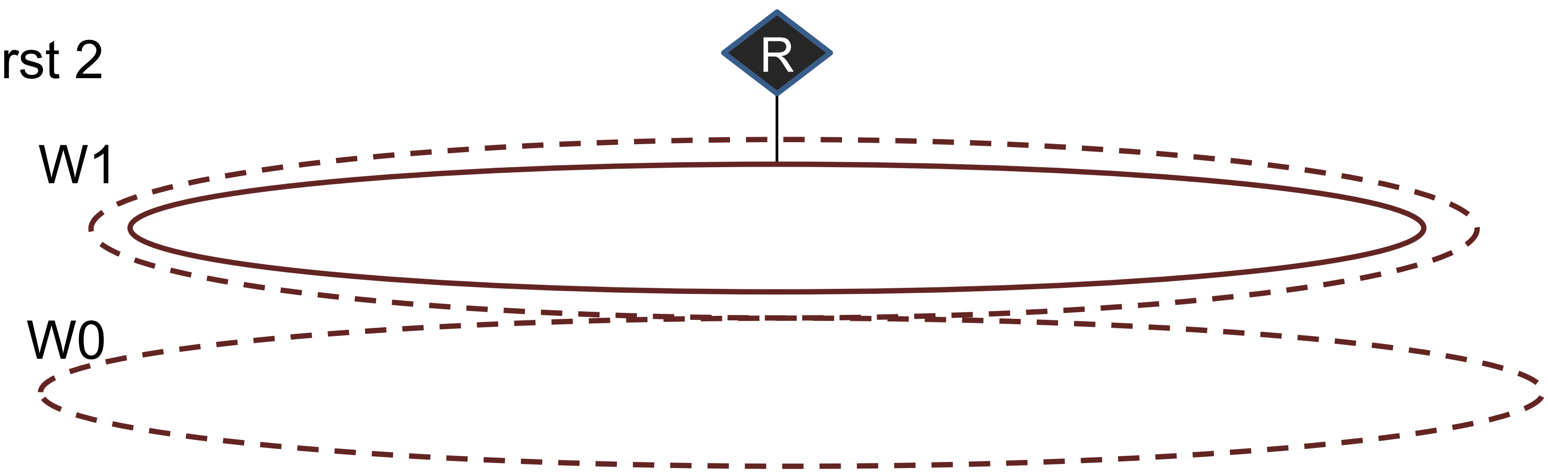


Grab first 2



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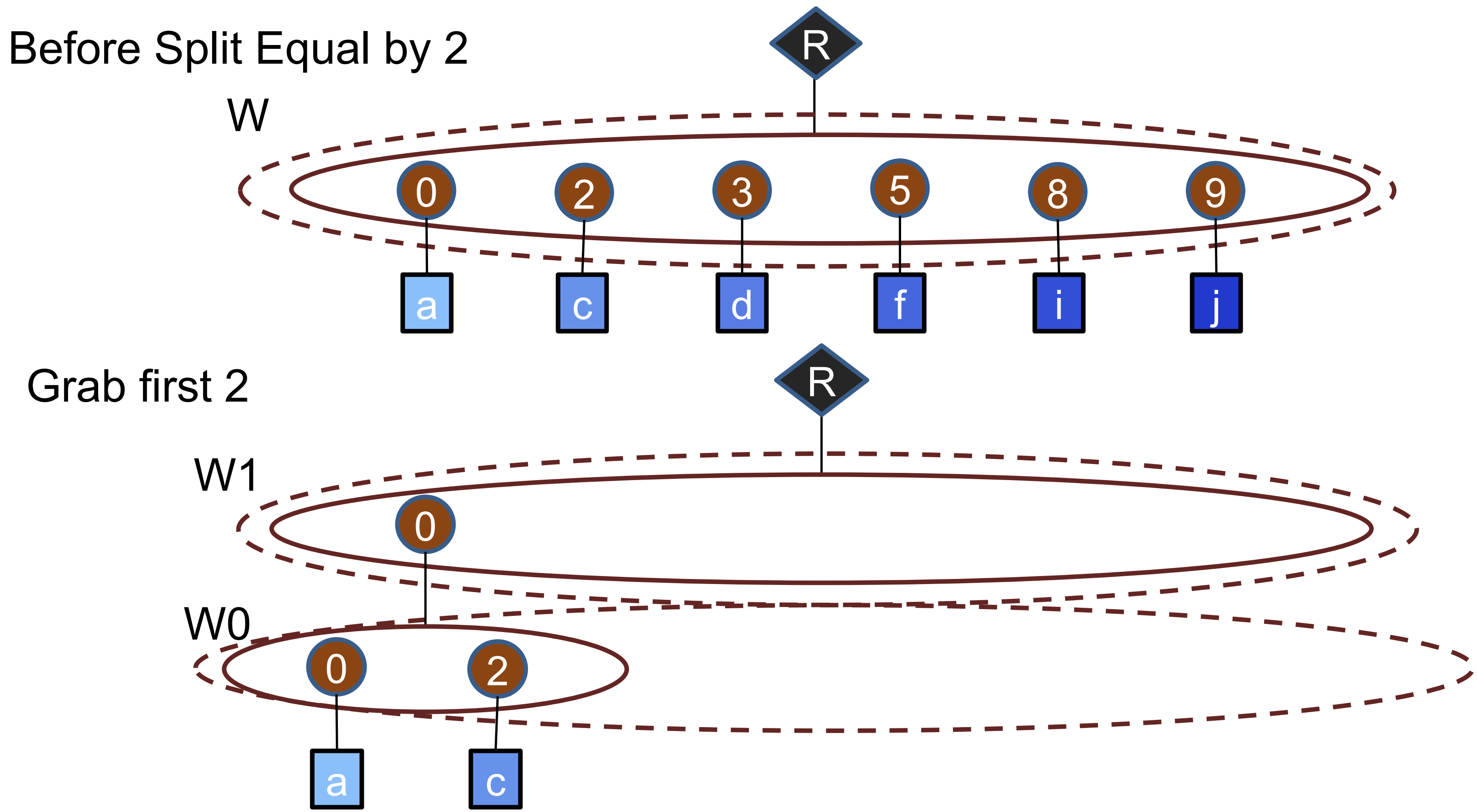

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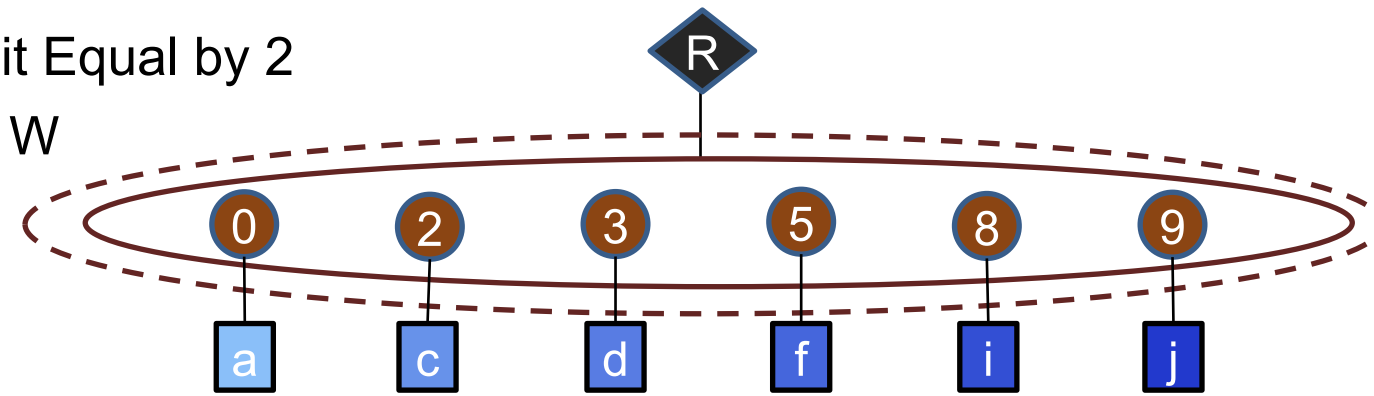

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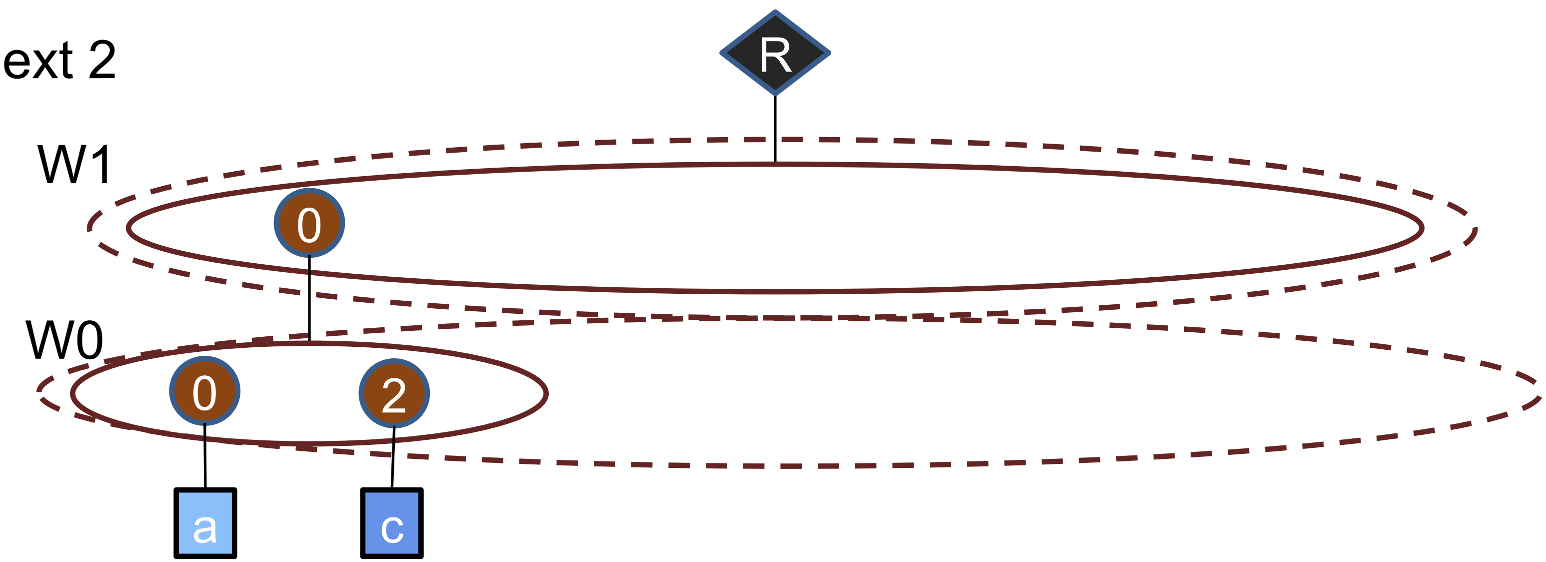

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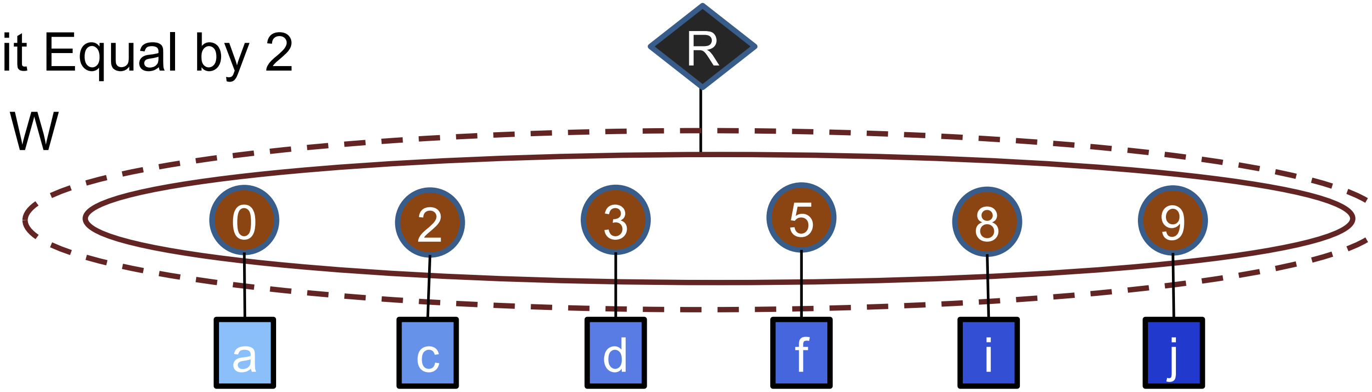

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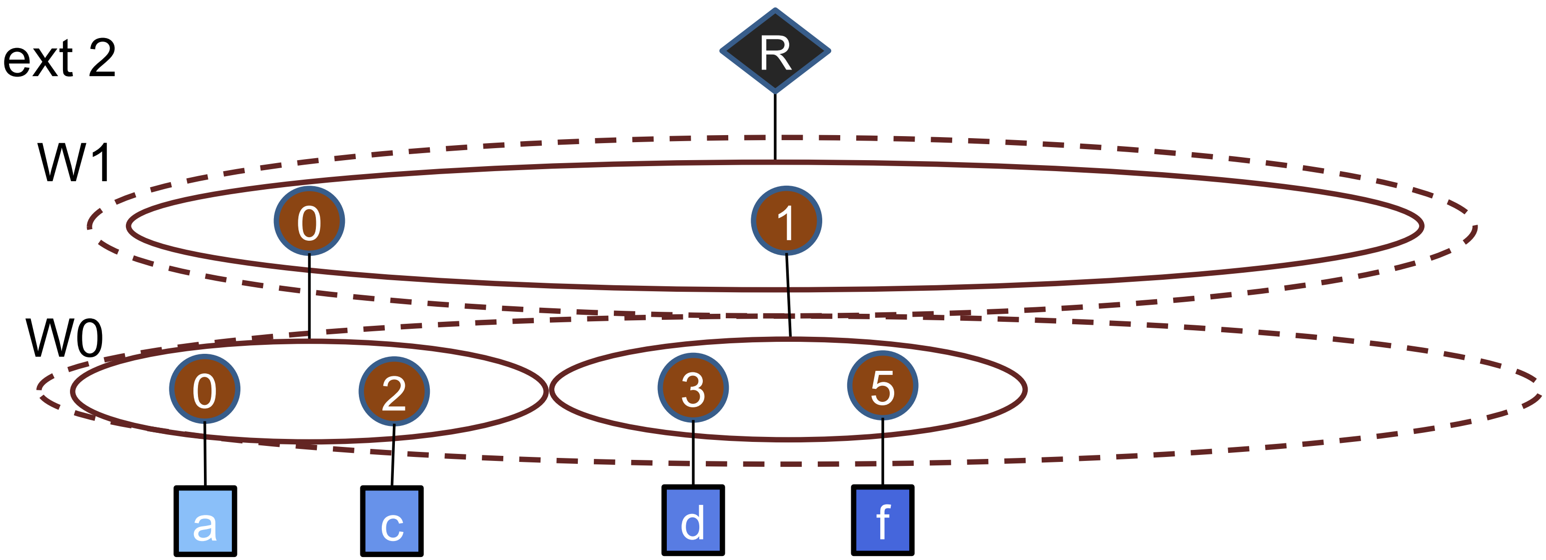

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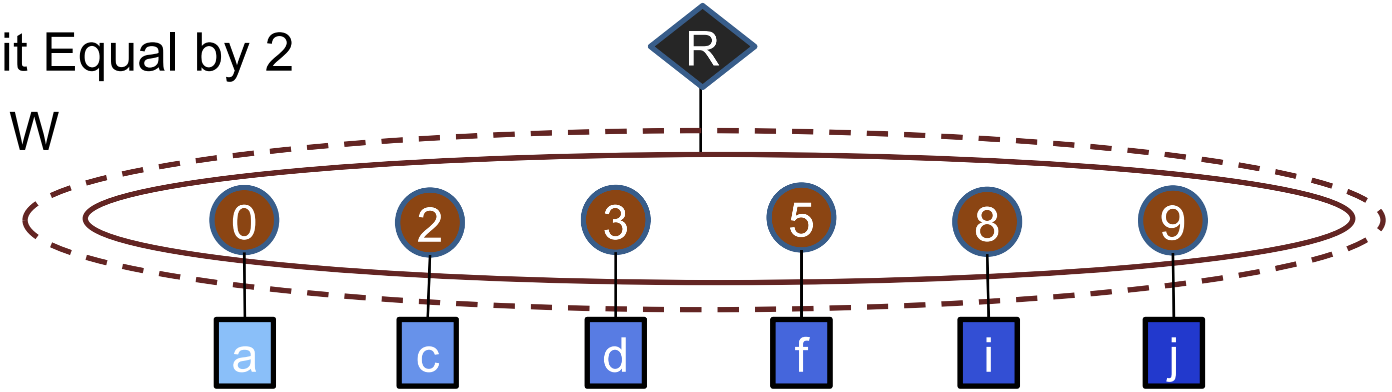

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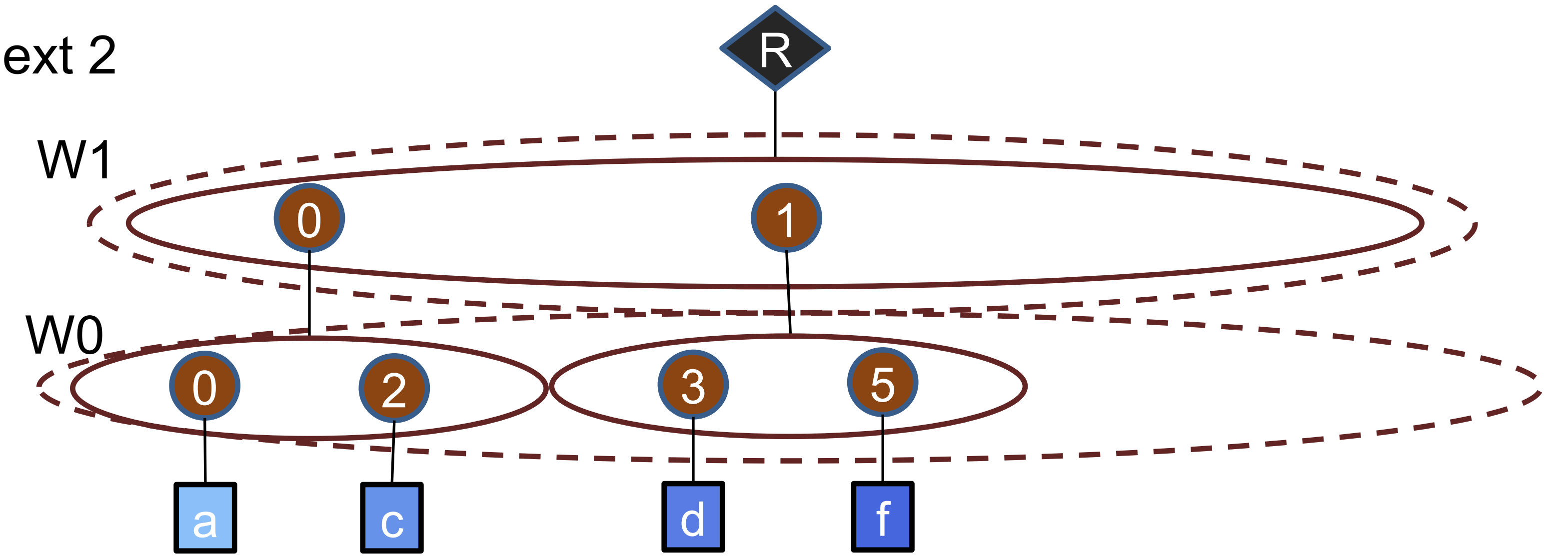

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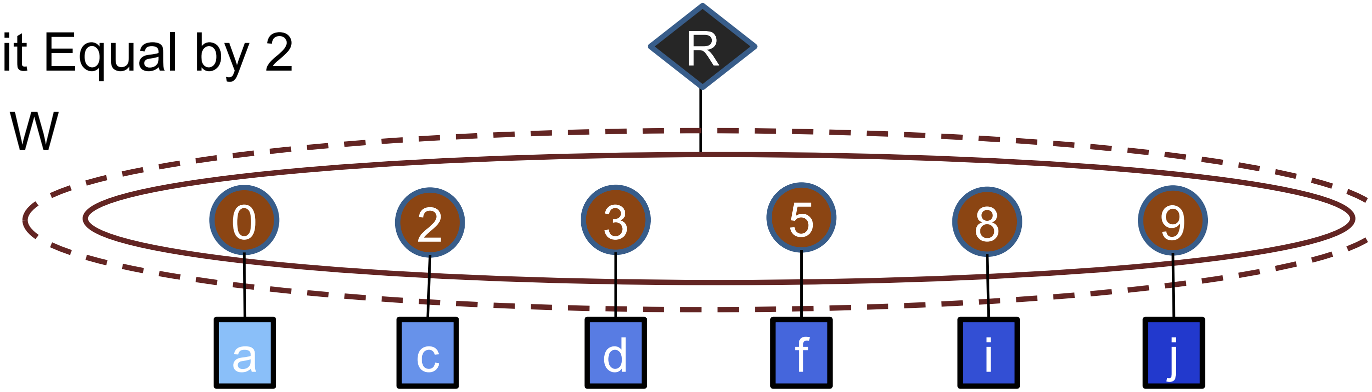

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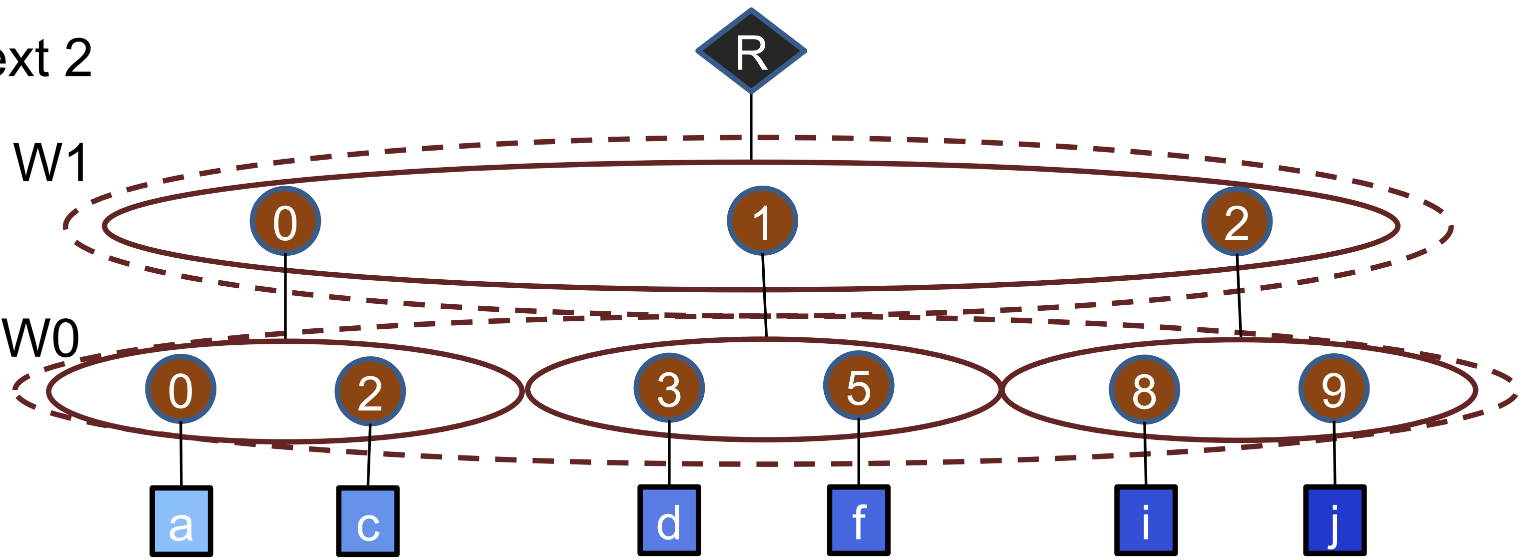

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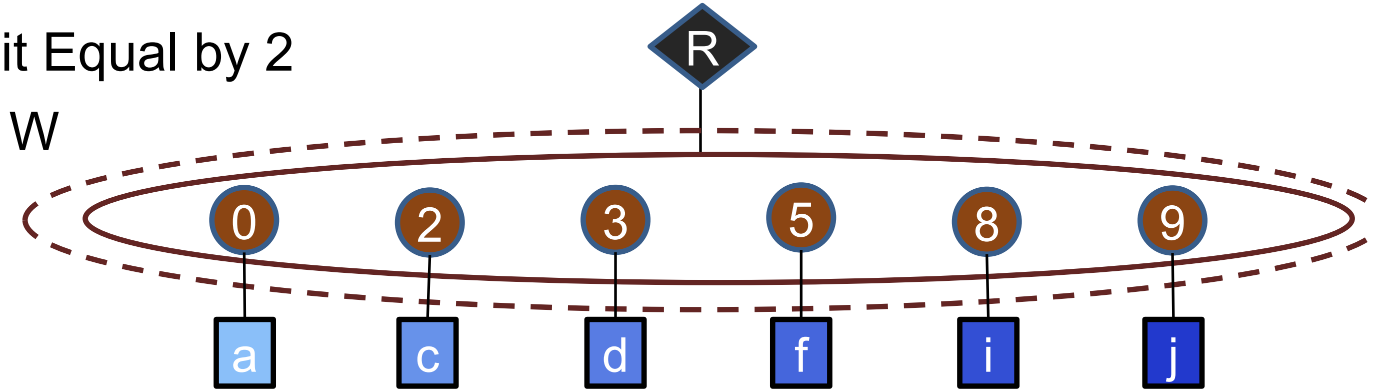

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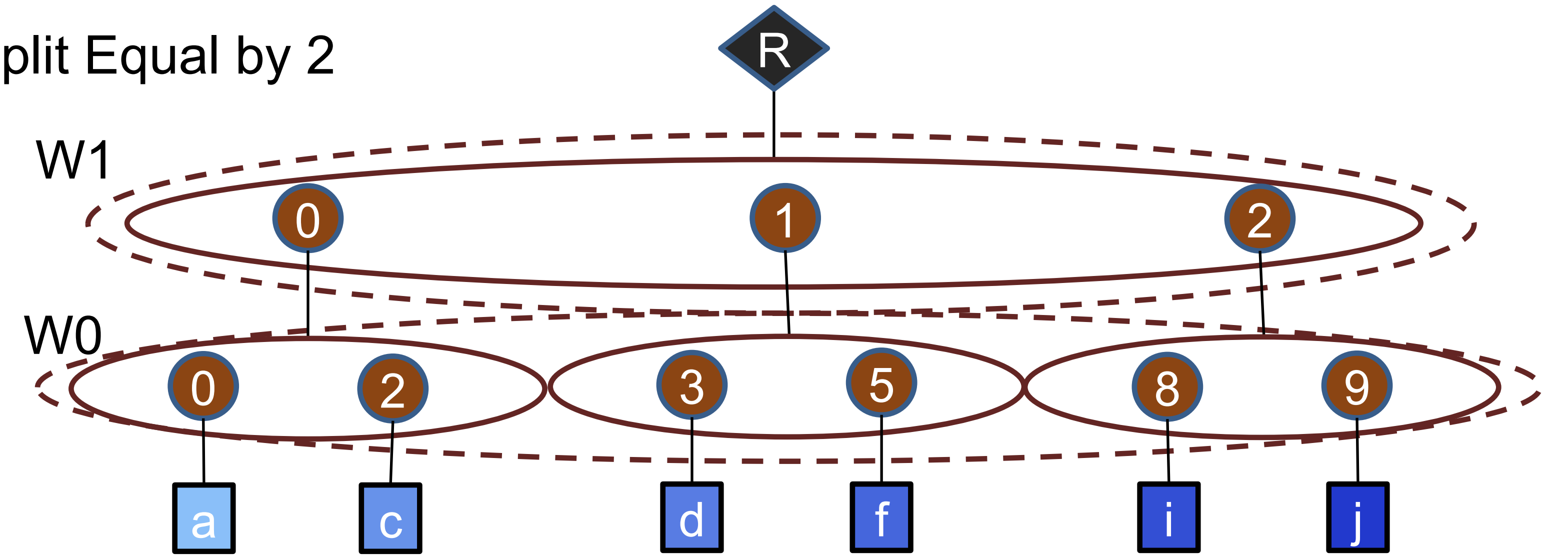

Parallel Weight Stationary - Sparse Weights

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f = Tensor(S)         # Filter weights
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for (s1, f_split) in f.splitEqual(2):
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Calculate coordinates

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       parallel-for q0 in [0, 4):
         q = q1*4 + q0
         w = q + s
         o[q] += i[w] * f_val
```

Get groups of two weights

Work on two weights in parallel

Work on four outputs at once

Calculate coordinates

Look up input activation

Parallel Weight Stationary - Sparse Weights

```
i = Array(W)          # Input activations
f = Tensor(S)         # Filter weights
o = Array(Q)          # Output activations

for (s1, f_split) in f.splitEqual(2):
  for q1 in [0, Q/4):
     parallel-for (s0, f_val) in f_split:
       parallel-for q0 in [0, 4):
         q = q1*4 + q0
         w = q + s
         o[q] += i[w] * f_val
```

Get groups of two weights

Work on two weights in parallel

Work on four outputs at once

Calculate coordinates

Accumulate multiple outputs each spatially

Look up input activation

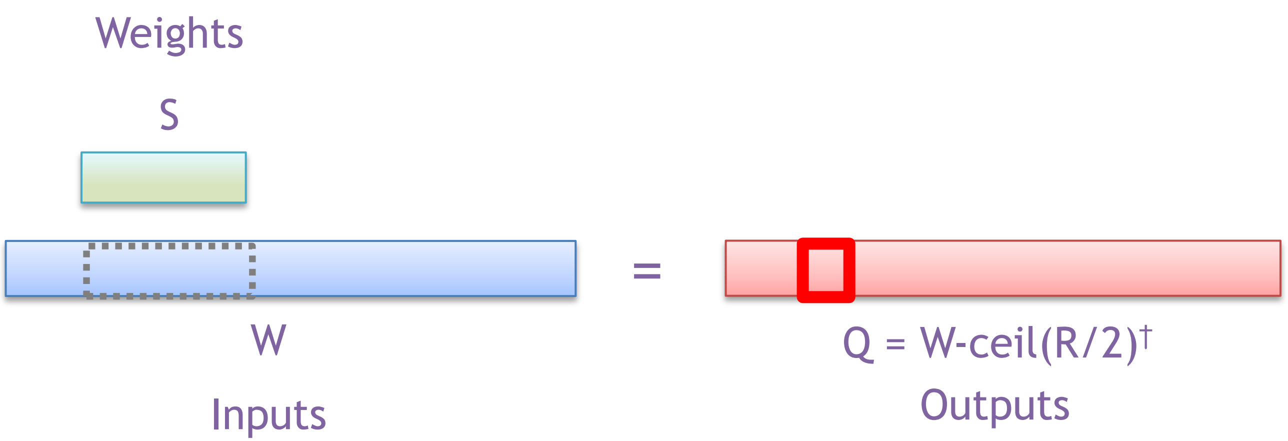
CONV: Exploiting Sparse Inputs & Sparse Weights

Output Stationary - Sparse Weights & Inputs

```
i = Tensor(W)          # Input activations
f = Tensor(S)          # Filter weights
o = Array(Q)           # Output activations

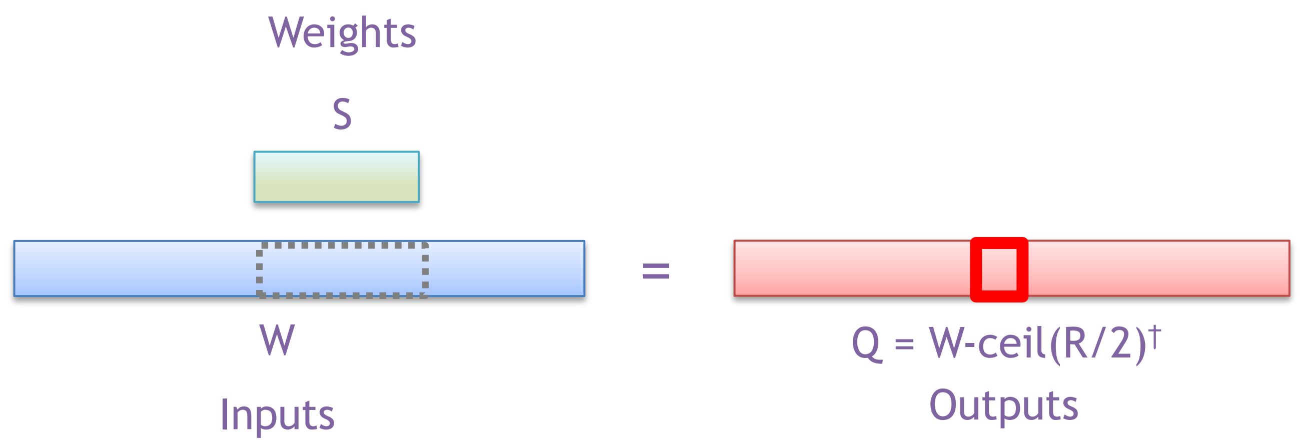
for q in [0,Q):
  for (s, (f_val, i_val)) in f.project(+q) & i:
    o[q] += i_val * f_val
```

Fiber Coordinate Projection



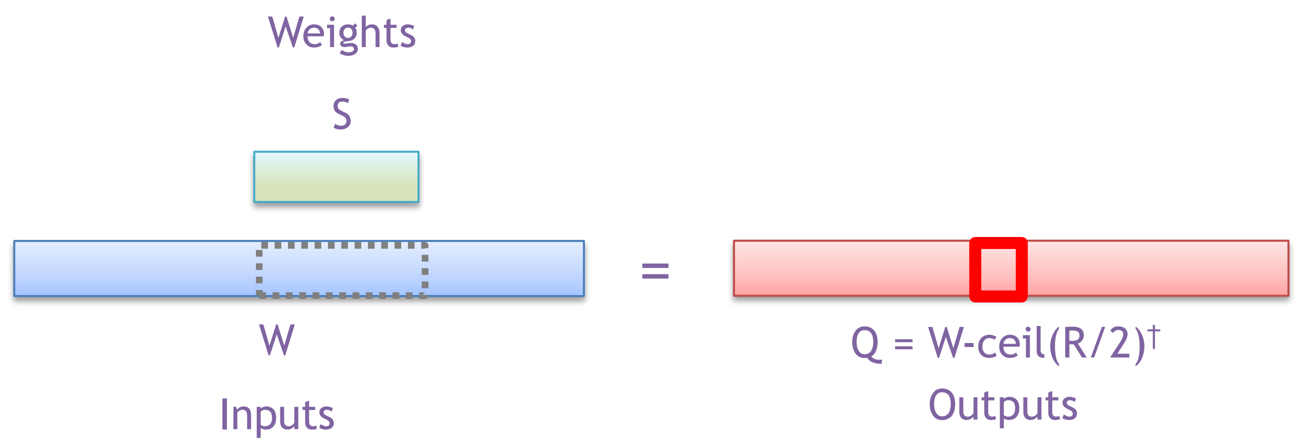
fiber-projection

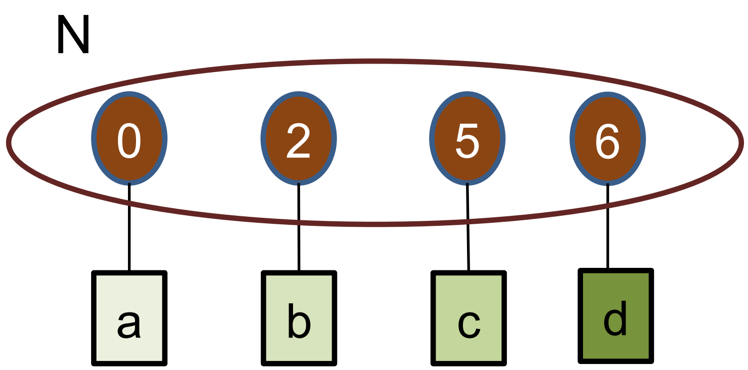
Fiber Coordinate Projection



fiber-projection

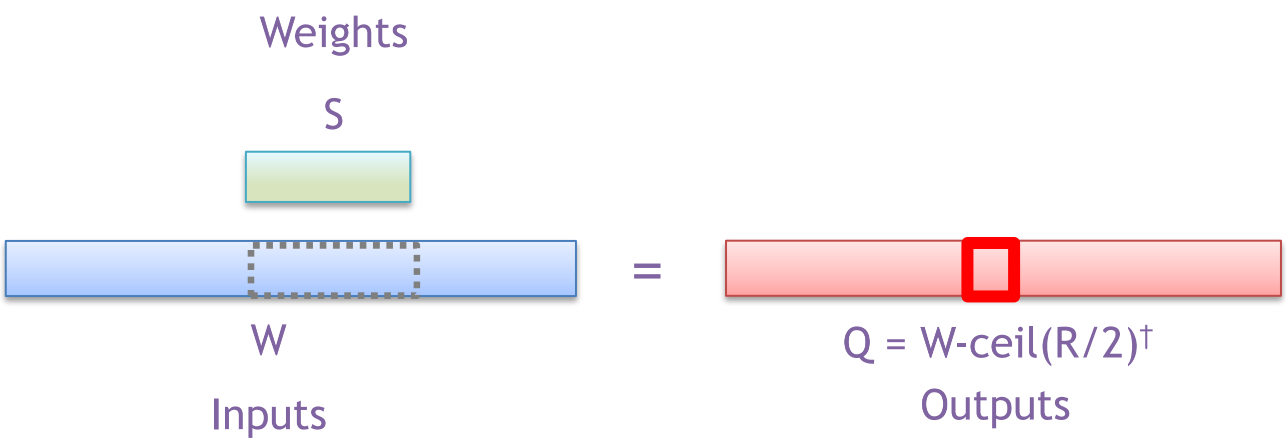
Fiber Coordinate Projection

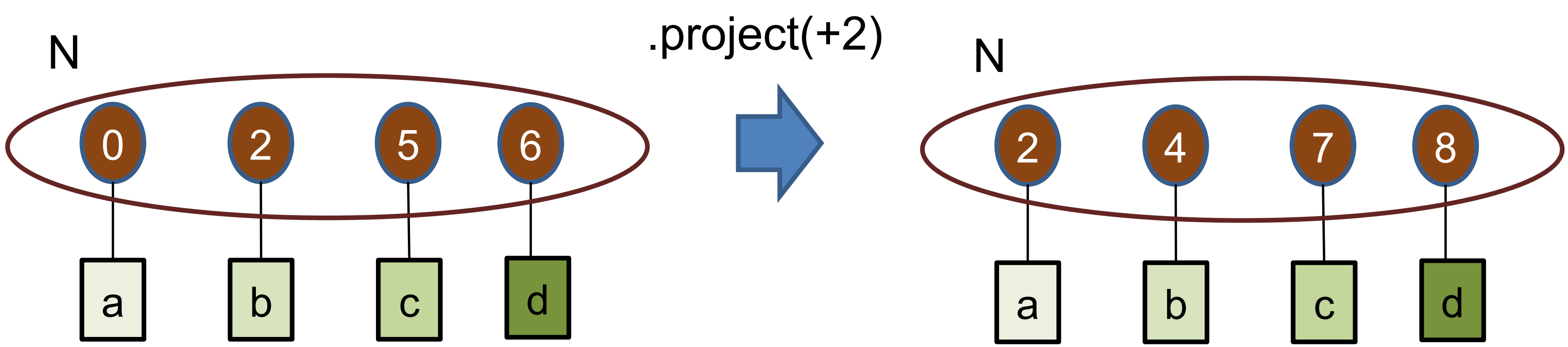




fiber-projection

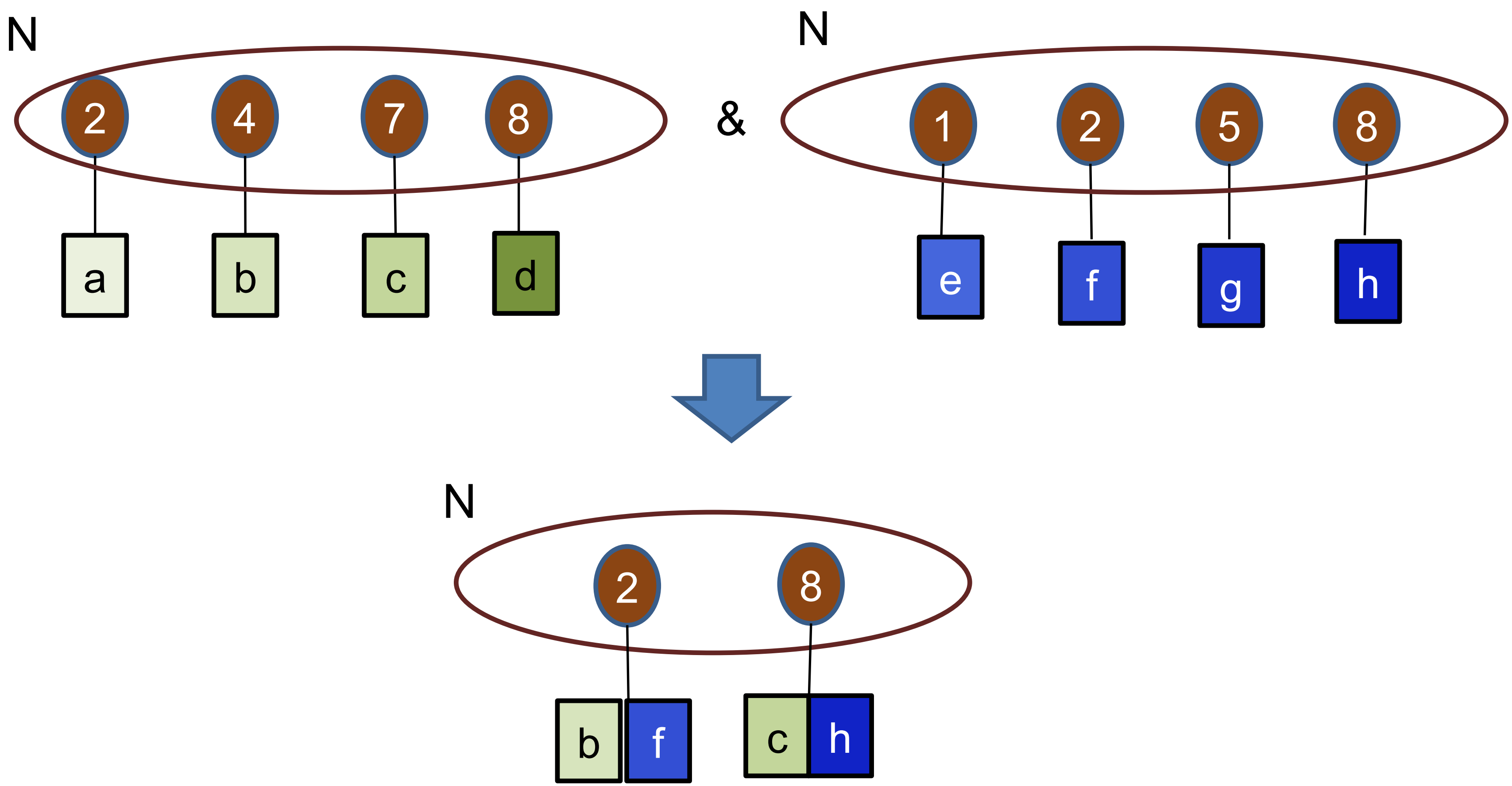
Fiber Coordinate Projection



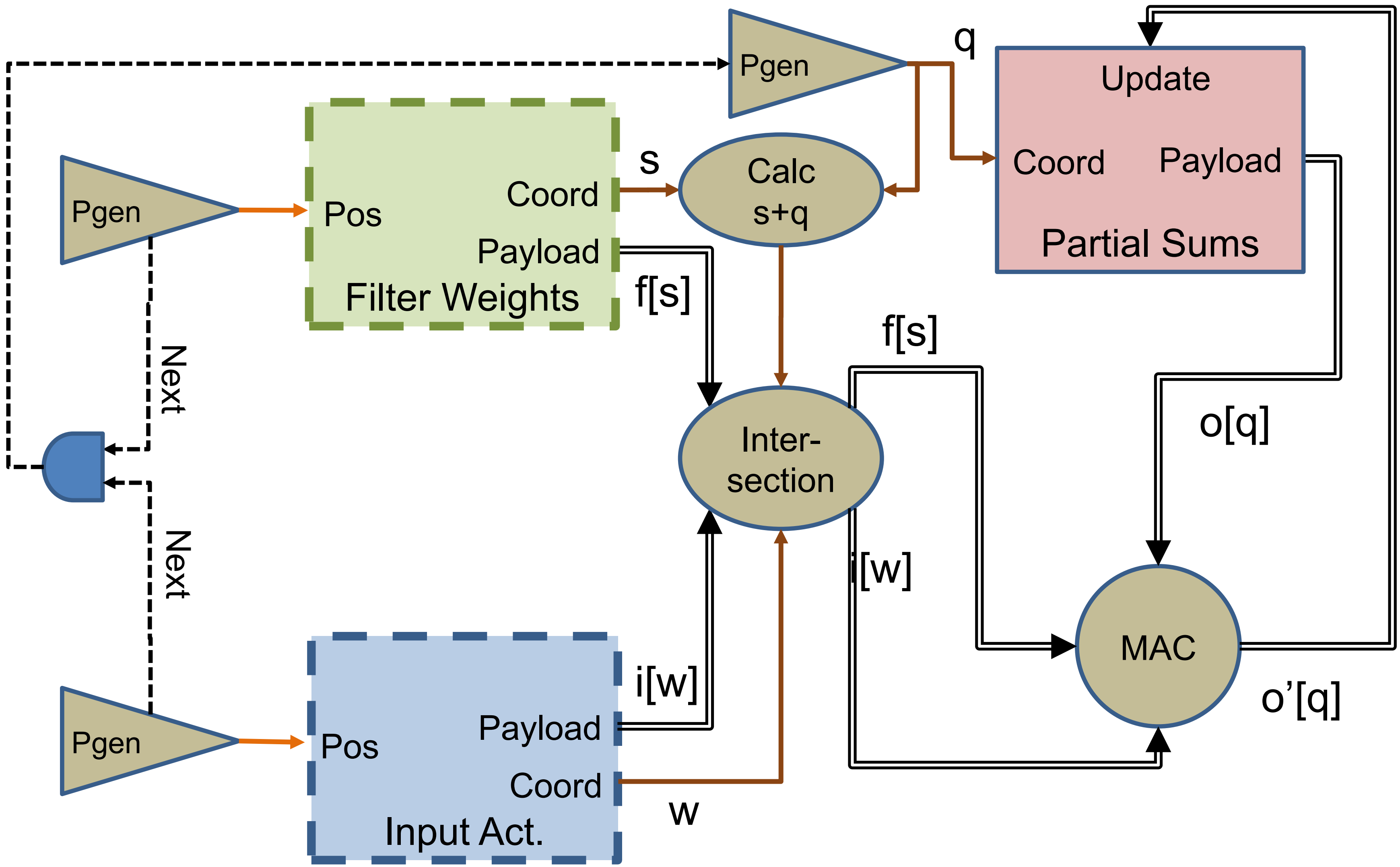


fiber-projection

Fiber Intersection



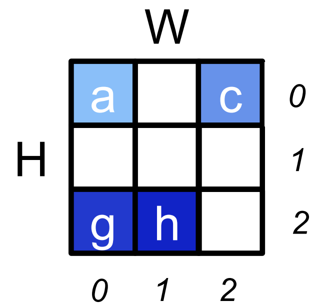
Output Stationary - Sparse Weights & Inputs

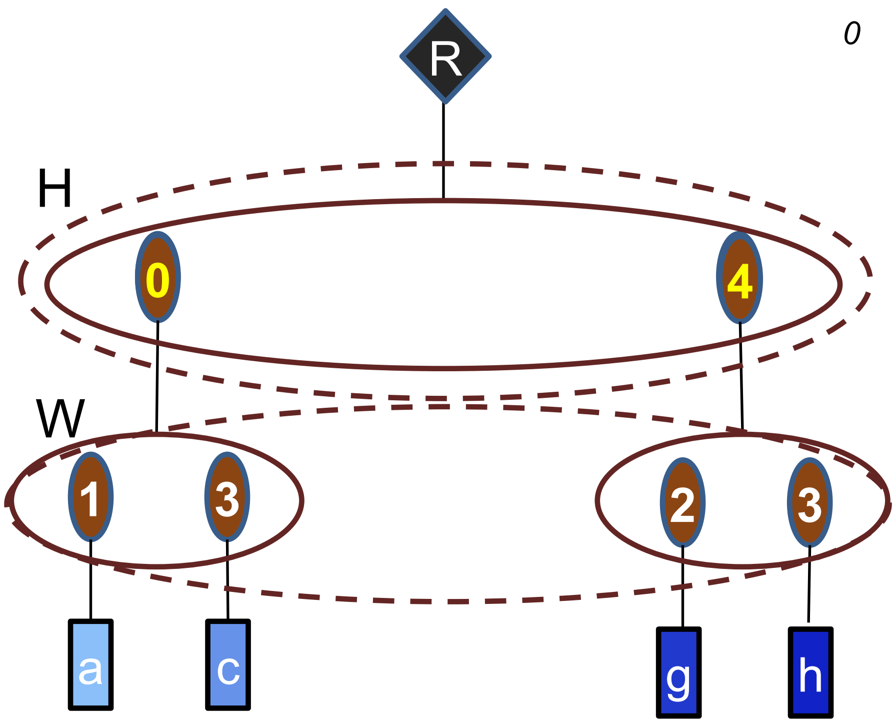


Flattening Ranks

For efficiency one can form new representations where the data structure for two or more ranks are combined.

Tensor-<CC>(HW)





Flattening Ranks

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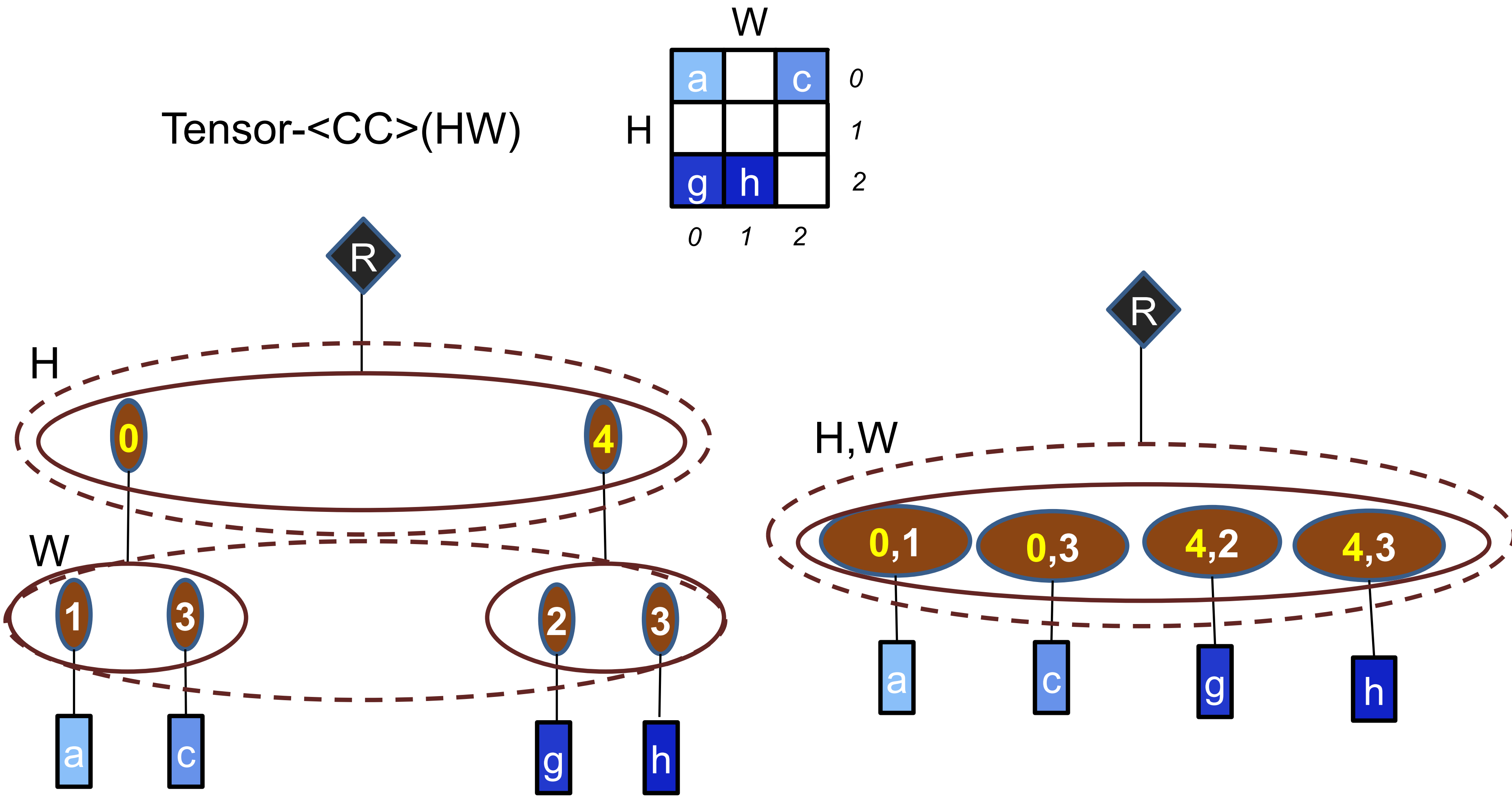
Tensor-<CC>(HW)



Flattening Ranks

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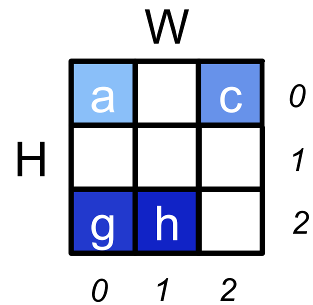
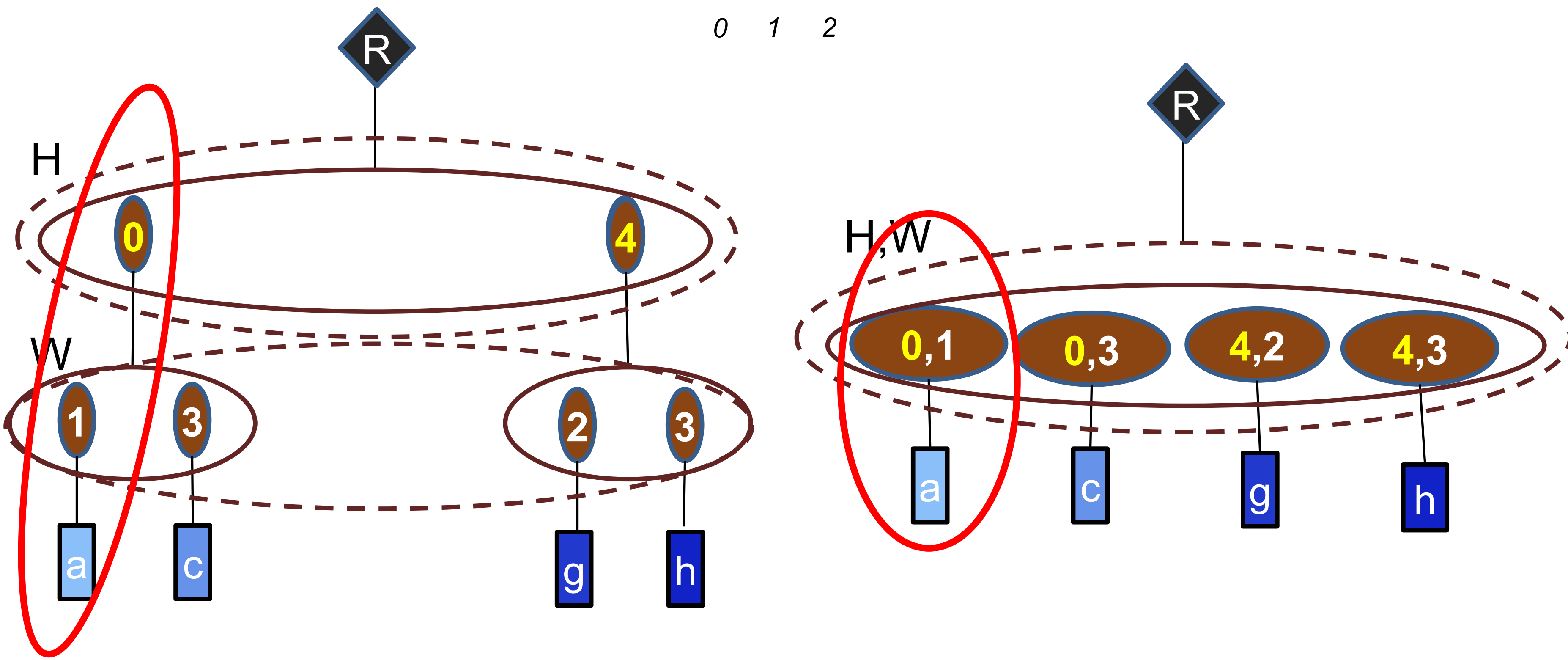
Tensor-<CC>(HW)





Flattening Ranks

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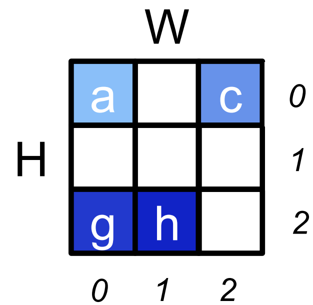
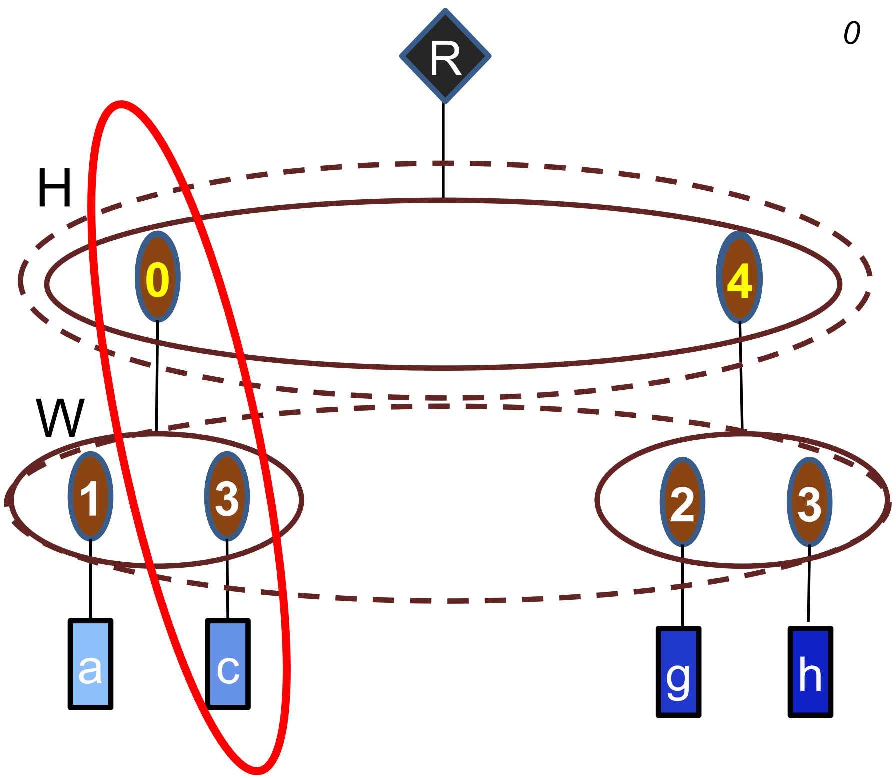
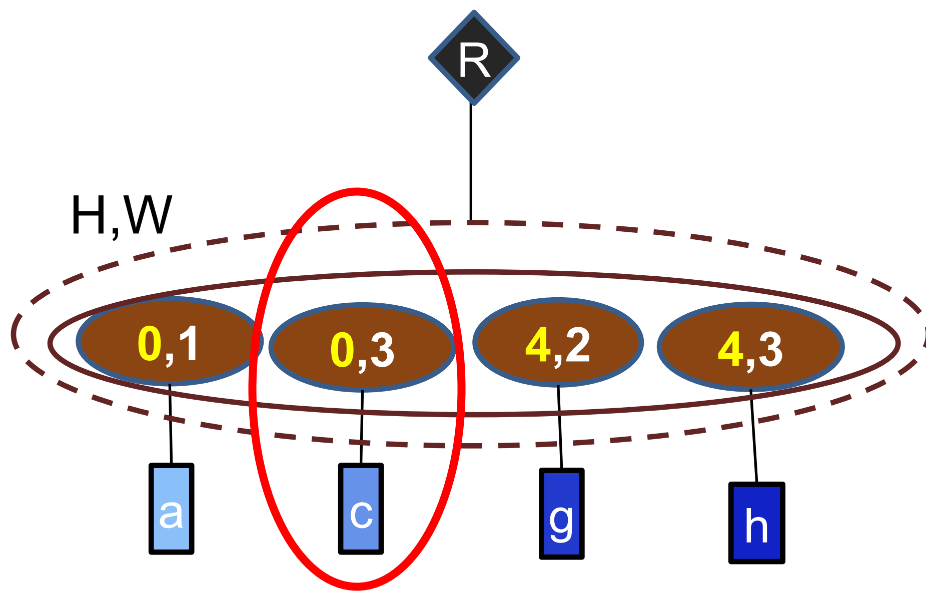
Tensor-<CC>(HW)

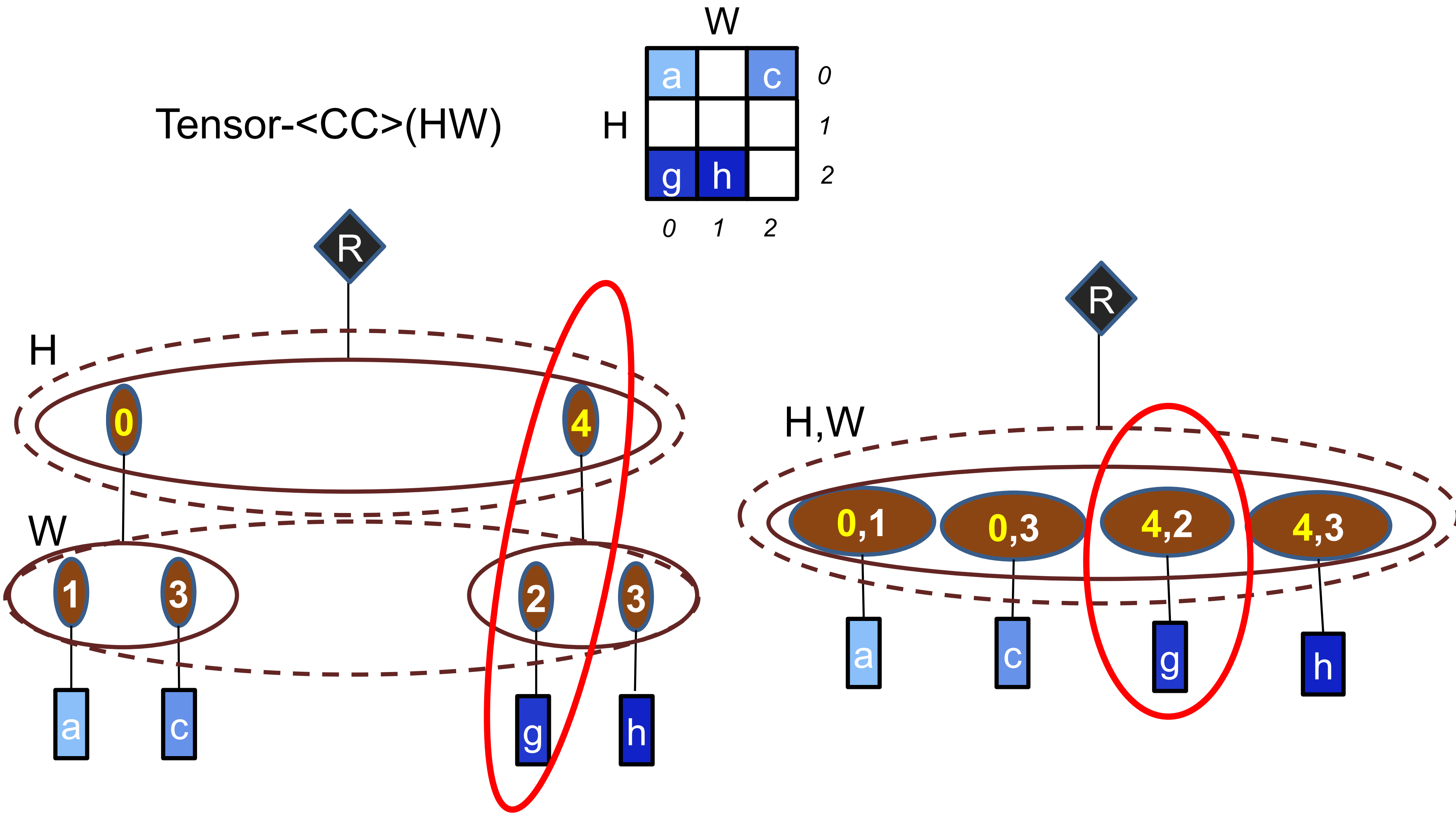






Flattening Ranks

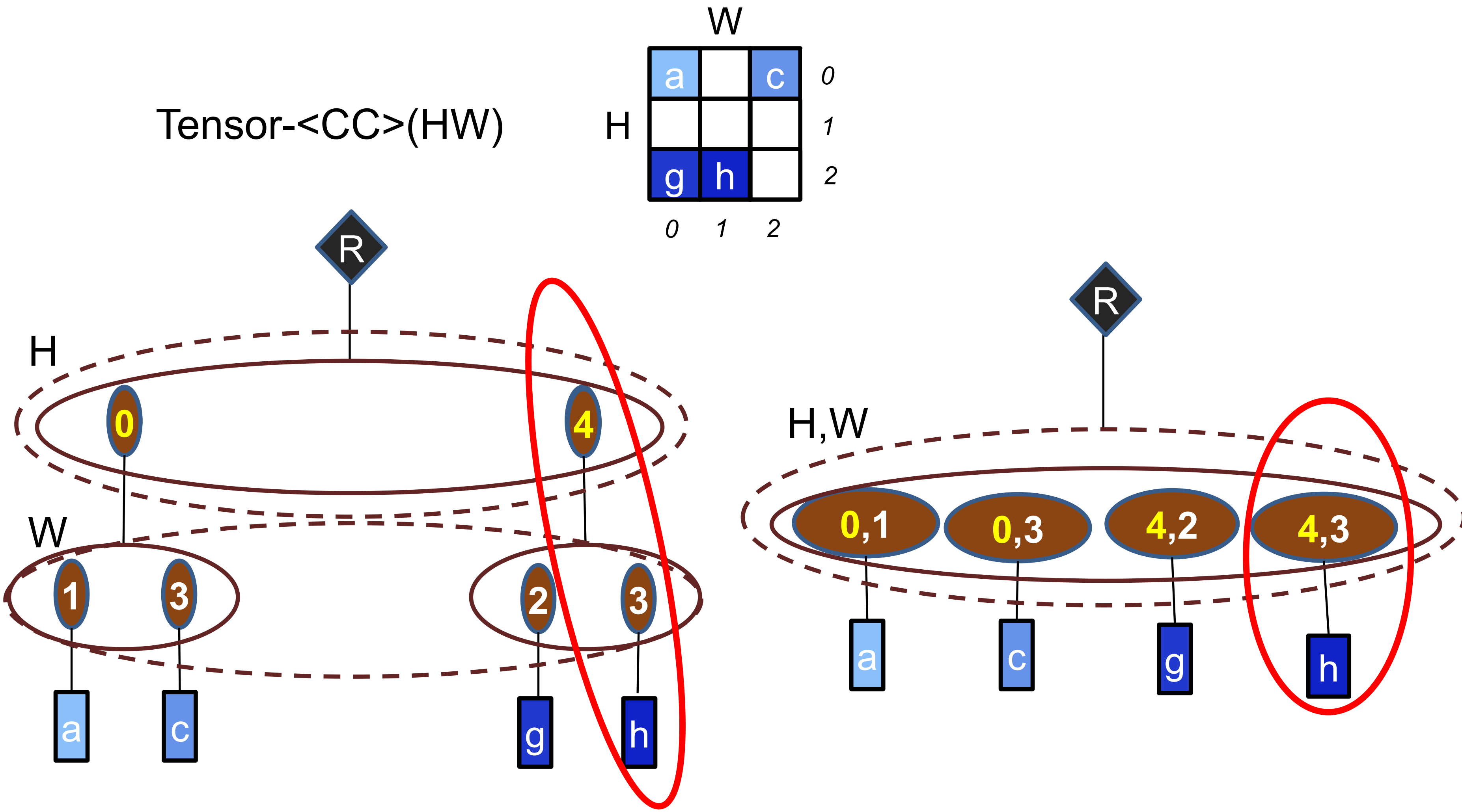
For efficiency one can form new representations where the data structure for two or more ranks are combined.



Flattening Ranks

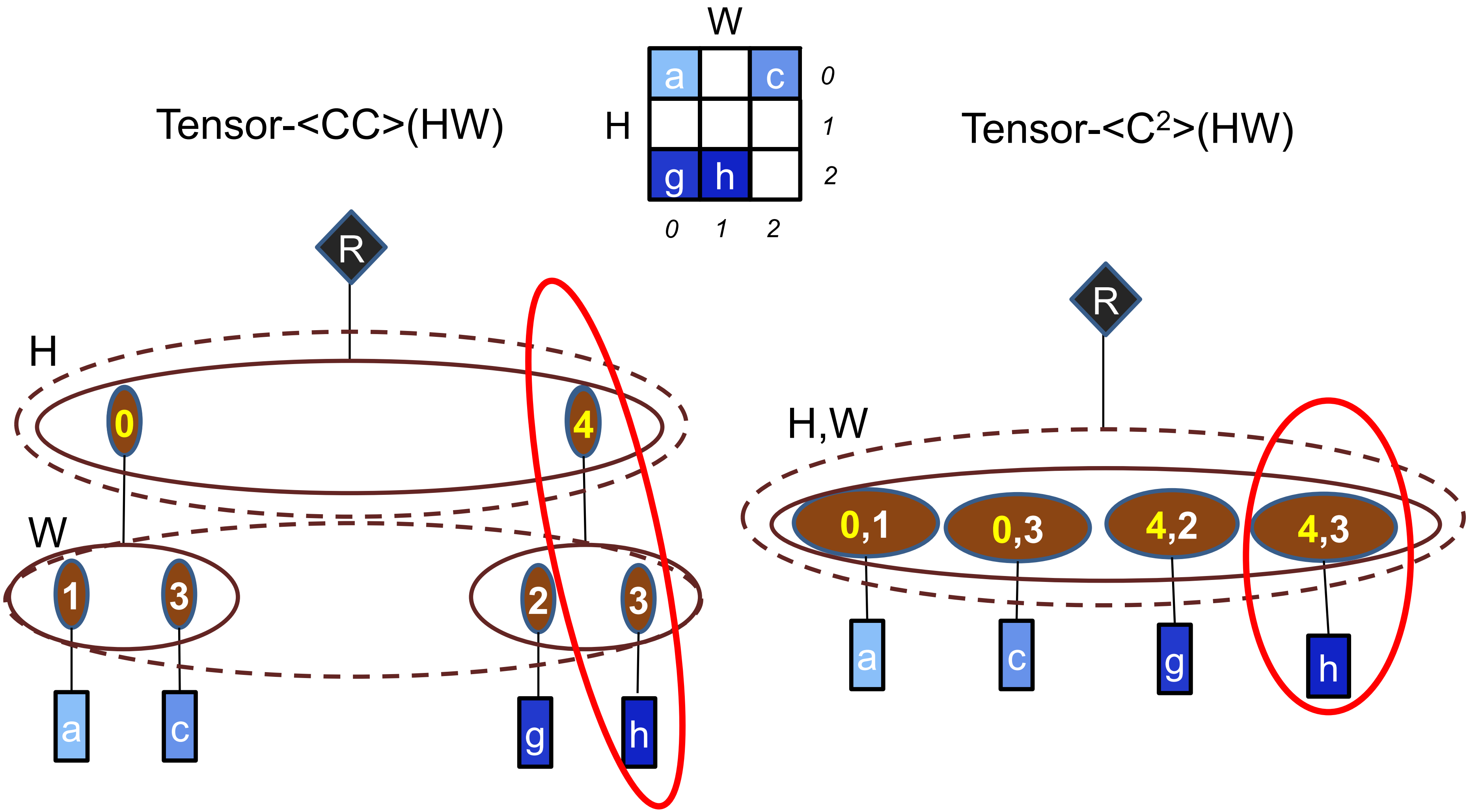
For efficiency one can form new representations where the data structure for two or more ranks are combined.

Tensor-<CC>(HW)



Flattening Ranks

For efficiency one can form new representations where the data structure for two or more ranks are combined.



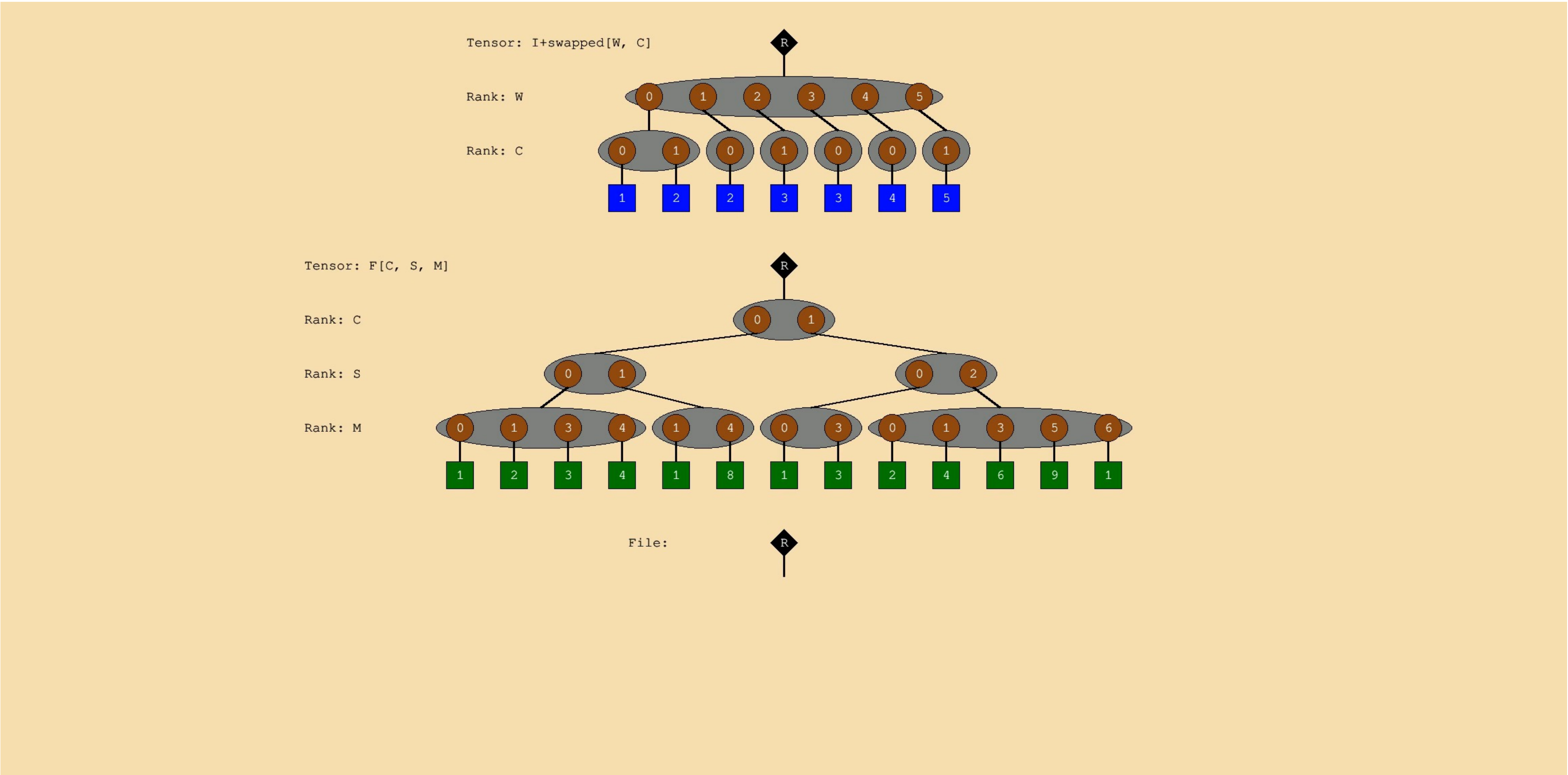
Row Stationary – Sparse Inputs & Activations

```
i = Tensor(CW)          # Input activations (CW flattened)
f = Tensor(C,SM)        # Filter weights (SM flattened)
o = Array(M, Q)         # Output activations

for ((c, w), i_val) in i:
  f_c = f.getPayload(c)
  f_c_split = f_c.splitEven(2)
  parallel-for (_, f_sm) in f_c_split:
    for ((s, m), f_val) in f_sm if w-Q <= s < w:
      q = w - s
      o[m, q] += i_val * f_val
```

Eyeriss V2 – Chen et.al., JETCAS 2018

Row Stationary – Sparse Inputs & Activations



Eyeriss V2 – Chen et.al., JETCAS 2018

Thank you!

Next Lecture:
Transactional Memory