## Problem M13.1: Sequential Consistency

## Problem M13.1.A

Can X hold value of 4 after all three threads have completed? Please explain briefly.
Yes / No
C1-C4, B1-B3, A1-A4, B4- B6

## Problem M13.1.B

Can X hold value of 5 after all three threads have completed?
Yes / No
All results must be even!

## Problem M13.1.C

Can X hold value of 6 after all three threads have completed?
Yes / No
All of C, All of A, All of B

Problem M13.1.D

For this particular program, can a processor that reorders instructions but follows local dependencies produce an answer that cannot be produced under the SC model?

Yes / No

All stores/loads must be done in order because they're to the same address, so no new results are possible.

## Problem M13.2: Relaxed Memory Models [? Hours]

We will study the interaction between two processes on different processors on such a system:

| P1 | P2 |  |
| :---: | :---: | :---: |
| P1.1: LW R2, 0 (R8) | P2.1: LW R4, 0 (R9) |  |
| P1.2: SW R2, 0 (R9) | P2.2: SW R5, 0 (R8) |  |
| P1.3: LW R3, 0 (R8) | P2.3: SW R4, 0 (R8) |  |

Problem M13.2.A

| Memory | contents |
| :--- | :--- |
| M[R8] | 7 |
| M[R9] | 6 |

Yes No

P1.1 P2.1 P1.2 P1.3 P2.2 P2.3

## Problem M13.2.B

| memory | Contents |
| :--- | :--- |
| M[R8] | 6 |
| M[R9] | 7 |

## Yes <br> 

The result would require that the memory contents don't change. Since each thread reads a data value and writes it to another address, this simply impossible here.

## Problem M13.2.C

## Is it possible for M[R8] to hold 0 ?

Yes


The only way that $\mathrm{M}[\mathrm{R} 8]$ could end up with 0 is if P 2.3 is completed before P 2.1 and P 2.2 . This violates Weak Ordering, so it is not possible.

Now consider the same program, but with two MEMBAR instructions.

| P1 | P2 |
| :---: | :---: |
| P1.1: LW R2, 0 (R8) | P2.1: LW R4, 0 (R9) |
| P1.2: SW R2, 0 (R9) | MEMBAR $_{\text {PW }}$ |
| MEMBARWR | P2.2: SW R5, 0 (R8) |
| P1.3: LW R3, 0 (R8) | P2.3: SW R4, 0 (R8) |

We want to compare execution of the two programs on our system.
Here the intention was to keep the starting conditions the same as in first three questions, and ask about the final conditions. This wasn't clear, so we accepted both solutions. The yes/no answers don't actually change, but Questions 11 for 12 become simpler.

## Problem M13.2.D

If both $M[R 8]$ and $M[R 9]$ contain 6 , is it possible for $R 3$ to hold 8 ?

Without MEMBAR instructions?


No

With MEMBAR instructions?


No

Following sequence works with and without MEMBAR instructions:
P1.1 -> P1.2 -> P2.1 -> P2.2 -> P1.3 -> P2.3

Problem M13.2.E

If both $M[R 8]$ and $M[R 9]$ contain 7, is it possible for $R 3$ to hold 6 ?

Without MEMBAR instructions?
Yes


With MEMBAR instructions?
Yes


If $\mathrm{M}[\mathrm{R} 8]$ and $\mathrm{M}[\mathrm{R} 9]$ are to end up with 7, we have to execute P 2.3 before we execute P 1.1 Since P1.3 has to come after P1.1 (Weak Ordering), R3, has to end up with 7 not 6.

## Problem M13.2.F

Is it possible for both $\mathrm{M}[\mathrm{R} 8]$ and $\mathrm{M}[\mathrm{R} 9]$ to hold 8 ?

Without MEMBAR instructions?


P2.2 P1.1 P1.2 P2.1 P2.3 P1.3

With MEMBAR instructions?
Yes
No

The sequence above violates the MEMBAR in P2—P2.2 executes before P2.1. That is the only way to get 8 into both memory locations, thus the result is impossible with MEMBARs insterted.

## Problem M13.3: Memory Models

Consider a system which uses Sequential Consistency (SC). There are three processes, P1, P2 and P3, on different processors on such a system (the values of $R_{A}, R_{B}, R_{C}$ were all zeros before the execution):

| P1 | P2 | P3 |
| :---: | :---: | :---: |
| P1.1: ST (A), 1 | P2.1: ST (B), 1 | P3.1: ST (C), 1 |
| P1.2: LD $R_{C}, ~(C)$ | $P 2.2: L D R_{A},(A)$ | $P 3.2: L D R_{B}, ~(B)$ |

## Problem M13.3.A

After all processes have executed, it is possible for the system to have multiple machine states. For example, $\left\{R_{A}, R_{B}, R_{C}\right\}=\{1,1,1\}$ is possible if the execution sequence of instructions is $P 1.1 \rightarrow P 2.1 \rightarrow P 3.1 \rightarrow P 1.2 \rightarrow P 2.2 \rightarrow P 3.2$. Also, $\left\{R_{A}, R_{B}, R_{C}\right\}=\{1,1,0\}$ is possible if the sequence is $\mathrm{P} 1.1 \rightarrow \mathrm{P} 1.2 \rightarrow \mathrm{P} 2.1 \rightarrow \mathrm{P} 3.1 \rightarrow \mathrm{P} 2.2 \rightarrow \mathrm{P} 3.2$.

For each state of $\left\{R_{A}, R_{B}, R_{C}\right\}$ below, specify the execution sequence of instructions that results in the corresponding state. If the state is NOT possible with SC, just put X.
$\{0,0,0\}: \mathrm{X}$
$\{0,1,0\}:$ P2.1 P2.2 P1.1P1.2P3.1 P3.2
$\{1,0,0\}:$ P1.1 P1.2 P3.1 P3.2 P2.1 P2.2
$\{0,0,1\}:$ P3.1 P3.2 P2.1 P2.2 P1.1 P1.2

## Problem M13.3.B

Now consider a system which uses Weak Ordering(WO), meaning that a read or a write may complete before a read or a write that is earlier in program order if they are to different addresses and there are no data dependencies.

Does WO allow the machine state(s) that is not possible with SC? If yes, provide an execution sequence that will generate the machine states(s).

Yes. $\{0,0,0\}$ by $\mathrm{P} 1.2 \rightarrow \mathrm{P} 2.2 \rightarrow \mathrm{P} 3.2 \rightarrow \mathrm{P} 1.1 \rightarrow \mathrm{P} 2.1 \rightarrow \mathrm{P} 3.1$

## Problem M13.3.C

The WO system in Problem M13.3.B provides four fine-grained memory barrier instructions. Below is the description of these instructions.

- MEMBAR $R_{R R}$ guarantees that all read operations initiated before the $M E M B A R_{R R}$ will be seen before any read operation initiated after it.
- MEMBAR $R_{\text {RW }}$ guarantees that all read operations initiated before the $M E M B A R_{R W}$ will be seen before any write operation initiated after it.
- MEMBAR $R_{\text {WR }}$ guarantees that all write operations initiated before the $M E M B A R_{W R}$ will be seen before any read operation initiated after it.
- MEMBAR ${ }_{\text {ww }}$ guarantees that all write operations initiated before the MEMBAR $_{\text {Ww }}$ will be seen before any write operation initiated after it.

Using the minimum number of memory barrier instructions, rewrite P1, P2 and P3 so the machine state(s) that is not possible with SC by the original programs is also not possible with WO by your programs.

| P1 | P2 | P3 |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { P1.1: } \operatorname{ST} \text { (A), } 1 \\ & \text { MEMBAR }_{W R} \\ & \text { P1.2: LD } R_{C}, \quad(C) \end{aligned}$ | $\begin{aligned} & \text { P2.1: ST (B), } 1 \\ & M E M B A R_{W R} \\ & \text { P2.2: LD } R_{A}, \text { (A) } \end{aligned}$ | P3.1: ST (C), 1 MEMBAR ${ }_{\text {WR }}$ <br> P3.2: LD $\mathrm{R}_{\mathrm{B}}$, (B) |

## Problem M13.4: Memory consistency models (Spring 2016 Quiz 3, Part B)

Consider two processes P 1 and P 2 running on two different processors.
Assume that memory locations $X$ and $Y$ contain initial value 0.

| P1 | P2 |  |
| :--- | :--- | :--- |
|  |  |  |
| P1.1: $\quad$ LD R1 $\leftarrow(Y)$ | P2.1: | ST $(X) \leftarrow 1$ |
| P1.2: $\quad$ LD R2 $\leftarrow(X)$ | P2.2: | ST $(Y) \leftarrow 1$ |

## Problem M13.4.A

Out of the following possible final values of $(\mathrm{X}, \mathrm{Y}, \mathrm{R} 1, \mathrm{R} 2)$, circle the ones that could occur if the system is Sequentially Consistent (SC).
$(\mathbf{0 , 0 , 0 , 0})$
(0,0,0,1)
(0,0,1,0)
(0,0,1,1)
(1,1,0,0)
(1,1,0,1)
(1,1,1,0)
(1,1,1,1)

## Problem M13.4.B

Out of the following possible final values of $(\mathrm{X}, \mathrm{Y}, \mathrm{R} 1, \mathrm{R} 2)$, circle the ones that could occur if the system enforces RMO, a weak memory model where loads and stores can be reordered after prior loads or stores.
$(0,0,0,0)$
(0,0,0,1)
(0,0,1,0)
(0,0,1,1)
(1,1,0,0)
(1,1,0,1)
$(1,1,1,0)$
$(1,1,1,1)$

The RMO machine has the following fine-grained barrier instructions:

- MEMBAR $R_{R R}$ guarantees that all reads initiated before $M E M B A R_{R R}$ will be performed before any read initiated after it.
- MEMBAR ${ }_{\mathbf{R W}}$ guarantees that all reads initiated before MEMBAR $_{\text {RW }}$ will be performed before any write initiated after it.
- MEMBAR ${ }_{W R}$ guarantees that all writes initiated before $M E M B A R_{W R}$ will be performed before any read initiated after it.
- MEMBARww guarantees that all writes initiated before MEMBARww will be performed before any write initiated after it.

Use the minimum number of memory barrier instructions, rewrite P1 and P2 such that the RMO machine produces the same outputs as the SC machine for the given code.

| P1 | P2 |
| :---: | :---: |
| $\begin{array}{ll} \text { P1.1: } & \text { LD } R 1 \leftarrow(Y) \\ & \text { MEMBARRR } \\ \text { P1.2: } & \text { LD } R 2 \leftarrow(\mathrm{X}) \end{array}$ | $\begin{array}{ll} \text { P2.1: } & \text { ST }(X) \leftarrow 1 \\ & \text { MEMBARWW } \\ \text { P2.2: } & \text { ST }(Y) \leftarrow 1 \end{array}$ |

Again, consider two processes P1 and P2 running the code below on two different processors. Assume that memory locations $X, Y$, and $Z$ contain initial value 0 .

| P1 | P2 |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| P1.1: | LD R1 $\leftarrow(Z)$ | P2.1: | ST $(X) \leftarrow 1$ |
| P1.2: | ST $(Y) \leftarrow 1$ | P2.2: | LD R3 $\leftarrow(Y)$ |
| P1.3: | LD R2 $\leftarrow(X)$ | P2.3: | ST $(Z) \leftarrow 1$ |

## Problem M13.4.D

Out of the following possible final values of $(R 1, R 2, R 3)$, circle the ones that could occur if the system is Sequentially Consistent (SC).
$(\mathbf{0 , 0 , 0})$
$(0,1,0)$
$(\mathbf{1 , 0 , 0})$
$(1,1,0)$
$(0,0,1)$
$(0,1,1)$
$(1,0,1)$
$(1,1,1)$

## Problem M13.4.E

Out of the following possible final values of (R1, R2, R3), circle the ones that could occur if the system enforces RMO (loads and stores can be reordered after prior loads or stores).
$(0,0,0)$
(0,1,0)
$(1,0,0)$
$(1,1,0)$
$(0,0,1)$
$(0,1,1)$
$(1,0,1)$
$(1,1,1)$

## Problem M13.4.F

Using the minimum number of memory barrier instructions (given in Question 3), rewrite P1 and P2 such that the RMO machine produces the same outputs as the SC machine for the given code.


