// 2:1 multiplexer
always @(sel or a or b) begin
  if (sel) z <= b;
  else z <= a;
end
Some History ...

In late 70’s Mead-Covway showed how to lay out transistors systematically to build logic circuits.

**Tools:**
- Layout editors, for manual design;
- Design rule checkers, to check for legal layout configurations;
- Transistor-level simulators;
- Software generators to create dense transistor layouts;

1980: Circuits had 100K transistors

In 80’s designers moved to the use of gate arrays and standardized cells, pre-characterized modules of circuits, to increase productivity.

**Tools:**
- To automatically place and route a netlist of cells from a predefined cell library
- The emphasis in design shifted to gate-level schematic entry and simulation
History continued...

By late 80’s designers found it very tedious to move a gate-level design from one library to another because libraries could be very different and each required its own optimizations.

Tools:

- Logic Synthesis tools to go from Gate netlists to a standard cell netlist for a given cell library.
- Powerful optimizations!
- Simulation tools for gate netlists, RTL;
- Design and tools for testability, equivalence checking, ...

IBM and other companies had internal tools that emphasized top down design methodology based on logic synthesis.

Two groups of designers came together in 90’s: Those who wanted to quickly simulate their designs expressed in some HDL and those who wanted to map a gate-level design in a variety of standard cell libraries in an optimized manner.
Synthesis Tools

Idea: once a behavioral model has been finished why not use it to *automatically synthesize* a logic implementation in much the same way as a compiler generates executable code from a source program?

a.k.a. “silicon compilers”

Synthesis programs process the HDL then

1. infer logic and state elements
2. perform technology-independent optimizations (e.g., logic simplification, state assignment)
3. map elements to the target technology
4. perform technology-dependent optimizations (e.g., multi-level logic optimization, choose gate strengths to achieve speed goals)
Simulation vs Synthesis

In a HDL like Verilog or VHDL not every thing that can be simulated can be synthesized.

There is a difference between simulation and synthesis semantics. Simulation semantics are based on sequential execution of the program with some notion of concurrent synchronous processes. Not all such programs can be synthesized. It is not easy to specify the synthesizable subset of an HDL.

So in today's lecture we will gloss over 1, briefly discuss 2 and emphasize 3 and 4.

1 infer logic and state elements
2 perform technology-independent optimizations (e.g., logic simplification, state assignment)
3 map elements to the target technology
4 perform technology-dependent optimizations (e.g., multi-level logic optimization, choose gate strengths to achieve speed goals)
Logic Synthesis

\textbf{assign} z = (a & b) | c;

// dataflow
\textbf{assign} z = sel ? a : b;
Logic Synthesis (II)

wire [3:0] x,y,sum;
wire cout;
assign {cout,sum} = x + y;

As a default + is implemented as a ripple carry editor
module parity(in,p);
    parameter WIDTH = 2; // default width is 2
    input [WIDTH-1 : 0] in;
    output p;
    // simple approach: assign p = ^in;
    // here's another, more general approach
    reg p;
    always @(in) begin: loop
        integer i;
        reg parity = 0;
        for (i = 0; i < WIDTH; i = i + 1)
            parity = parity ^ in[i];
        p <= parity;
    end
endmodule

wire [3:0] word;
wire parity;
parity #(4) ecc(word,parity); // specify WIDTH = 4
Synthesis of Sequential Logic

```verilog
reg q;

// D-latch
always @(g or d) begin
  if (g) q <= d;
end
```

If q were simply a combinational function of d, the synthesizer could just create the appropriate combinational logic. But since there are times when the always block executes but q isn’t assigned (e.g., when \( g = 0 \)), the synthesizer has to arrange to remember the value of “old” value q even if d is changing → it will infer the need for a **storage element** (latch, register, ...). Sometimes this inference happens even when you don’t mean it to - you have to be careful to always ensure an assignment happens each time through the block if you don’t want storage elements to appear in your design.

```verilog
reg q; // this time we mean it!

// D-register
always @(posedge clk) begin
  q <= d;
end
```
Sequential Logic (II)

reg q;
// register with synchronous clear
always @(posedge clk) begin
    if (!reset)  // reset is active low
        q <= 0;
    else // implicit posedge clk
        q <= d;
end

reg q;
// register with asynchronous clear
always @(posedge clk or negedge reset)
begin
    if (!reset)  // reset is active low
        q <= 0;
    else // implicit posedge clk
        q <= d;
end
// warning! async inputs are dangerous!
// there’s a race between them and the
// rising edge of clk.
Technology-independent* optimizations

- Two-level boolean minimization: based on the assumption that reducing the number of product terms in an equation and reducing the size of each product term will result in a smaller/faster implementation.

- Optimizing finite state machines: look for equivalent FSMs (i.e., FSMs that produce the same outputs given the same sequence of inputs) that have fewer states.

- Choosing FSM state encodings that minimize implementation area (= size of state storage + size of logic to implement next state and output functions).

* None of these operations is completely isolated from the target technology. But experience has shown that it’s advantageous to reduce the size of the problem as much as possible before starting the technology-dependent optimizations. In some places (e.g. the ratio of the size of storage elements to the size logic gates) our assumptions will be valid for several generations of the technology.
Two-Level Boolean Minimization

Two-level representation for a multiple-output Boolean function:
- Sum-of-products

Optimization criteria:
- number of product terms
- number of literals
- a combination of both

Minimization steps for a given function:

1. **Generate** the set of prime product-terms for the function
2. **Select** a minimum set of prime terms to cover the function.

State-of-the-art logic minimization algorithms are all based on the Quine-McCluskey method and follow the above two steps.
Prime Term Generation

Express your Boolean function using 0-terms (product terms with no don't care care entries).

Include only those entries where the output of the function is 1 (label each entry with its decimal equivalent).

Look for pairs of 0-terms that differ in only one bit position and merge them in a 1-term (i.e., a term that has exactly one '-' entry).

Next 1-terms are examined in pairs to see if they can be merged into 2-terms, etc. Mark k-terms that get merged into (k+1) terms so we can discard them later.

2-terms: 8, 9, 10, 11 10-- [D]
10, 11, 14, 15 1-1- [E]

Label unmerged terms:
these terms are prime!

3-terms:

Example due to Srini Devadas
# Prime Term Table

An “X” in the prime term table in row $R$ and column $C$ signifies that the 0-term corresponding to row $R$ is contained by the prime corresponding to column $C$.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>0101</td>
<td>.</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>0111</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1000</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>.</td>
</tr>
<tr>
<td>1001</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1010</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
</tr>
<tr>
<td>1011</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
</tr>
<tr>
<td>1110</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>.</td>
</tr>
<tr>
<td>1111</td>
<td>.</td>
<td>X</td>
<td>.</td>
<td>X</td>
<td>.</td>
</tr>
</tbody>
</table>

Each row with a single X signifies an essential prime term since any prime implementation will have to include that prime term because the corresponding 0-term is not contained in any other prime.

In this example the essential primes “cover” all the 0-terms.
Dominated Columns

Some functions may not have essential primes (Fig. 1), so make arbitrary selection of first prime in cover, say A (Fig. 2). A column U of a prime term table dominates V if U contains every row contained in V. Delete the dominated columns (Fig. 3).

1. Prime table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>X</td>
</tr>
<tr>
<td>0001</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>0101</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>0111</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1000</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1010</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1110</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1111</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

2. Table with A selected

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>0101</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>0111</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1000</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1010</td>
<td>.</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1110</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1111</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

3. Table with B & H removed

<table>
<thead>
<tr>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>0101</td>
<td>X</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>0111</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1000</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
</tr>
<tr>
<td>1010</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
</tr>
<tr>
<td>1110</td>
<td>.</td>
<td>X</td>
<td>X</td>
<td>.</td>
</tr>
<tr>
<td>1111</td>
<td>X</td>
<td>X</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

C dominates B, G dominates H

Selecting C and G shows that only E is needed to complete the cover.

This gives a prime cover of \{A, C, E, G\}. Now backtrack to our choice of A and explore a different (arbitrary) first choice; repeat, remembering minimum cover found during search.
The Quine-McCluskey Method

The input to the procedure is the prime term table $T$.

1. Delete the dominated primes (columns) in $T$. Detect essential primes in $T$ by checking to see if any 0-term is contained by a single prime. Add these essential primes to the selected set. Repeat until no new essential primes are detected.

2. If the size of the selected set of primes equals or exceeds the best solution thus far return from this level of recursion. If there are no elements left to be contained, declare the selected set as the best solution recorded thus far.

3. Heuristically select a prime.

4. Add the chosen prime to the selected set and create a new table by deleting the prime and all 0-terms that are contained by this prime in the original table. Set $T$ to this new table and go to Step 1.

Then, create a new table by deleting the chosen prime from the original table without adding it to the selected set. No 0-terms are deleted from the original table. Set $T$ to new table and go to Step 1.

The good news: this technique generalizes to multi-output functions (see the QM handout on the website for details). The bad news: the search time grows as $2^{2^N}$ where $N$ is the number of inputs. So most modern minimization systems use heuristics to make dramatic reductions in the processing time.
Mapping to target technology

• Once we’ve minimized the logic equations, the next step is mapping each equation to the gates in our target gate library.

Popular approach: DAG covering (K. Keutzer)
Mapping Example

Problem statement: find an “optimal” mapping of this circuit:

Into this library:

Example due to Kurt Keutzer
DAG Covering

• Represent input netlist in normal form (“subject DAG”).
• Represent each library gate in normal form (“primitive DAGs”).
• Goal: find a minimum cost covering of the subject DAG by the primitive DAGs.
  • If the subject and primitive DAGs are trees, there is an efficient algorithm (dynamic programming) for finding the optimum cover.
  • So: partition subject DAG into a forest of trees (each gate with fanout > 1 becomes root of a new tree), generate optimal solutions for each tree, stitch solutions together.
## Primitive DAGs for library gates

<table>
<thead>
<tr>
<th>Element/Area Cost</th>
<th>Tree Representation (normal form)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INVERTER</strong></td>
<td>![INVERTER Diagram]</td>
</tr>
<tr>
<td>2</td>
<td>![INVERTER Diagram]</td>
</tr>
<tr>
<td><strong>NAND2</strong></td>
<td>![NAND2 Diagram]</td>
</tr>
<tr>
<td>3</td>
<td>![NAND2 Diagram]</td>
</tr>
<tr>
<td><strong>NAND3</strong></td>
<td>![NAND3 Diagram]</td>
</tr>
<tr>
<td>4</td>
<td>![NAND3 Diagram]</td>
</tr>
<tr>
<td><strong>NAND4</strong></td>
<td>![NAND4 Diagram]</td>
</tr>
<tr>
<td>5</td>
<td>![NAND4 Diagram]</td>
</tr>
<tr>
<td><strong>AOI21</strong></td>
<td>![AOI21 Diagram]</td>
</tr>
<tr>
<td>4</td>
<td>![AOI21 Diagram]</td>
</tr>
<tr>
<td><strong>AOI22</strong></td>
<td>![AOI22 Diagram]</td>
</tr>
<tr>
<td>5</td>
<td>![AOI22 Diagram]</td>
</tr>
</tbody>
</table>
Possible Covers

Hmmm. Seems promising but is there a systematic and efficient way to arrive at the optimal answer?
Use dynamic programming!

Principle of optimality: Optimal cover for a tree consists of a best match at the root of the tree plus the optimal cover for the sub-trees starting at each input of the match.

Complexity:
To determine the optimal cover for a tree we only need to consider a best-cost match at the root of the tree (constant time in the number of matched cells), plus the optimal cover for the subtrees starting at each input to the match (constant time in the fanin of each match) → $O(N)$.
Optimal tree covering example

Step 1.

Step 2.

Step 3.

Step 4.

Cover with ND2 or ND3?

1 NAND2  
+ subtree 5  
Area cost 8

1 NAND3  
= 4

Cover with INV or AO21?

1 AO21  
+ subtree 1  
+ subtree 2  
Area cost 13
Area cost 9

1 Inverter  
+ subtree 1  
= 4
Example (II)

Step 5.
Cover with ND2 or ND3?

NAND2
- subtree 1: 9
- subtree 2: 4
- 1 NAND2: 3
Area cost 16

NAND3
- subtree 1: 8
- subtree 2: 2
- subtree 3: 4
- 1 NAND2: 4
Area cost 18

Step 6.
Cover with INV or AOI21?

INV
- subtree 1: 16
- 1 INV: 2
Area cost 18

AOI21
- subtree 1: 11
- 1 AOI21: 5
Area cost 20

Step 7.
Cover with ND2 or ND3 or ND4?

Step 7a.
Cover with ND2?

Area cost 21

subtree 1: 18
subtree 2: 0
1 NAND2: 3
Example (III)

Yes Regis, this is our final answer. This matches our earlier intuitive cover, but accomplished systematically.

Refinements: timing optimization incorporating load-dependent delays, optimization for low power.
Technology-dependent optimizations

- Additional library components: more complex cells may be slower but will reduce area for logic off the critical path.
- Load buffering: adding buffers/inverters to improve load-induced delays along the critical path
- Resizing: Resize transistors in gates along critical path
- Retiming: change placement of latches/registers to minimize overall cycle time
- Increase routability over/through cells: reduce routing congestion.

Verilog → Logic Synthesis → Gate netlist → Place & route → Mask

- HDL → logic
- map to target library
- optimize speed, area

- create floorplan blocks
- place cells in block
- route interconnect
- optimize (iterate!)