MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Laboratory for Computer Science

Computation Structures Group Memo 180

# An Abstract Implementation for Concurrent Computation With Streams

by

Jack Dennis
Ken Weng

This paper will appear in the Proceedings of the 1979 International Conference on Parallel Processing.

July 1979

# AN ABSTRACT IMPLEMENTATION FOR CONCURRENT COMPUTATION WITH STREAMS[a]

Jack B. Dennis
Ken K.-S. Weng
Laboratory for Computer Science
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

*Abstract* -- This paper is a contribution toward developing practical general-purpose computer systems embodying data flow principles. We outline a hardware structure capable of high concurrency and present an abstract model of data flow program execution which could be implemented within the proposed hardware structure. Our abstract model supports a user programming language that includes recursive function modules and provides streams of values for inter-module communication.

## Introduction

We present here a conceptual model of program execution that can serve as the functional specification for a distributed or highly concurrent computer system based on data flow principles. The programming language supported by our conceptual model or "abstract implementation" is an applicative or value-oriented language that includes streams of values as a basic programming tool. Streams are attractive because use of streams for communication between program modules leads to programs whose modules have functional semantics and whose overall meaning can be expressed as functional components combined using composition and a fixpoint operator [12] - thus avoiding use of side effects. In the present discussion we only consider determinate programs. The extension of this work to nondeterminate computation is a subject of current research.

Specifically, we introduce a value-oriented language and discuss representation of its semantics by translation into recursive data flow schemas [9]. We sketch an operational semantics (formal interpreter) for these data flow schemas and outline the structure of a hardware system capable of highly concurrent execution of value-oriented programs. A more detailed and complete presentation of this work is given in the thesis of Weng [17].

(a) This research was supported in part by the National Science Foundation under research grant MCS75-04060 A01 and in part by the Lawrence Livermore Laboratory of the University of California under contract 8545403.

## A Simple Value-Oriented Language

Our textual language departs from conventional languages in several ways. There is no notion of sequential control flow and there are no explicit primitives for introducing parallelism. The concurrency of a computation is determined by the data dependency within the program rather than by explicit creation of concurrent processes.

The language is value-oriented in the sense that each syntactic unit defines a mathematical function that maps input values into result values: there are no side effects or other spurious interactions in the evaluation of expressions.

The language does not have the notion of memory locations or variables commonly found in conventional sequential programming languages; instead names are used to denote values defined by expressions in much the same way as in mathematics. With value-oriented semantics, it is natural to write programs in a form that exhibits the inherent concurrency of an algorithm. The data types of the language[a] are <u>integer</u>, <u>real</u>, <u>boolean</u>, <u>character-string</u>, <u>structure</u>, and <u>procedure</u>. We shall call these data types *simple data types*. The operations for types <u>integer</u>, <u>real</u>, <u>boolean</u>, and <u>character-string</u> are the usual operations and need no comment. The operations for values of type <u>structure</u> are defined below. The only operation for <u>procedure</u> values is procedure application.

The syntax of the language is given in Fig. 1. A procedure consists of a set of procedure definitions followed by an expression. A procedure definition is of the form

P = <u>procedure</u> ( $a_1$:$T_1$,..., $a_m$:$T_m$) <u>yields</u> $R_1$,...,$R_n$;
<procedure def>
.
.
.
<procedure def>
<expression>
<u>end</u> P;

(a) The language described here is closely related to the language called VAL in development at MIT [3].

```
Notation :   { < E > }+  means  < E > | < E >, { < E > }+
             { < E > }  means  { < E > }+ | empty

< program > ::=  program { < procedure def > } < expression > end
< procedure def > ::=  < name > = procedure ( < input list > )
                               yield < output list >;
                               { < procedure def > };  < expression >
                               end < name >
< input list > ::=  { < type declaration > }
< type declaration > ::=  < name > : < type >
< output list > ::=  { < type > }

< expression > ::=  < primitive expression >
                  | { < expression > }+
                  | < let-block expression >
                  | < conditional expression >
                  | < application expression >
< let-block expression > ::=
                  let  { < type declaration > };  { < name def > }; in < expression >  end
< name def > ::=  { < name > } = < expression >
< conditional expression > ::=
                  if < expression > then < expression > else < expression > end

< application expression > ::=  < name > ( < expression > )
< primitive expression > ::=
                  < expression > < primitive operation > < expression >
                  | < primitive operation > ( < expression > )
                  | < name >
                  | < constant >
< simple data type > ::=  integer | real | boolean | character-string | structure
< type > ::=  < simple data type > | stream of < simple data type >
```

Figure 1. Syntax of the language

This defines a procedure P that requires m input values $a_1,\dots,a_m$ of types $T_1,\dots,T_m$ respectively. The names $a_1,\dots,a_m$ must be distinct and can appear free in <expression>. The evaluation of the procedure yields an ordered set of values of types $R_1,\dots,R_n$ resulting from <expression>.

Each expression denotes an ordered set (n-tuple) of values whose *arity* is n. We give a recursive definition of the arity A(E) of each of the five types of expressions as follows:

A( <primitive expression> ) = 1

A( $\langle exp_1\rangle, \dots, \langle exp_k\rangle$ )
$= A(\langle exp_1\rangle) + \dots + A(\langle exp_k\rangle)$

A( <let-block expression> )
= A( let <definitions> in <exp> end )
= A( <exp> )

A( <conditional expression> )
= A( if <exp> then $\langle exp_t\rangle$ else $\langle exp_f\rangle$ end )
= A( $\langle exp_t\rangle$ )
= A( $\langle exp_f\rangle$ )

A( <procedure application> )
= A( <name> ( <expression> ) )
= the number of elements in the <output list> of procedure <name>.

For a <procedure def> to be correct, the arity of the expression which is its body must match the number of result types specified in its <output list>.

Often it is convenient to introduce names for expressions because they are common subexpressions of larger expressions. The let-block expression is used for introducing names such that each name stands for an expression of arity one. A let-block expression is of the form:

```
let { <type declaration> };
        <name-list1> = <exp1>;
              .
              .
              .
        <name-listk> = <expk>;
in <exp> end;
```

The names in type declarations of a let-block are local names meaningful only within the block; these names must be distinct from each other and may appear free in $\langle exp_1 \rangle, \ldots, \langle exp_k \rangle$, and $\langle exp \rangle$. Name conflicts in nested let-blocks are resolved by the scope rule that inner definitions take precedence over outer definitions.

We require that the number of names in a name-list be equal to the arity of the expression to the right of the equality sign. The *value* of a name in a name-list is the value of the corresponding expression appearing on the right hand side of the equal sign, and must be of the type specified by the type declaration. The value of a let-block expression is the value of $\langle exp \rangle$.

A conditional expression is of the form:

```
if <exp1> then <exp2> else <exp3> end;
```

The expression $\langle exp_1 \rangle$ is a boolean value of arity one. The expressions $\langle exp_2 \rangle$ and $\langle exp_3 \rangle$ have the same arity and the corresponding value in each expression must be of the same type. The value of a conditional expression is the value of $\langle exp_2 \rangle$ if $\langle exp_1 \rangle$ is the boolean value true; otherwise it is the value of $\langle exp_3 \rangle$.

A procedure application expression is of the form:

```
P( <exp> );
```

where the expression $\langle exp \rangle$ has the same arity as the number of input values required by the procedure P and the type of each value matches that of the input specification. The result of the procedure application is an expression of the arity and types defined by the yield clause of the procedure heading.

As a simple example of a program in our value-oriented language, Fig. 2 shows a procedure that defines a parallel computation of the factorial function.

## Data Structures

For the purpose of the present exposition, we will introduce a simple but very general data structure type. A data structure can be either nil which denotes the structure having no components, or a structure having n component values $v_1, \ldots, v_n$ whose *selector* names are respectively $s_1, \ldots, s_n$. The selectors are either character strings or integers and each selector name must be different from all

```
Factorial = procedure ( n : integer )
        yields integer;

    Product = procedure ( n1 : integer, n2 : integer )
            yields integer;
                if n2 =< n1 then n1
                else let middle : integer;
                        middle = (n1 + n2) quotient 2;
                        in Product( n1, middle )
                        * Product( middle+1, n2 ) end
                end
    end Product;

    if n < 0 then error else Product(1, n) end;

end Factorial;
```

Figure 2. An Example Program

others in the same data structure. We represent such a structure value by the notation

$$(s_1 : v_1, \ldots, s_n : v_n).$$

The operations on data structures are defined below, where d and d' are data structures, s is a selector name, and c is a value of any type:

(1) create ( )
The create operation yields the nil data structure.

(2) append (d, s, c)
The result is a data structure d' which is identical to d except that the s component is c regardless of whether d already contains a component with selector name s.

(3) delete (d, s)
The result is a data structure d' which does not have an s component.

(4) select (d, s)
If d has an s component, the result is the value of that component. Otherwise, the result is the value undefined.

(5) nil-structure (d)
This is a predicate whose value is true if d is nil; otherwise its value is false.

Notice that the effects of

delete (d, s)

and

append (d, s, nil)

are different, since the the delete operation would remove the component (s, d') while the append operation would replace it with (s, nil). It should be mentioned that an array

```
reverse = procedure ( x : structure )
            yields structure;

    if nil-structure ( x )  then x else
            let  left, right : structure;
                    left = reverse( select( x, "r") );
                    right = reverse( select( x, "l") );
            in  append( append
                    ( create( ), "l", left), "r", right)
            end
    end
end reverse;
```

Figure 3. reverse

is simply a data structure whose selector names are all integers.

The data structure operations are illustrated by the recursive procedure "reverse" in Fig. 3, which interchanges the role of selector names l and r in a given data structure of arbitrary depth.

### Streams

A stream is a sequence of values, all of the same type, that are passed in succession, one-at-a-time between program modules.

The use of streams of data in programming is an alternative way of expressing computations that have conventionally been expressed as coroutines or a set of cooperating processes. For example, a compiler may be organized into phases which are implemented as a set of coroutines [6].

The operations on values of type stream of T are defined below where s and s' are streams, and c is a value of type T.

(1) [ ]
    The result is the empty stream which is the sequence of length zero.

(2) cons ( c, s )
    The result is a stream s' whose first element is c and whose remaining elements are the elements of the stream s.

(3) first ( s )
    The result is the value c which is the first element of s. If s is empty, the result is undefined.

(4) rest ( s )
    The result is the stream left after removing the first element of s. If s = [ ], the result is undefined.

(5) empty ( s )
    The result is true if s = [ ], and is false otherwise.

```
prime_generator = procedure ( n : integer )
        yields  stream of integer;

    generate = procedure ( i, n : integer )
            yields  stream of integer;
        if i < n then [ ]
        else cons ( i, generate( i+1, n ) ) end;
    end generate;

    sieve = procedure ( s : stream of integer )
            yields stream of integer;
                if empty ( s ) then [ ]
                else  let x : integer,
                        s2, s3 : stream of integer;
                            x, s2 = first ( s ), rest ( s );
                            s3 = delete ( x, s2 );
                            in  cons ( x, sieve( s3 ) ) end;
                end;
    end sieve;

    delete = procedure ( x : integer,
            s : stream of integer )
            yields  stream of integer;
                if empty ( s ) then [ ]
                else  let y : integer,
                        s2, s3 : stream of integer;
                            y, s2 = first ( s ), rest ( s );
                            s3 = delete ( x, s2 );
                            in  if divide ( x, y ) then s3
                                    else cons ( y, s3 ) end;
                        end;
                end;
    end delete;

    sieve (  generate ( 2, n ) );

end prime_generator;
```

Figure 4. A Prime Number Generator

The following identity is satisfied by the stream operations:

```
if  empty( s )  then  s = [ ]
              else  s = cons( first( s ), rest( s ) )
              end
```

The problem of generating all prime numbers less than a given integer n is a good example of the use of streams in constructing a modular program so as to expose many independent actions for concurrent execution. The sieve of

Eratosthenes expressed in our textual language is presented in Fig. 4. The procedure "generate" produces the sequence of successive integers beginning with 2. This stream is processed by "sieve" to remove nonprime elements. Procedure "sieve" operates by taking the first element of its input and removing all multiples of the first element (using "delete") and applying "sieve" recursively to the remaining elements. (The first use of stream concepts for the prime number sieve, as far as we know, was in [16]. It seems the example has been discovered independently by several authors.)

## Data flow schemas

A data flow schema is an operational model of concurrent computation. The form of schemas used here derives from the work of Dennis and Fosseen [9] and Dennis [7]. A data flow schema is a directed graph composed of nodes called *actors* and arcs connecting them. An arc pointing to an actor is called an *input arc* of the actor; and an *output arc* is an arc emanating from the actor. Each actor has an ordered set of input arcs and output arcs. There are five types of actors: link, operator, switch, merge and sink. The five types of actors are shown in Fig. 5. An *(m, n) data flow schema* must have m links which do not have input arcs, and n links not having output arcs. These links are respectively called *input links* and *output links* of the (m, n) schema. Further, we require that the schema must be proper in the sense that all other actors must have the required arcs of its actor type, and each arc must be connected at both ends.

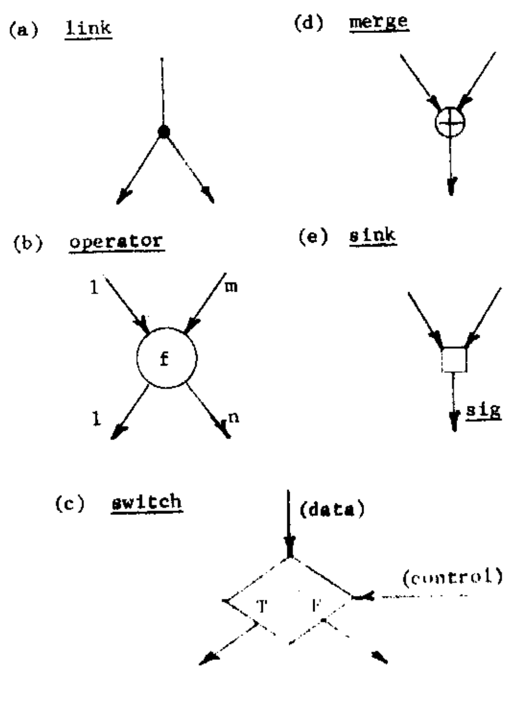



Figure 5. Data flow actors.

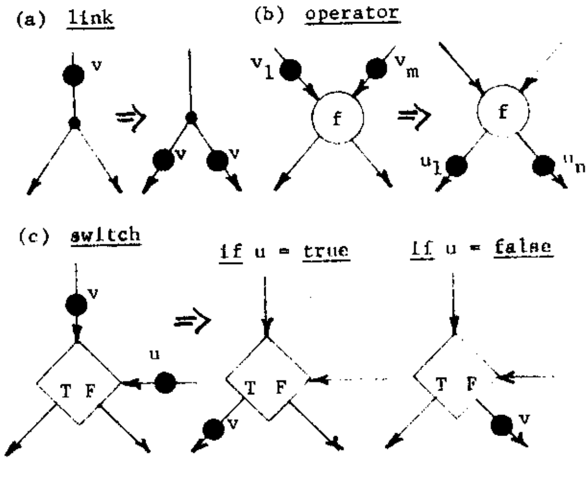



Figure 6. Examples of firing rules.

Stating the operational semantics of data flow schemas requires additional concepts. A *configuration* of a data flow schema is the graph of the schema together with an assignment of labeled *tokens* to some arcs of the graph. An assignment of a token to an arc is represented by the presence of a solid circle on the arc. The label denotes the value carried by the token and may be omitted when the particular value is irrelevant to the discussion. Informally, the presence of a token on an arc means that a value is made available to the actor to which the arc points. For the present, tokens carry values of type integer, real, boolean, structure, or stream.

### Firing Rules

Execution of an (m, n) schema advances it from one configuration to another through the *firing* of some actor that is *enabled*. The firing rules for the principal actor types are specified in Fig. 6. A necessary condition for any actor to be enabled is that each output arc does not hold a token. An actor is enabled when a token is present on each input arc -- with the exception of a merge actor. The firing of an actor causes the tokens to be absorbed from the input arcs and completes by placing a token on each of the output arcs. The values of the output tokens are functionally related to the values of the input tokens. A link simply replicates the value received and distributes it to the destination actors indicated by output arcs. The effect of firing an operator is to apply to the inputs $v_1,\ldots,v_m$ the function associated with the operation name written inside the operator to yield the outputs $u_1,\ldots,u_n$. The switch and merge are used for controlling the flow of tokens. A switch requires a data input and a control input which is a boolean value. The firing of a switch replicates the input token on one of the output arcs according to the boolean control value. The arrival of a token on either input arc enables a merge, and upon firing, a token conveying the same value is placed on the output arc. The behavior of a merge is inherently nondeterminate: when two input tokens reside on the input arcs, the firing rule does not specify in which order the output tokens will be generated. A sink absorbs the input tokens upon firing and places a special token signal on the output arc. The purpose of a sink actor is to absorb unwanted values; the signal output token is necessary for the implementation of schema application to be described.

The set of functions commonly associated with an operator includes the scalar arithmetic operations and constant functions.

## Well Formed Data Flow Schemas

Unrestricted use of actors in data flow schemas is undesirable since an arbitrary interconnection of these actors may form a schema which deadlocks or has nondeterminate behavior. Because these properties are undesirable for reliable programming we choose a subclass of schemas which will satisfy the needs of programming.

An *(m, n) well formed* data flow schema is an (m, n) data flow schema formed by any acyclic composition of component data flow schemas, where each component is either a link, a sink, an operator, or a *conditional subschema*.

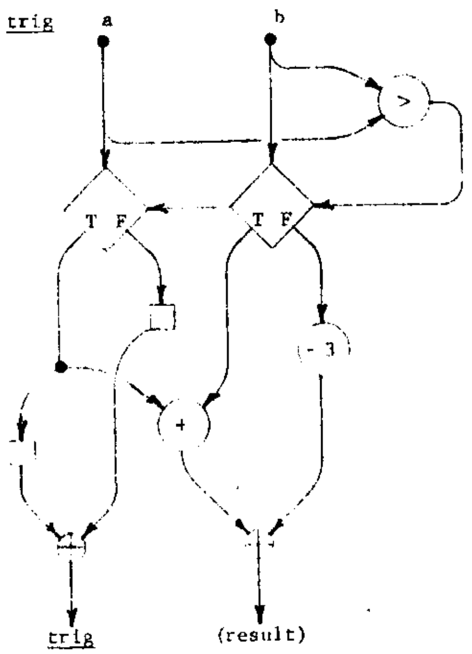



Figure 7. A conditional schema.

Fig. 7 is an example of a conditional schema which computes the value of the expression

if a > b then a + b else b - 3

Here, the trig output provides a *completion signal* indicating that the sink actor has absorbed the unused copy of a. The structure of a conditional schema corresponds in an obvious way to conditional expressions.

## The Apply Actor

The class of well formed data flow schemas cannot express program features such as procedures, procedure applications, and iterations. We introduce an actor apply whose meaning is explained in Fig. 8. The first input to an apply actor is a token associated with an (m, n) well formed data flow schema. An apply actor is enabled when a token is present on each input arc. The effect of firing an apply actor is to replace the actor with the specified (m, n) schema as shown in the figure. The (m, n) schema replacing the apply actor may itself contain apply actors, allowing recursion to be expressed.

We have not included structures of data flow schemas which correspond to language constructs such as while loops in Algol 60 or Do statements in Fortran. Such structures necessarily involve cyclic connections of actors which do not correspond to actual data dependencies, and introduce unnecessary delays. Furthermore, the semantics of cyclic schemas is more complicated, since issues of safety and liveness must be dealt with. We choose to support these language features in the equivalent form of recursive application of data flow schemas. This allows simultaneous execution of instances of a data flow schema which correspond to successive iterations of a while loop.

An example of the use of apply actors is given in Fig. 9. This recursive schema implements the "reverse" function stated earlier in Fig. 3. The input link actor labeled trig is an input link whose function is to trigger those actors that generate constants, in this case the create actor that produces the empty data structure.

The apply actor presented requires that all input values be present on the input arcs to become enabled. A language implemented in terms of the apply actor will have "call by value" semantics, that is, the result of application is well defined only when the computations producing arguments to the procedure all terminate. This is in contrast with a more general form of procedure application which allows procedure application to begin even though computation of some arguments is not complete.

### Data Flow Processor

The structure of a data flow processor suitable for supporting execution of recursive data flow schemas is shown Fig. 10. It consists of six subsystems: Functional Units, Structure Controller, Execution Controller, the Arbitration and Distribution Networks, and the Packet

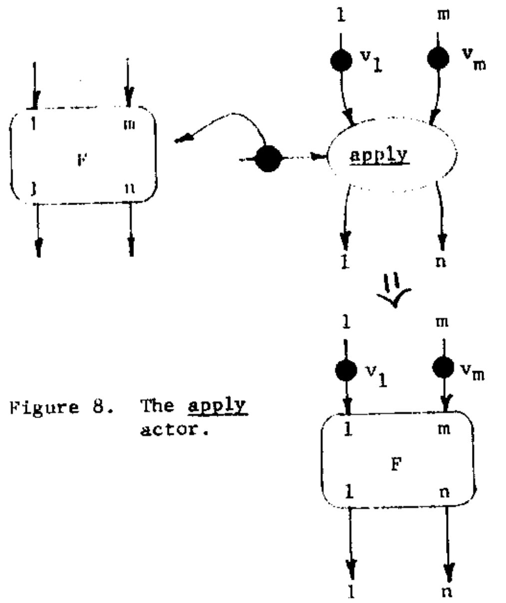



Figure 8. The apply actor.

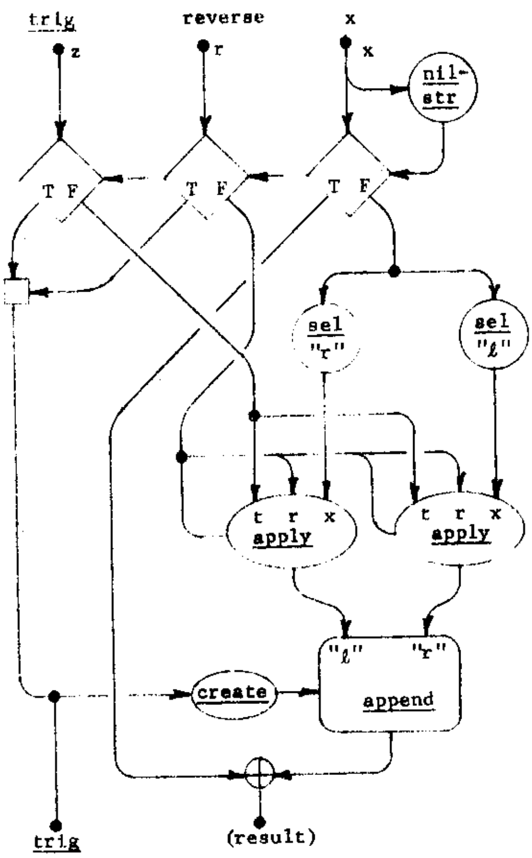



Figure 9. Recursive schema.

Memory. The Execution Controller fetches instructions and operands from the Packet Memory and forms them into operation packets. Each operation packet is passed to the Arbitration Network for transmission to an appropriate Functional Unit if a scalar operation is called for, or to the Structure Controller for the data structure operations create, append, and select. Instruction execution in the Structure Controller and Functional Units generate result packets which are sent through the Distribution Network to the Execution Controller where they will join with other operands to activate their target instructions. How this is done is explained in greater detail in the next section.

The Packet Memory holds the collection of data structures as a collection of *items* each being a one-level data structure having scalar values and unique identifiers of other items as its components [8]. This collection of items represents an acyclic directed graph where each arc corresponds to a unique identifier component of the item representing its origin node. The Packet Memory maintains a reference count for each item and reclaims physical storage space as items become inaccessible.

Data structures held in the Packet Memory have three roles in the execution of data flow schemas: (1) as operands for the data structure operations implemented by the Structure Controller; (2) as *procedure structures* that have as components the instructions of a data flow procedure; and (3) *activation records* which hold operand values for instructions waiting for their enabling condition to be satisfied.

Although the Execution Controller, Structure Controller and the Packet Memory are shown in Fig. 10 as single units, we imagine that each is in fact a collection of many identical units. For example, the Packet Memory subsystem would consist of separate systems, each holding all items whose unique identifiers belong to a well defined part of the address space of unique identifiers. The Execution Controller subsystem would consist of identical modules each of which would serve a distinct subset of procedure activations.

The concept of a Packet Memory System was introduced in [8], and the design issues for these systems and the Structure Controller have been studied in [1, 2].

## Implementation of Data Flow Schemas

### Procedure Structures

A data flow schema is represented in the machine by a kind of data structure called a *procedure structure* illustrated in Fig. 11a. A procedure structure corresponding to a data flow schema of n actors is a data structure having n components with integer selector names from 1 to n assigned to the actors. Each component, called an *instruction*, is an encoding of an actor and its output arcs.

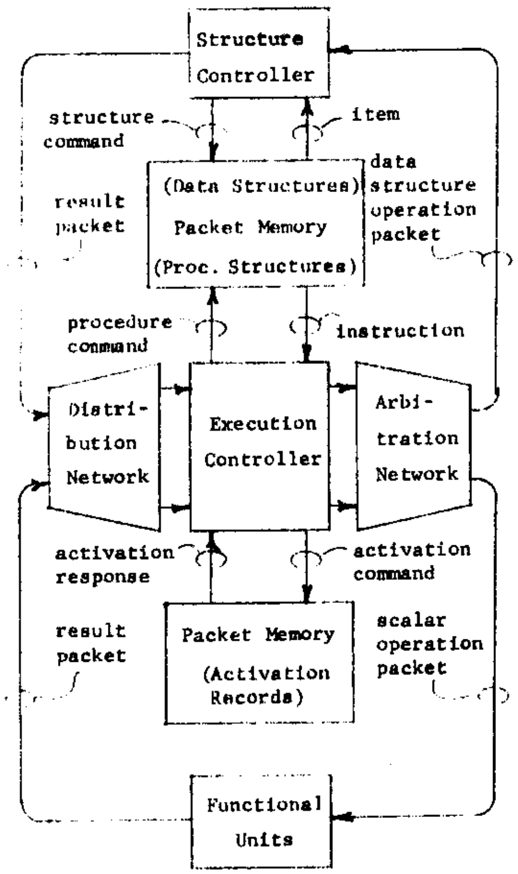



Figure 10. Data flow processor.

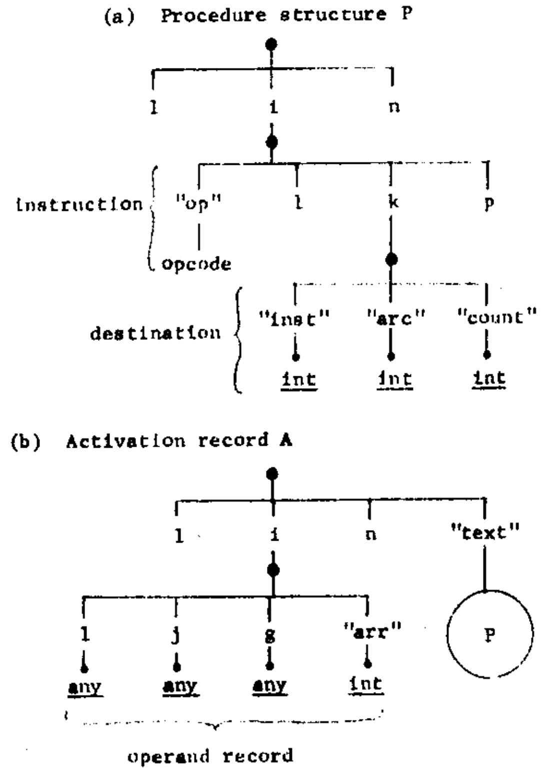



Figure 11. Procedure and activation structures.

The components of an instruction include an *operation* field which defines the function performed by the actor, and *destination* fields D1, ..., Dp corresponding to p output arcs. Each destination field has three subcomponents: the *inst* component is the integer selector name of the destination instruction; the *arc* component is an integer designation of an input arc of the destination; and the *count* component is the number of operand values required by the destination instruction.

## Activation Records

Since multiple instances of the same schema may be concurrently active in a computation, each activation (an instance of procedure execution) is represented by a separate *activation record* as shown in Fig. 11b. Each actor in an activation is uniquely identified by the tuple (A, I), where A is a uid allocated for the activation record and I is the integer assigned to the actor in the procedure structure. A token of value v on the k-th input arc of an actor (A, I) corresponds to a result packet that carries the information (A, I, k, v, count), where "count" is the number of tokens (operands) required for the enabling of the actor.

Enabling of an actor is detected by checking the number of result packets having arrived at the operand record -- the I component of the activation record A - against the count in the result packet. The detection of enabling is a function of the Execution Controller and the Packet Memory that store activation records. Upon enabling of actor instance (A, I), the instruction of the actor is fetched from the I component of the procedure structure. The following section describes how activation records might be manipulated.

An activation record has components with integer selectors for operand records and an additional "text" component that is the procedure structure for the activation. (In our implementation, this component is shared by other activations of the same schema.) An operand record may have as many integer subcomponents as input arcs of an actor, and also contains an "arrived" subcomponent indicating the number of arrived result packets. Since an activation record stores values of arrived result packets in its components, operations on an activation record modify its components. These operations are defined as follows:

(1) create-activation( P )
This returns the uid of a new activation record having P as its "text" component, but no other components.

(2) insert( A, i, k, v )
The insert operation adds the value v as the k-th operand of the i-th instruction in activation record A. In addition, the "arr" component of the operand record is incremented by one. To handle the first operand value to arrive, a missing "arr" component is interpreted as having the value zero.

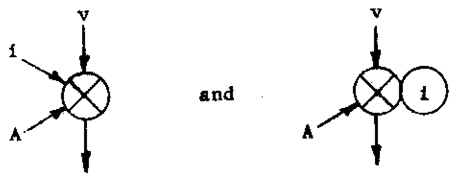
(3) remove( A, i )
This operation releases the i component of A; and is performed by the Execution Controller once it has generated the operation packet for actor instance (A, i).

(4) free( A )
This operation releases the entire activation record A by means of a command packet sent to the Packet Memory.

For each arriving result packet ( A, i, k, count, v ) the Execution Controller performs the operation insert( A, i, k, v ) and tests the updated value of the "arr" component against the "count" field of the result packet. If the values are equal, the instruction is fetched from the Packet Memory and used, together with the operand record, to construct an operation packet which is delivered to the Arbitration Network. The i component of activation record A is then released.

## Procedure Activation

Our implementation of the apply actor is illustrated in Fig. 12. The apply actor is replaced by the code diagrammed in Fig. 12b, and the applied graph F is augmented as in Fig. 12c. Here we use the notations




to mean insert ( A, i, 1, v ). The new actors extr-uid, const-ret and distribute will be explained below.

This implementation assumes the actors in each recursive schema are numbered according to this rule:

(1) Input link actors are numbered 1, ..., m.
(2) The link actors that receive the n-tuple of values resulting from a schema application are numbered J + 1, ..., J + n for some integer J.
(3) A link actor numbered 0 receives a packet ( A, J, n ) containing the information needed to construct result packets for returning values resulting from procedure execution.
(4) The remaining actors may be numbered arbitrarily.

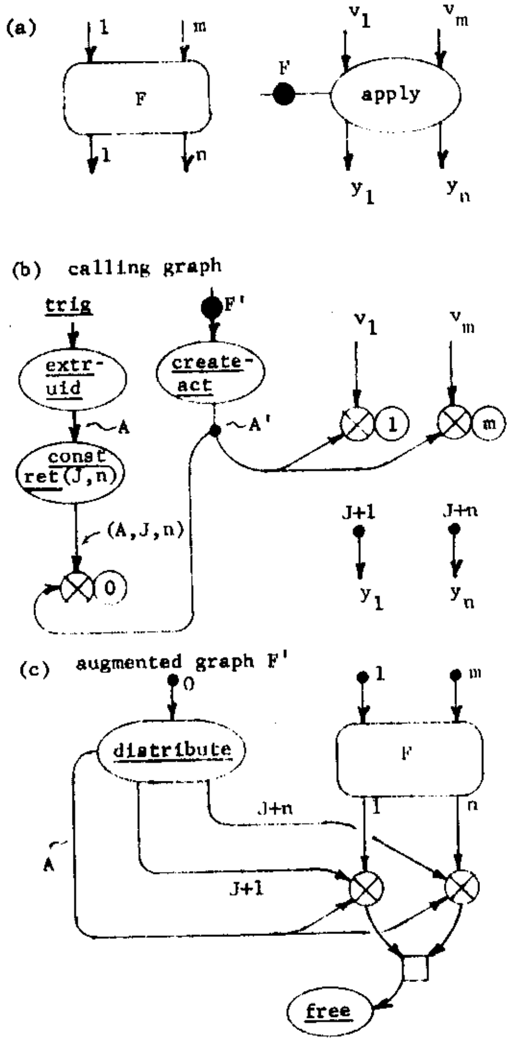



Figure 12. Implementation of apply.

The implementation scheme works as follows: The create-act actor produces the uid A' of a new activation record containing "text" component F' and passes it to the insert actors associated with input value $v_1$, ..., $v_m$. These actors cause result packets of the form ( A', i, 1, 1, $v_i$ ) to be generated which initiate execution of the new activation of F'. At the same time, the extr-uid and const-ref actors form the *return value* ( A, J, n ) and send it to link 0 of schema F'. Once result values $y_1$, ..., $y_n$ have been produced, the distribute and insert actors of F' generate result packets of the form ( A, J + i, 1, 1, $y_i$ ) which deliver result values to the calling schema. The free actor then releases the activation record, and its uid A' is returned to the pool of free uid's managed by the Packet Memory.

## Implementation of Stream Actors

In the implementation streams are represented as data structures. A stream is a data structure having an "f" component which is the first element of the stream, and an "r" component which is the data structure representing the rest of the stream. The empty stream is represented by nil. Operations on streams become operations on structure values; thus first( s ) and rest( s ) are implemented by select( s,"f" ) and select( s,"r" ), respectively.

We wish to make it possible for a stream to be processed by consuming modules while further stream elements are generated concurrently. To provide for this behavior, we must augment our concept of data structures so a data structure may be accessed before it is entirely constructed. We use the concept of *holes* which is based on the work of Henderson [11] who used the term "token". Our idea is related to but different from the idea of "suspensions" discussed by Friedman and Wise [10].

The idea is embodied in the implementation of the cons operation described in Fig. 13. Here the create-hole and write-hole actors are special data structure operators defined as follows:

> A create-hole actor returns a uid H allocated from the data structure address space. The free node is called a hole in that it has two states: *filled* and *unfilled*. In the unfilled state, all data structure operations on the hole are queued except the write-hole operation. Upon completion of the write-hole(H,v) operation, the hole H changes its state to filled and contains the value v. All previously queued and subsequent operations on H are processed without further delay; a subsequent write-hole operation on H is illegal.

To illustrate the concurrency provided by this implementation of streams, consider the recursive schema

s' = cons (v, s)

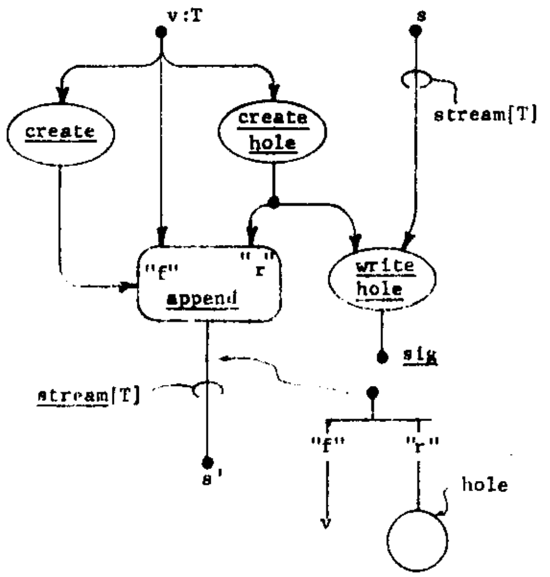



Figure 13. Implementation of cons.

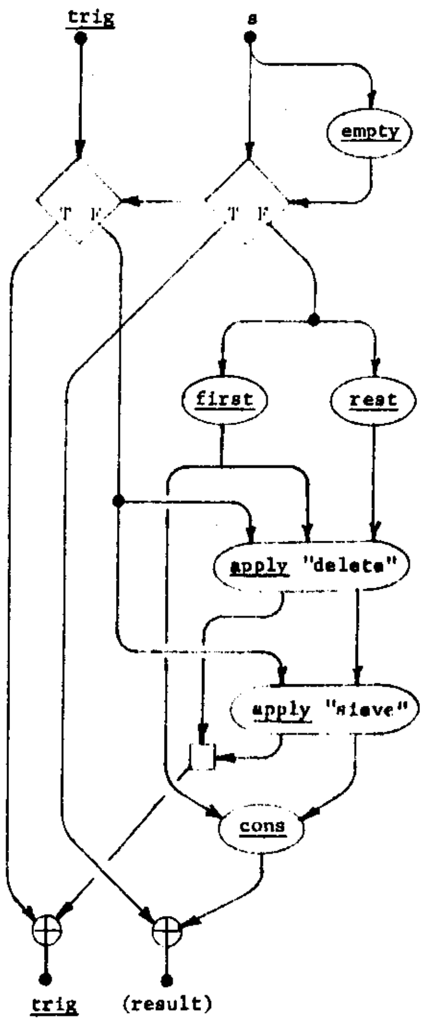



Figure 14. Data flow schema for "sieve".

shown in Fig. 14 for the "sieve" procedure of the prime number generator. Note that the output of the top activation of "sieve" will be a data structure containinng the first element of the result stream and a hole waiting to be filled in with the data structure generated by the recursive activation of "sieve". In this implementation each higher activation of "sieve" may be released as soon as it has completed its work (i.e., its hole has been filled), leaving the remaining work to be finished by deeper activations of the code.

## Remarks

The concept of stream has appeared in many forms [5, 12, 14, 15]. One of the earliest papers that discussed streams as a programming feature was an unpublished paper by McIlroy [15]. Despite the conceptual elegance of streams, programming has not yet departed from the sequential notion of coroutines and process synchronization primitives. Recent interest in concurrent programming languages and processors have motivated several other authors to investigate the feasibility of implementation of streams and related concepts of data structures with holes or with suspensions [4, 10, 13].

## References


[1] W. B. Ackerman, *A Structure Memory for Data Flow Computers*, Laboratory for Computer Science, Massachusetts Institute of Technology, TR-186, (August, 1977), 125 pp.

[2] W. B. Ackerman, "A Structure Processing Facility for Data Flow Computers," *Proceedings of the 1978 International Conference on Parallel Processing* (August, 1978), pp. 166-172.

[3] W. B. Ackerman, and J. B. Dennis, *VAL -- A Value-Oriented Algorithmic Language: Preliminary Reference Manual*, Laboratory for Computer Science, Massachusetts Institute of Technology, TR-218, (July, 1979), 80 pp.

[4] Arvind, and K. P. Gostelow, "Some Relationships Between Asynchronous Interpreters of A Dataflow Language," *Formal Description of Programming Concepts: Proceedings of the IFIP Working Conference on Formal Description of Programming Concepts* (August, 1977), pp. 95-119.

[5] W. H. Burge, "Stream Processing Functions," *IBM Journal of Research and Development 19* (January, 1975), pp. 12-25.

[6] M. E. Conway, "Design of a Separable Transition-Diagram Compiler," *Communications of the ACM 6* (July, 1963), pp. 396-408.

[7] J. B. Dennis, "First Version of a Data Flow Procedure Language", *Programming Symposium: Proceedings, Colleque sur la Programmation, Lecture Notes in Computer Science 19* (October, 1976), pp. 362-376.

[8] J. B. Dennis, "Packet Communication Architecture," *Proceedings of the 1975 Sagamore Computer Conference on Parallel Processing* (August, 1975), pp. 224-229.

[9] J. B. Dennis, and J. B. Fosseen, *Introduction to Data Flow Schemas*, Computation Structures Group, Laboratory for Computer Science, Massachusetts Institute of Technology, Memo 81-1 (September, 1973), 45 pp.

[10] D. P. Friedman, and D. S. Wise, "Aspects of Applicative Programming for Parallel Processing," *IEEE Transactions on Computers C-27* (April, 1978), pp. 289-296.

[11] D. A. Henderson, *The Binding Model: A Semantic Base for Modular Programming Semantics*, Laboratory for Computer Science, Massachusetts Institute of Technology, TR-145 (February, 1975), 282 pp.

[12] G. Kahn, "The Semantics of A Simple Language for Parallel Programming," *Information Processing 74: Proceedings of the IFIP Congress* (August, 1974), pp. 471-475.

[13] R. M. Keller, G. Lindstrom, and S. Patil, "A Loosely-Coupled Applicative Multi-Processing System," *1979 National Computer Conference, AFIPS Conference Proceedings 48* (June, 1979), pp. 613-622.

[14] P. J. Landin, "A Correspondence Between ALGOL 60 and Church's Lambda-Notation: Part I," *Communication of the ACM 8* (February, 1965), pp. 89-101.

[15] M. D. McIlroy, "Coroutines: Semantics In Search Of A Syntax," Unpublished Paper (1968).

[16] K.-S. Weng, *Stream-Oriented Computation in Recursive Data Flow Schemas*, Laboratory for Computer Science, Massachusetts Institute of Technology, TM-68, (October, 1975), 93 pp.

[17] K.-S. Weng, *An Abstract Implementation for a Generalized Data Flow Language*, Laboratory for Computer Science, Massachusetts Institute of Technology, Technical Report, forthcoming.