# Accelerators (II)

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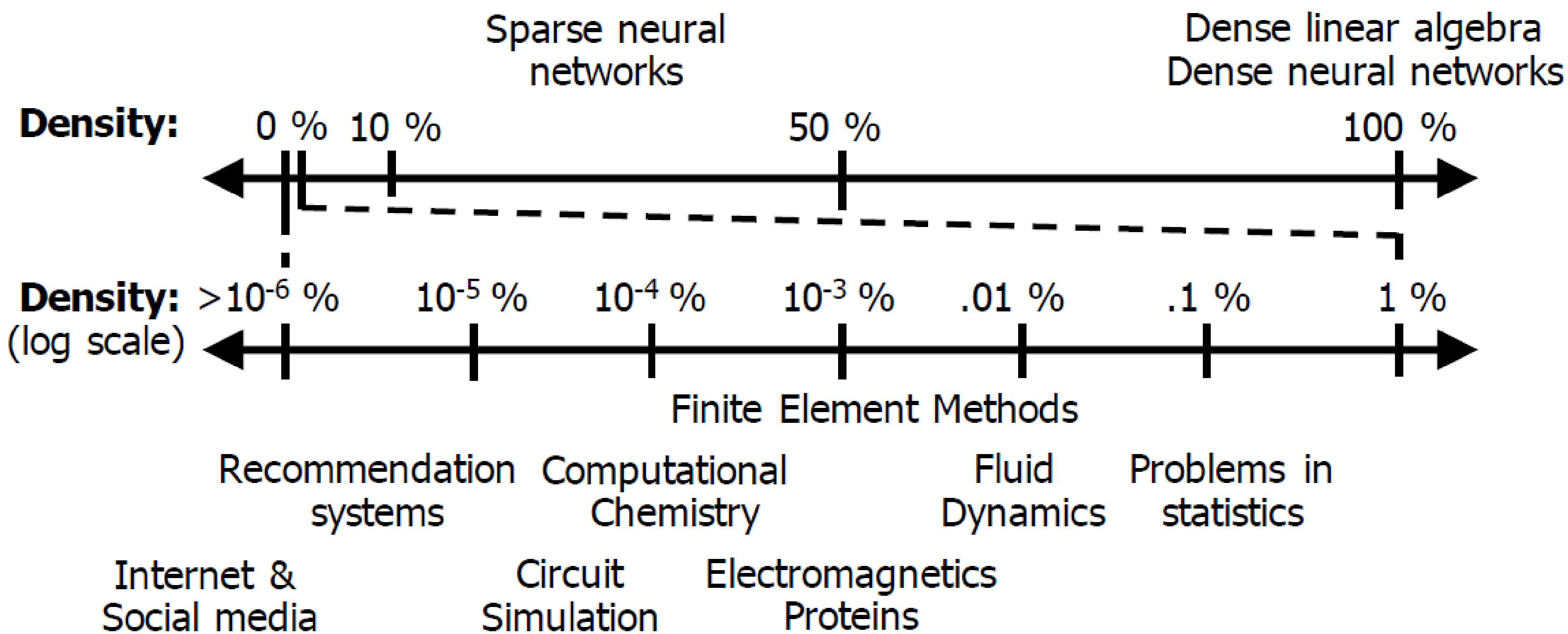
# Many problems use Sparse Tensors




[Extensor, Hegde, et.al., MICRO 2019]

# Exploiting Sparsity

Sparse data can be compressed

Can save space and energy by avoiding manipulation of zero values

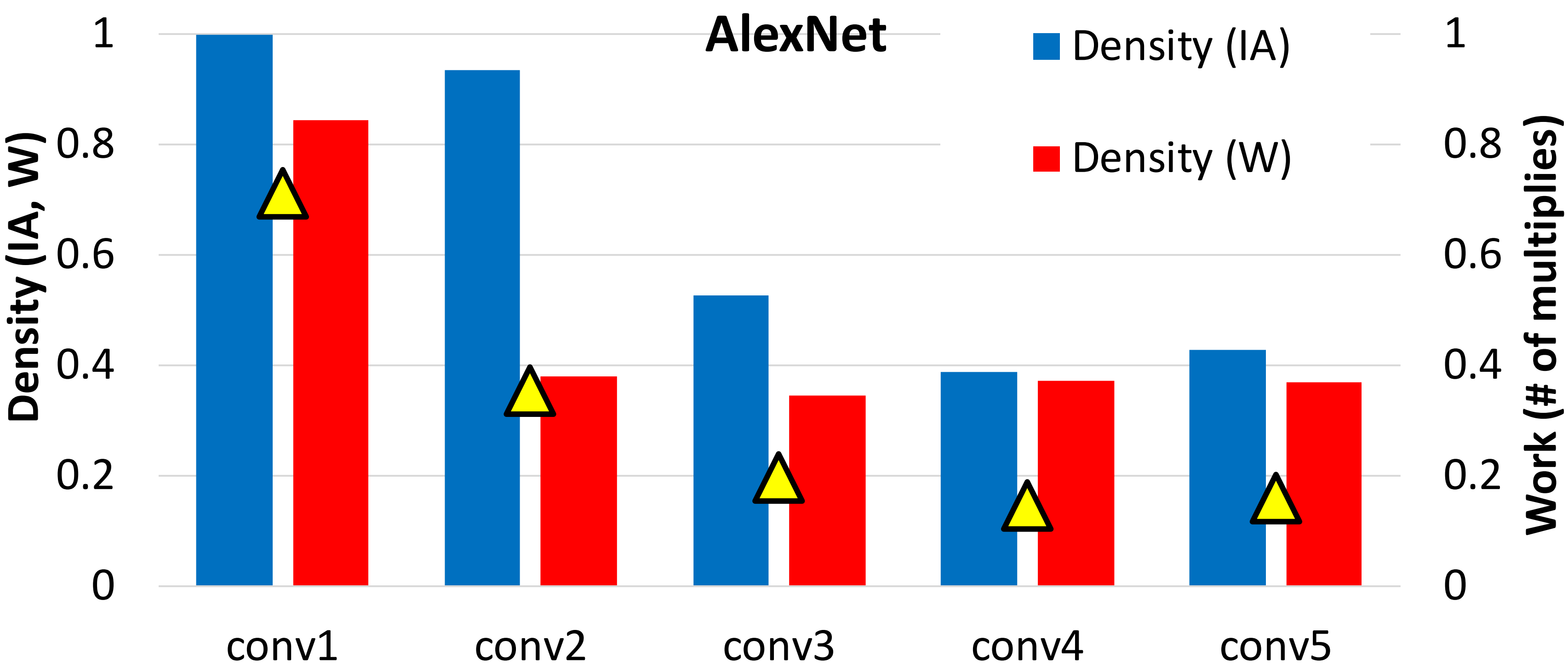
$$\boldsymbol{anything} \times \mathbf{0} \;\; == \mathbf{0}$$

$$\boldsymbol{anything} + \mathbf{0} = \boldsymbol{anything}$$

Can save time and energy by avoiding fetching unnecessary operands and avoiding **ineffectual** computations

# Motivation in DNNs

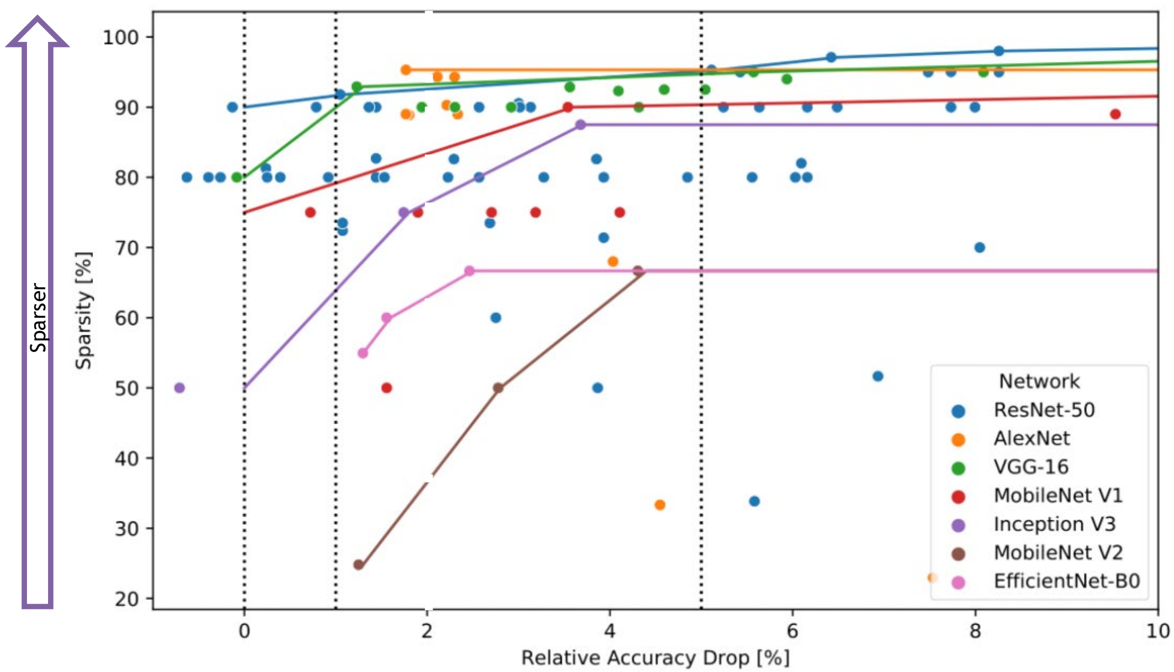
- Leverage CNN sparsity to improve energy-efficiency




SCNN, Parashar et.al., ISCA 2017

# Exploitable Sparsity

Acceptable sparsity depends on target task and error tolerance




**Error Tolerance**

| | ≤0% | ≤1%* | ≤2% |
|---|---|---|---|
| ResNet-50 | ~90% | ~90% | ~91% |
| AlexNet | | | ~93% |
| VGG-16 | ~80% | ~88% | ~92% |
| MobileNet V1 | ~72% | ~79% | ~82% |
| Inception V3 | ~50% | ~62% | ~73% |
| EfficientNet-B0 | | | ~52% |
| MobileNet V2 | | | ~25% |

**MLPerf error tolerance*

*Hoefler et al. arXiv, 2021*

# Hardware Sparse Acceleration Features

**Format:**
Choose tensor representations to save storage space and energy associated with zero accesses

**Gating:**
Explicitly eliminate ineffectual storage accesses and computes by letting the hardware unit staying idle for the cycle to save energy

**Skipping:**
Explicitly eliminate ineffectual storage accesses and computes by skipping the cycle to save energy and time

What is the chosen format?

Do all tensors share the same format?

When is a storage access gated?

How much is the compute able to skip ahead?

At which storage level is the skipping performed?

What is the criteria for skipping?

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# 1-D Output-Stationary Convolution


† Assuming: ‘valid’ style convolution

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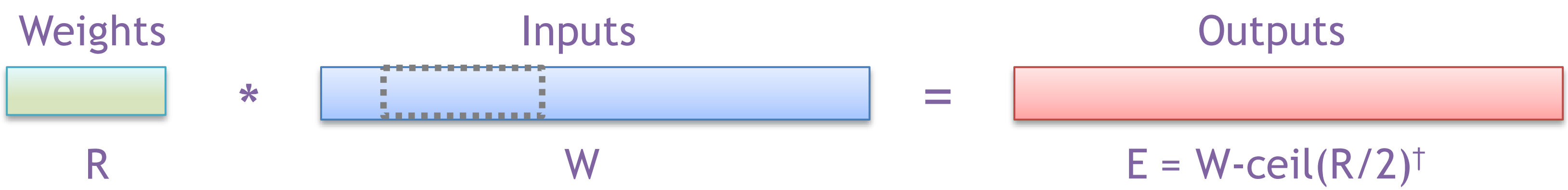


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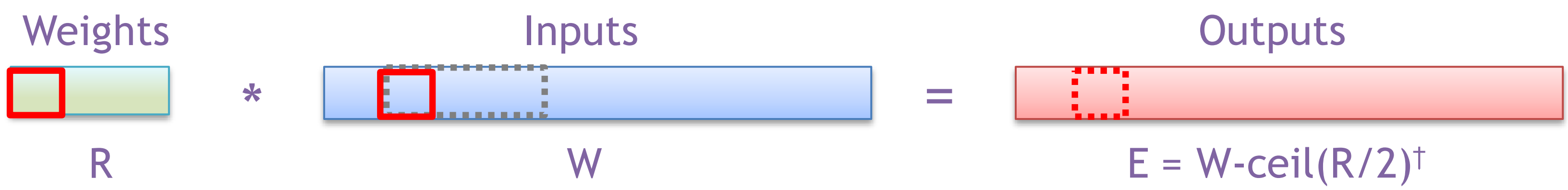


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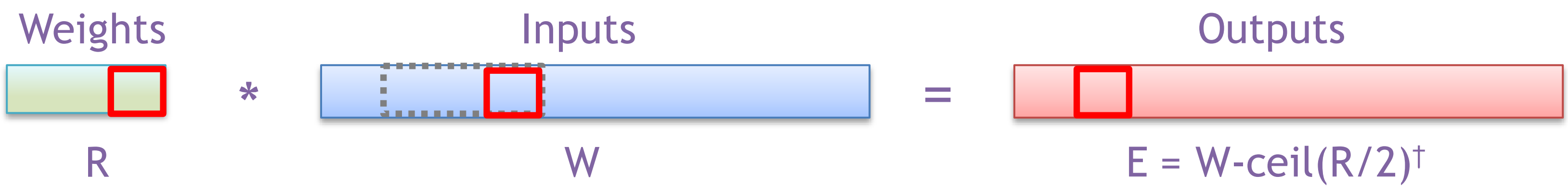


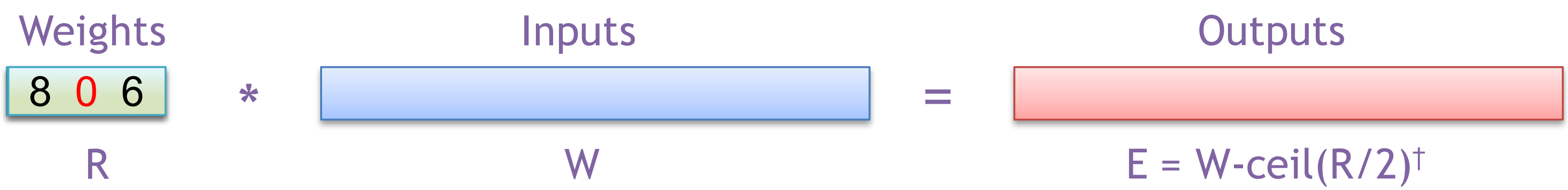

```
int i[W];       # Input activations
int f[S];       # Filter weights
int o[Q];       # Output activations

for q in [0..Q):
       for s in [0…S):
          o[q] += i[q+s]*f[s];
}}
```

What opportunity(ies) exist if some of the filter weights are zero?

† Assuming: 'valid' style convolution

# 1-D Output-Stationary Convolution




```
int i[W];       # Input activations
int f[S];       # Filter weights
int o[Q];       # Output activations

for q in [0..Q):
       for s in [0..S):
           if (!f[s]) o[q] += i[q+s]*f[r]
}}
```

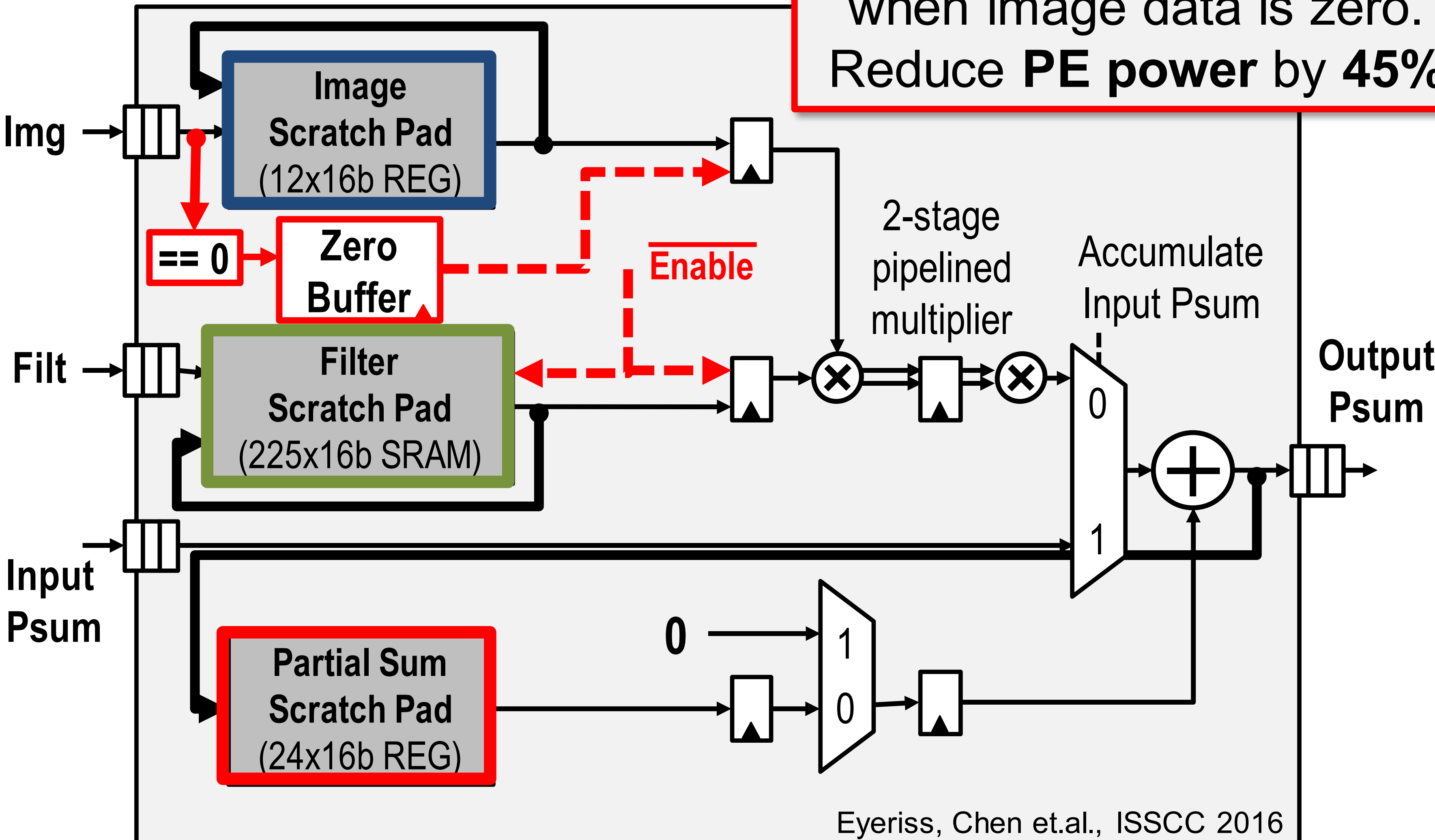
What did we save using the conditional execution?

What didn't we save using the conditional execution?

† Assuming: 'valid' style convolution

# Eyeriss – Clock Gating

Gate **mult** and **mem reads** when image data is zero. Reduce **PE power** by **45%**




Eyeriss, Chen et.al., ISSCC 2016

# Sparse Tensor Representation

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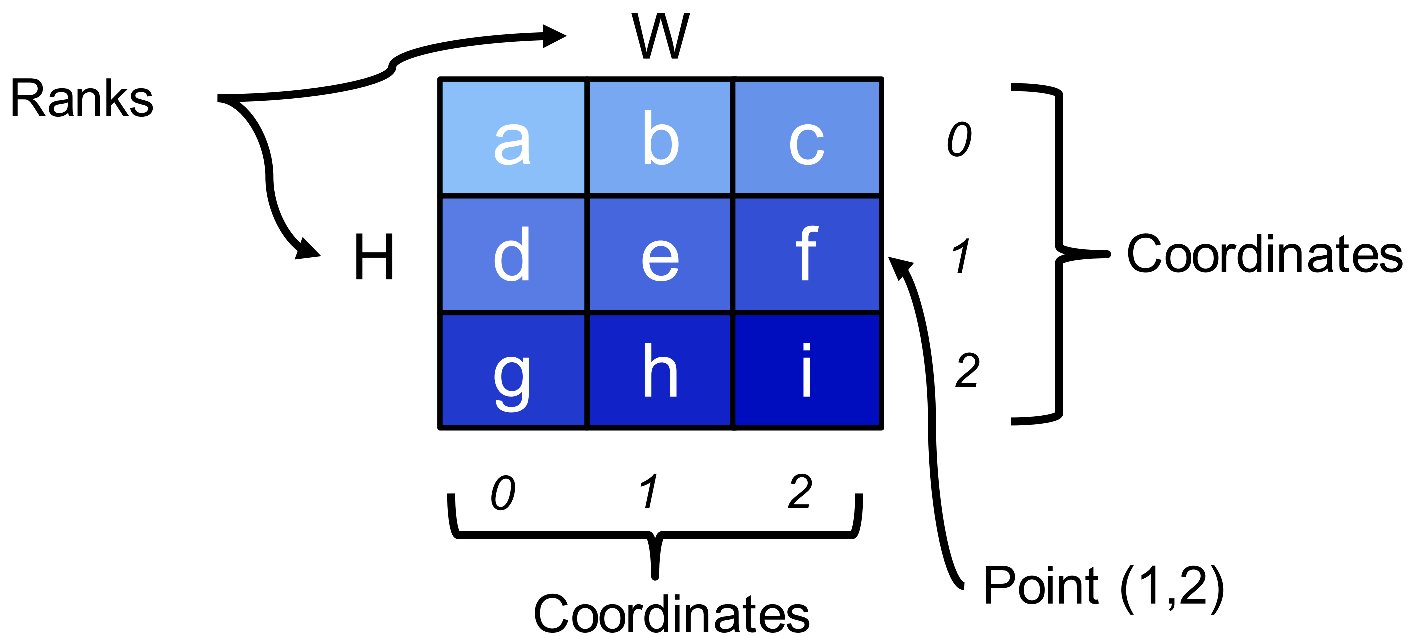
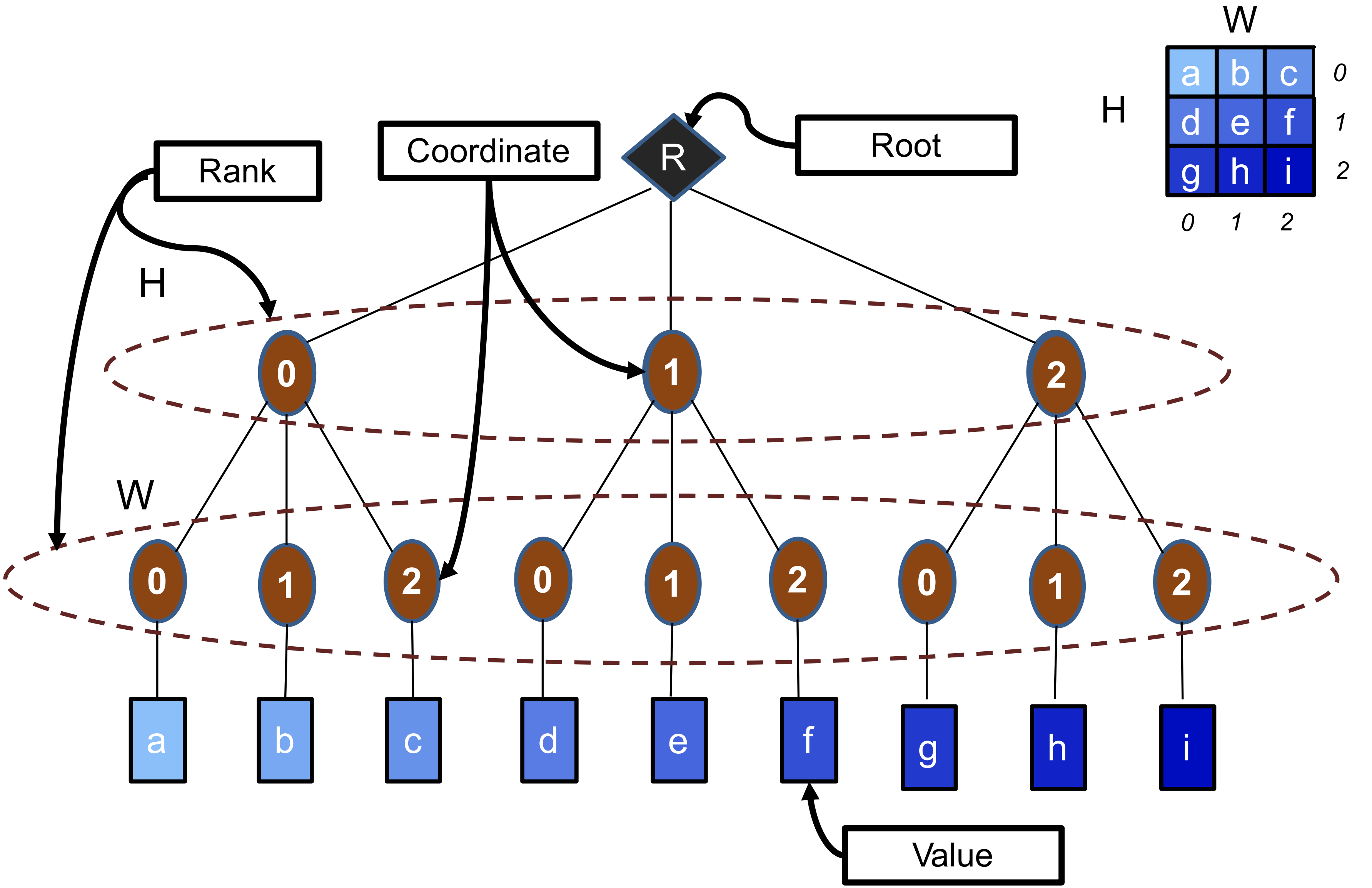
# Tensor Data Terminology




- The elements of each "rank" (dimension) are identified by their "coordinates", e.g., rank H has coordinates 0, 1, 2

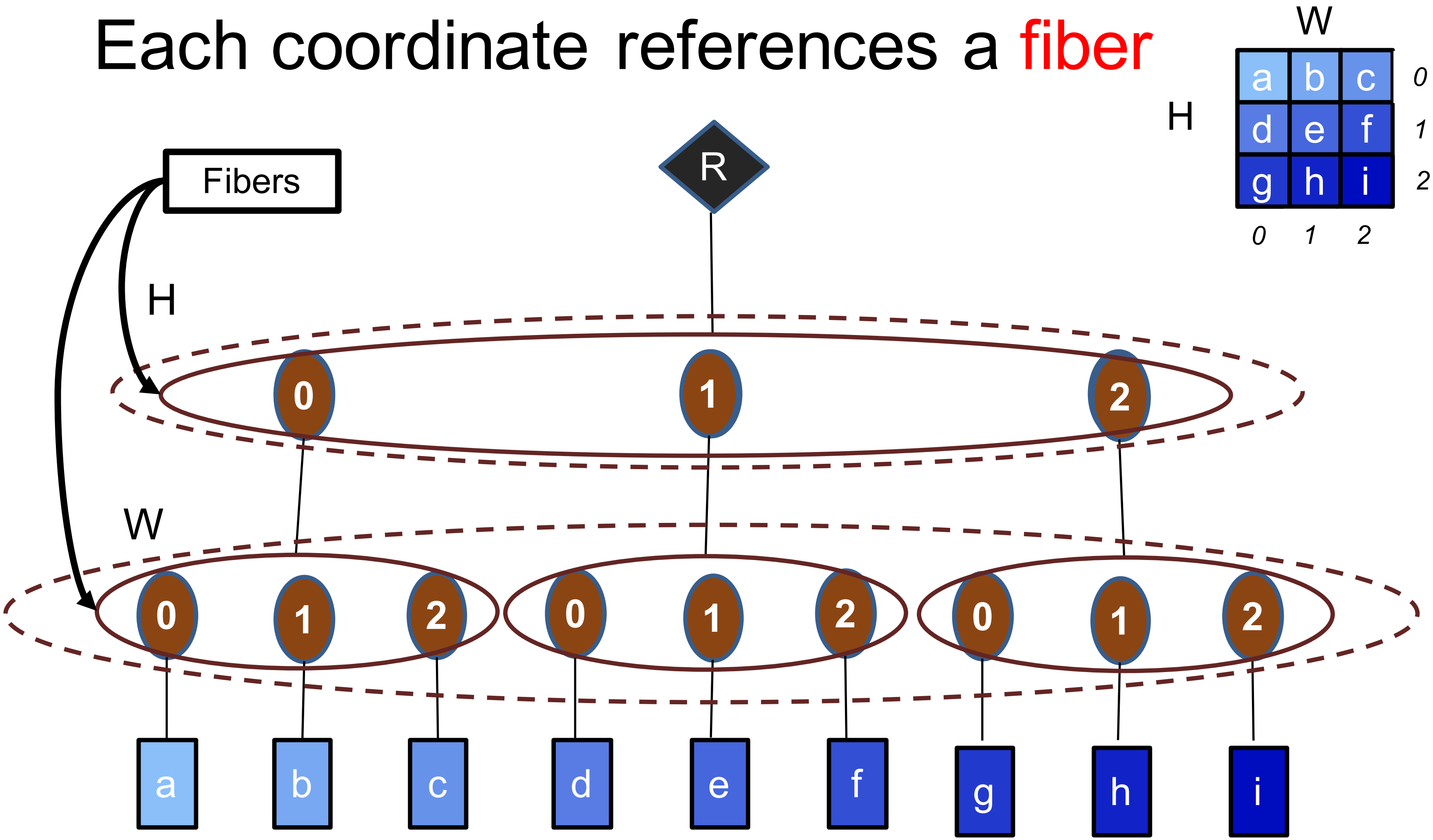
- Each element of the tensor is identified by the tuple of coordinates from each of its ranks, i.e., a "point".
  So (1,2) -> "f"

# Tree-based Tensor Abstraction



# Fibertree Tensor Abstraction

Each coordinate references a fiber

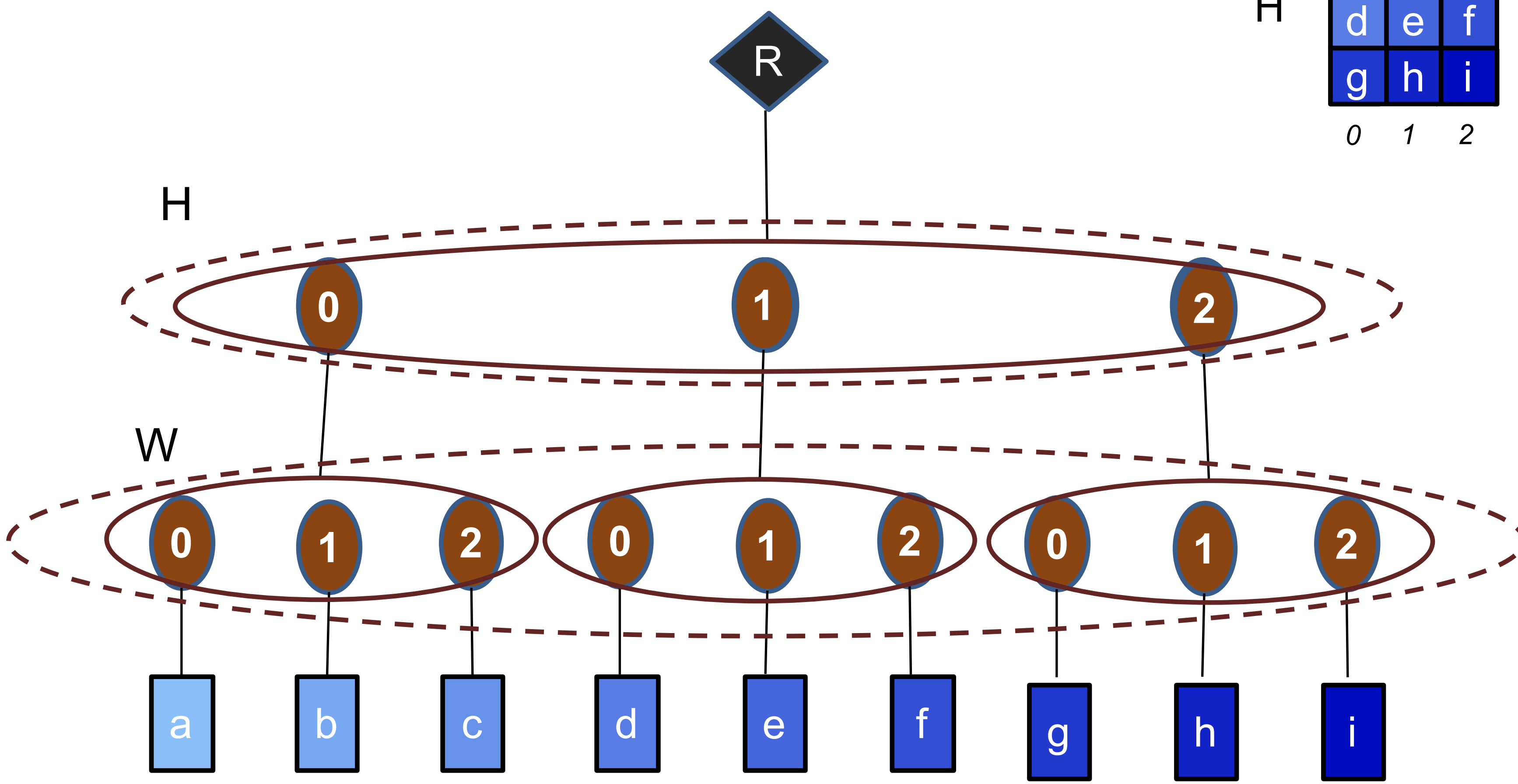


# Fibertree Tensor Abstraction

Finding point (2, 1)

W

| H | 0 | 1 | 2 |
|---|---|---|---|
| 0 | a | b | c |
| 1 | d | e | f |
| 2 | g | h | i |



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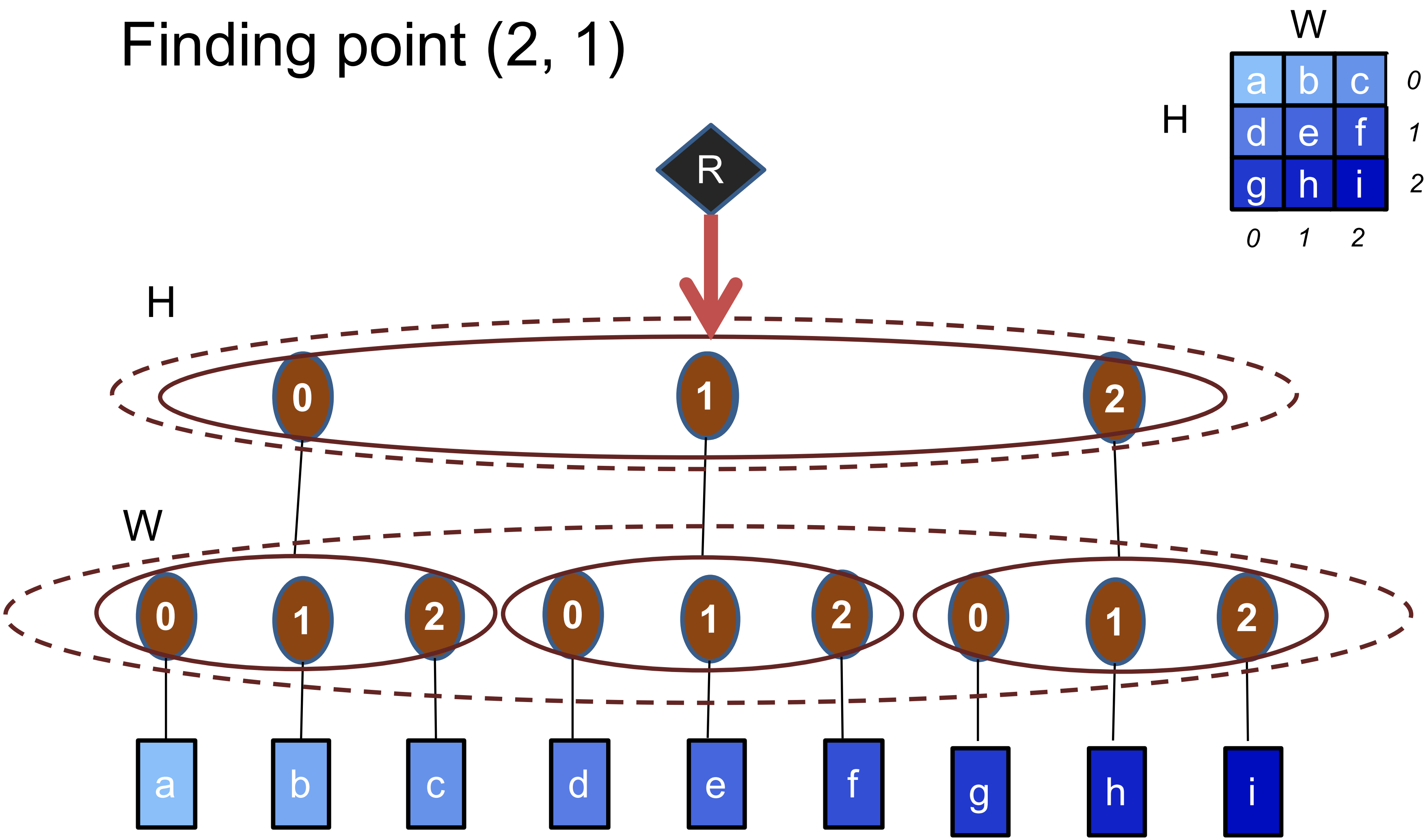

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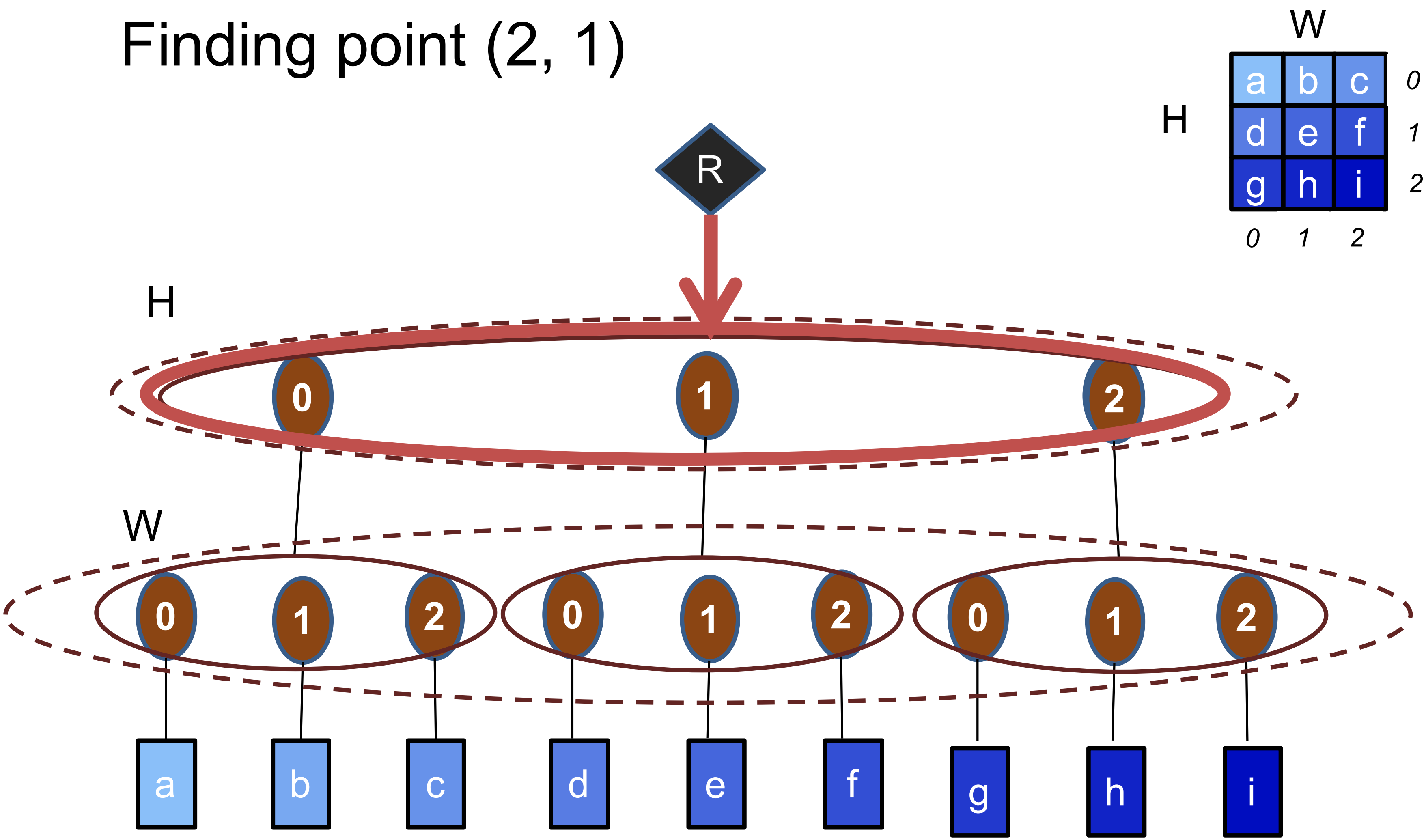

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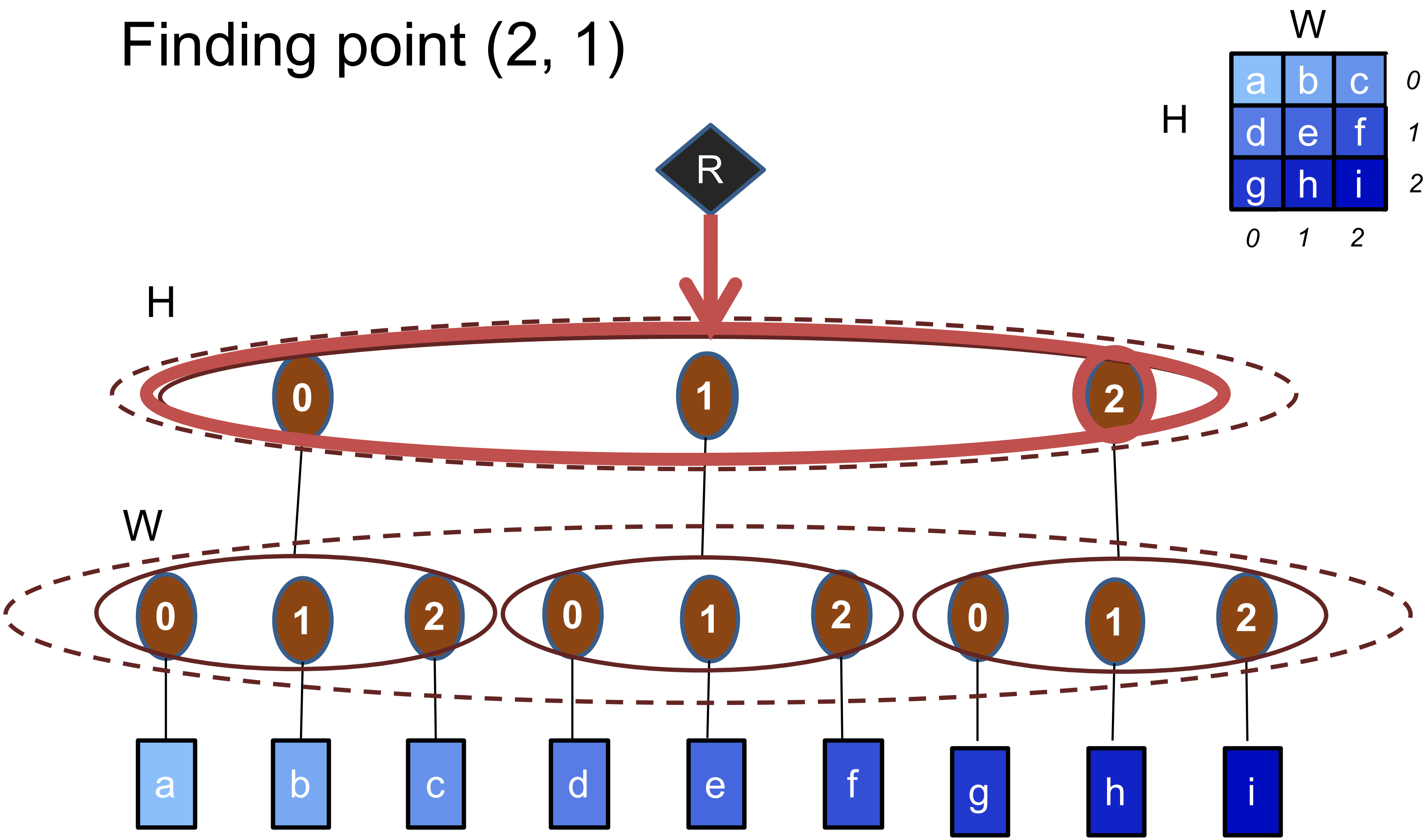

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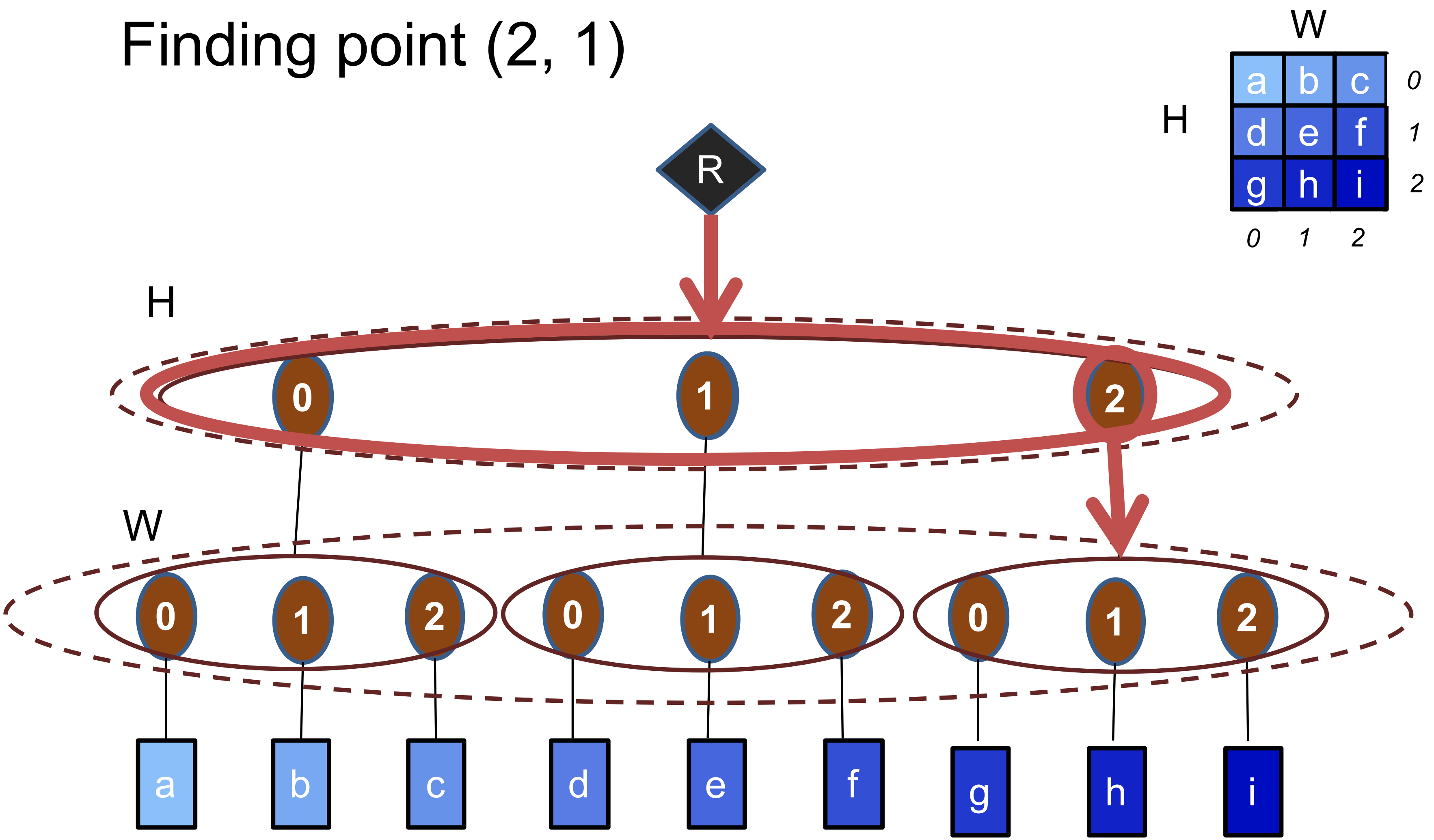

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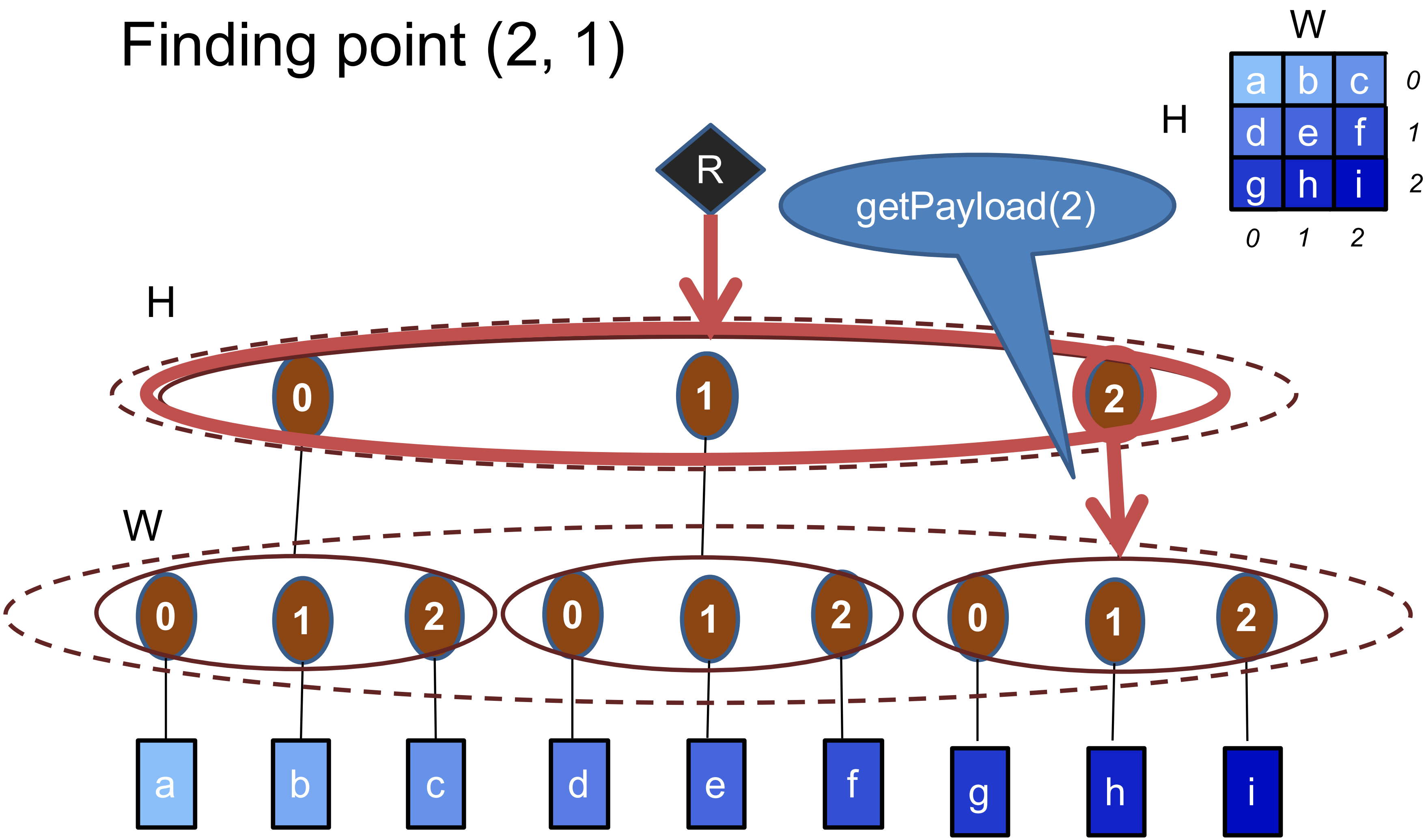

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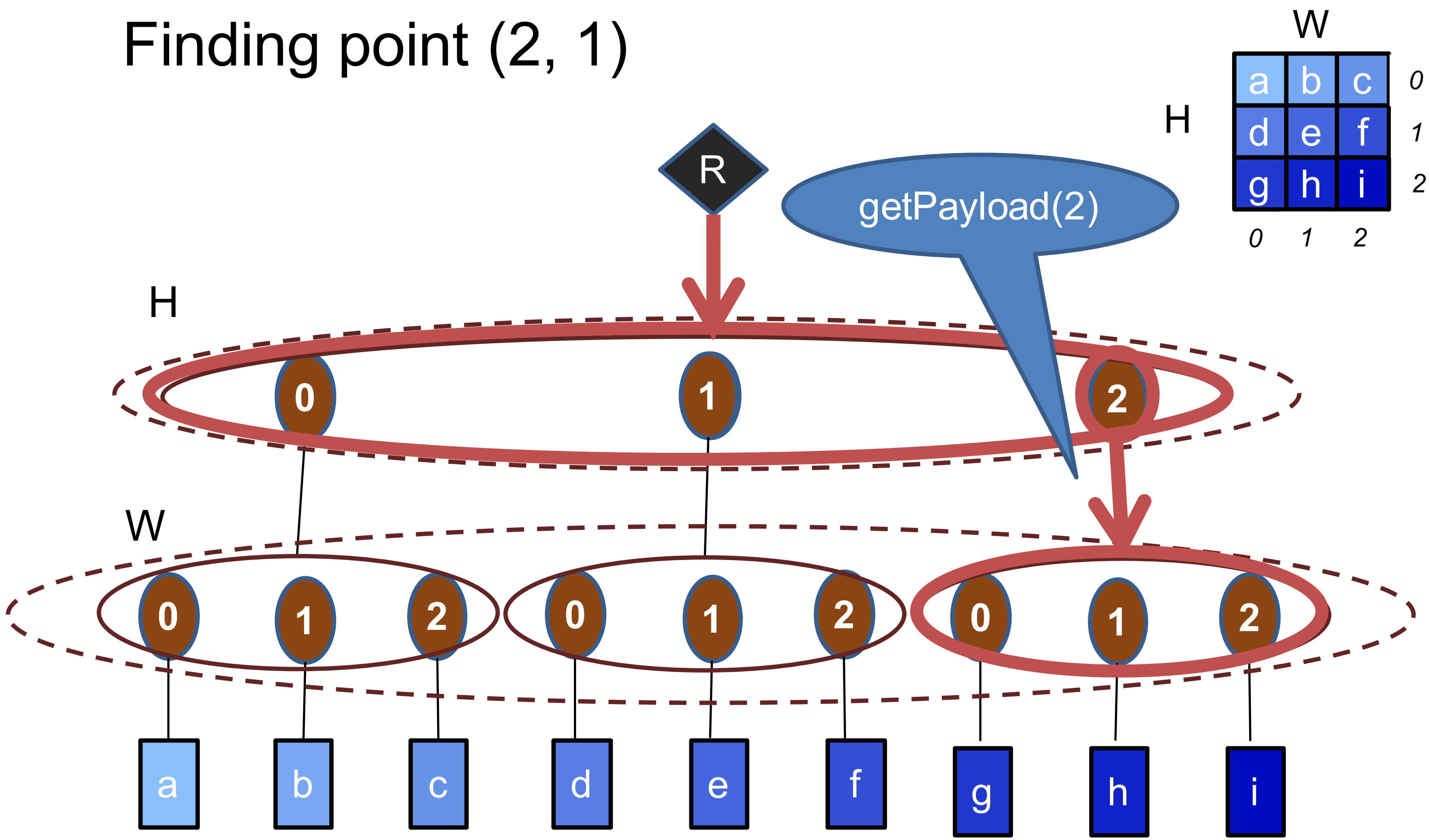

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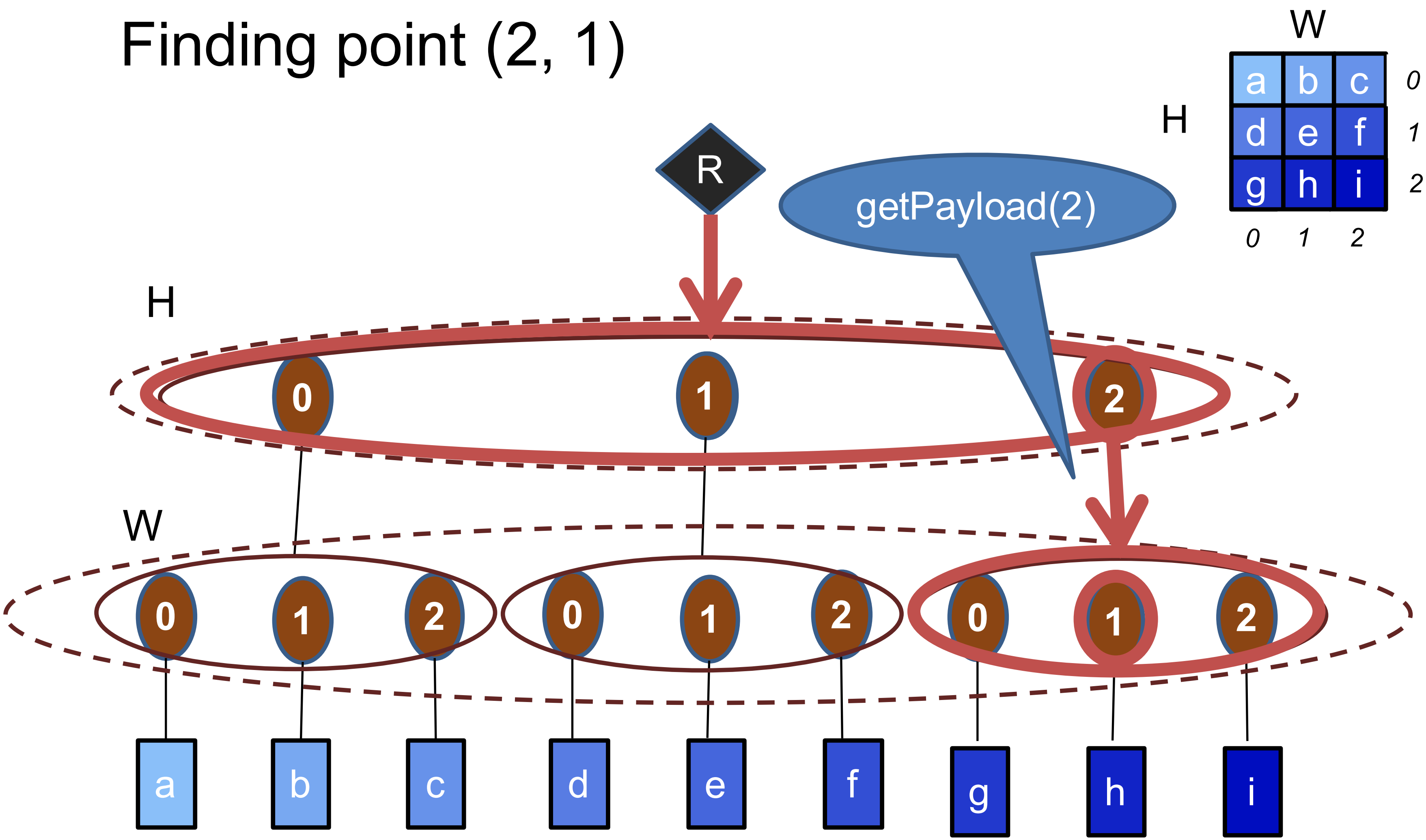

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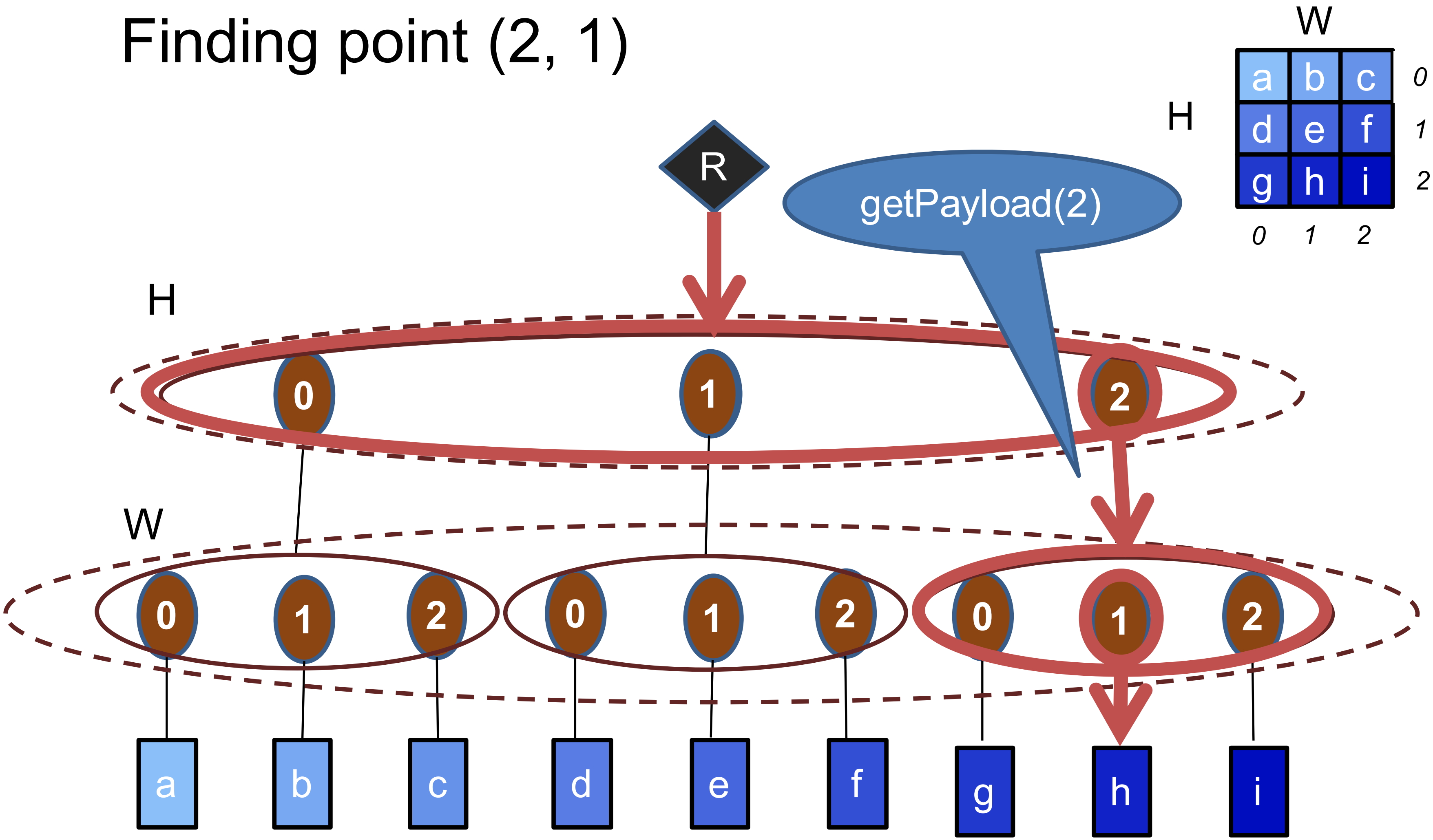

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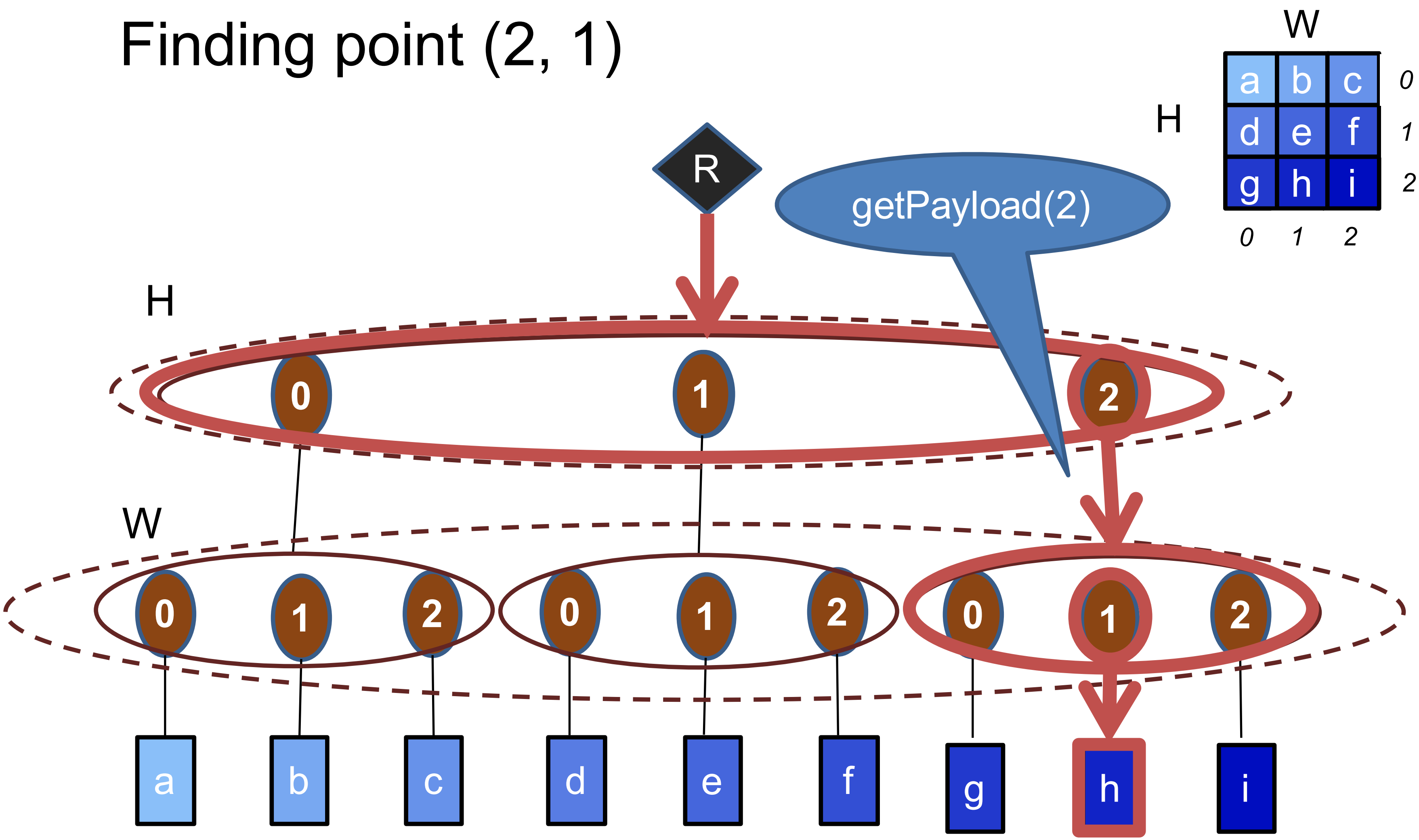

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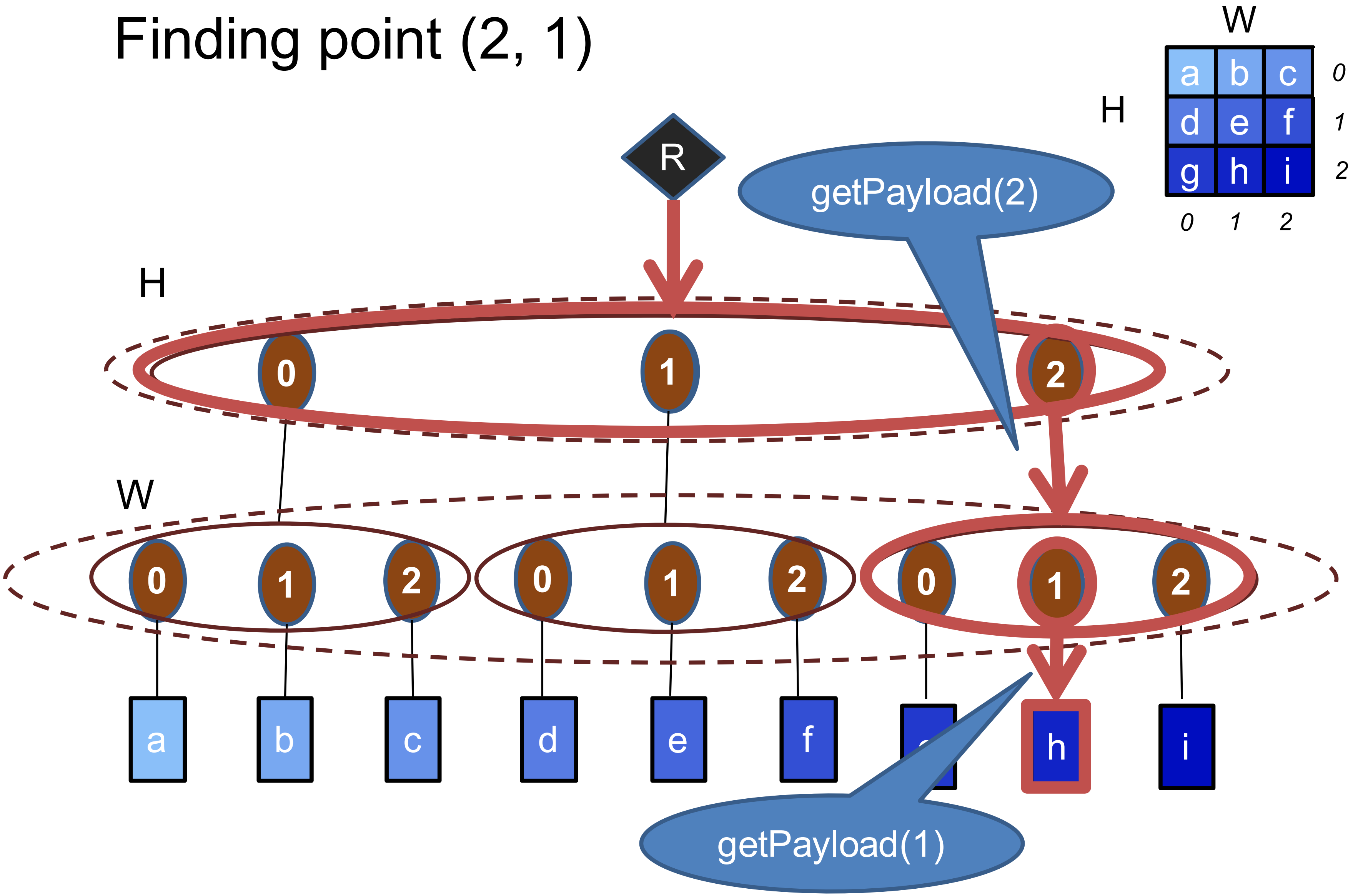

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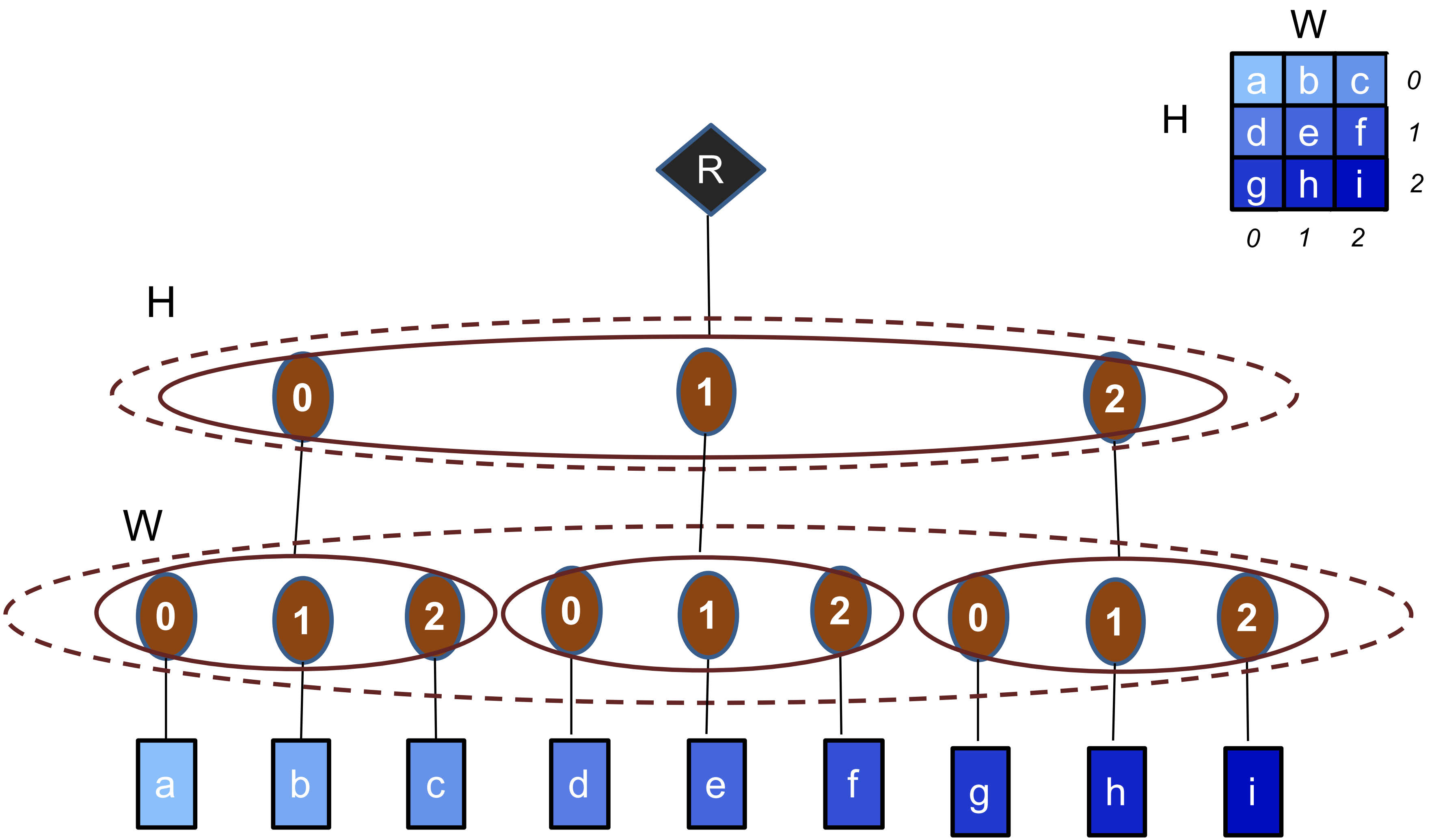
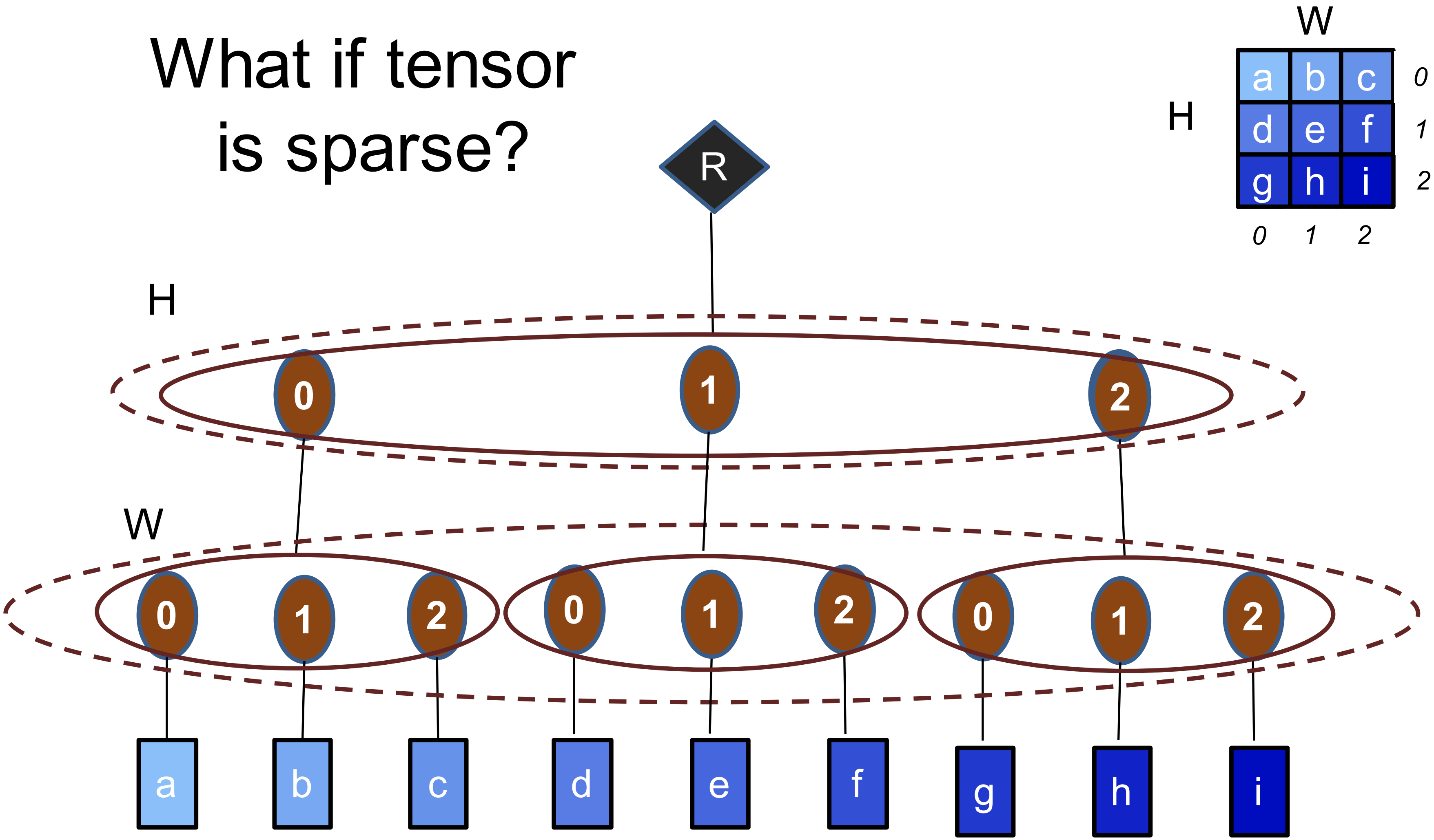


# Fibertree Tensor Abstraction

What if tensor is sparse?



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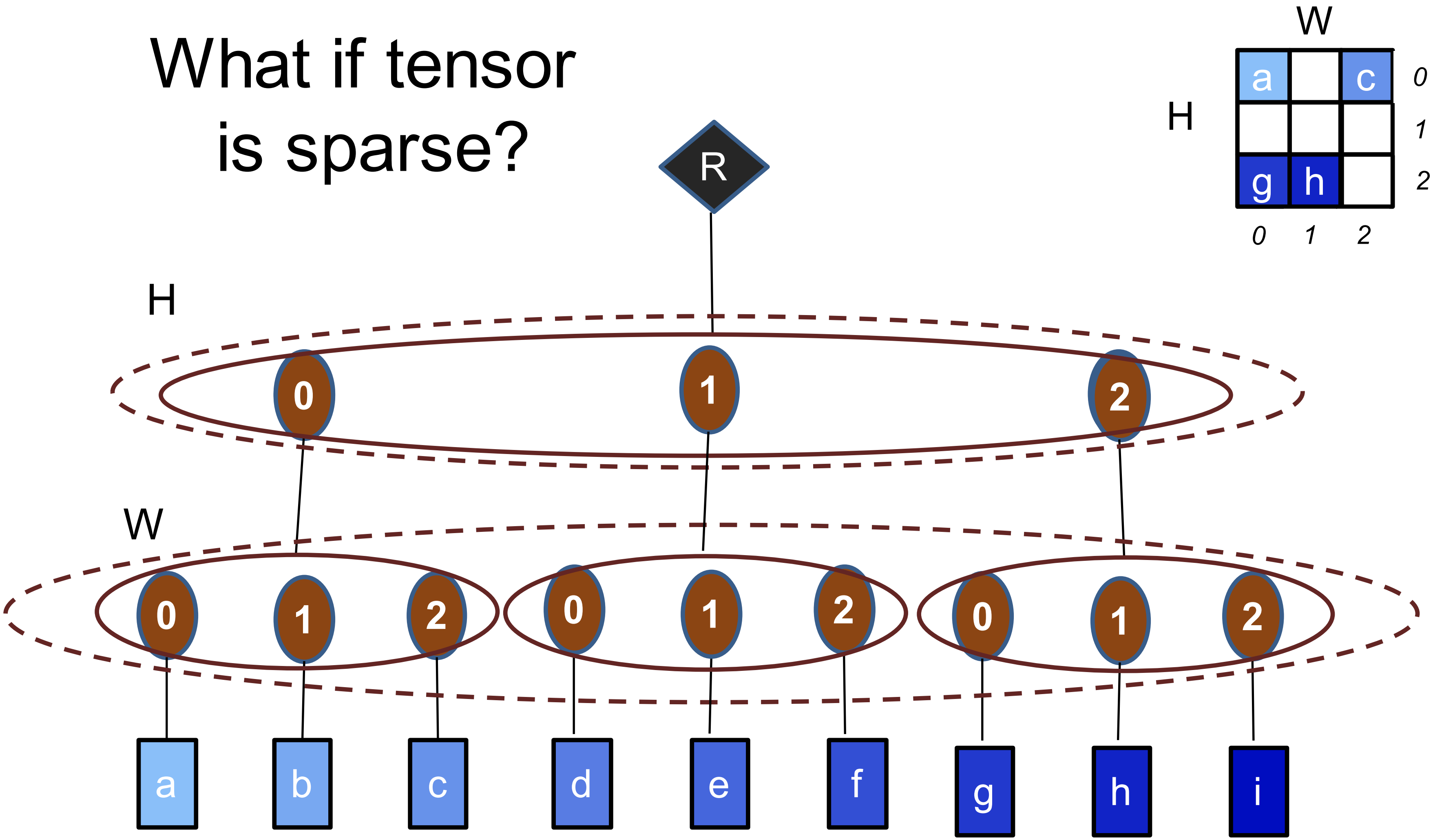

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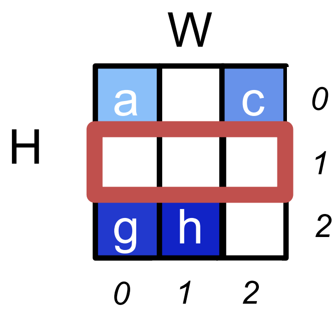


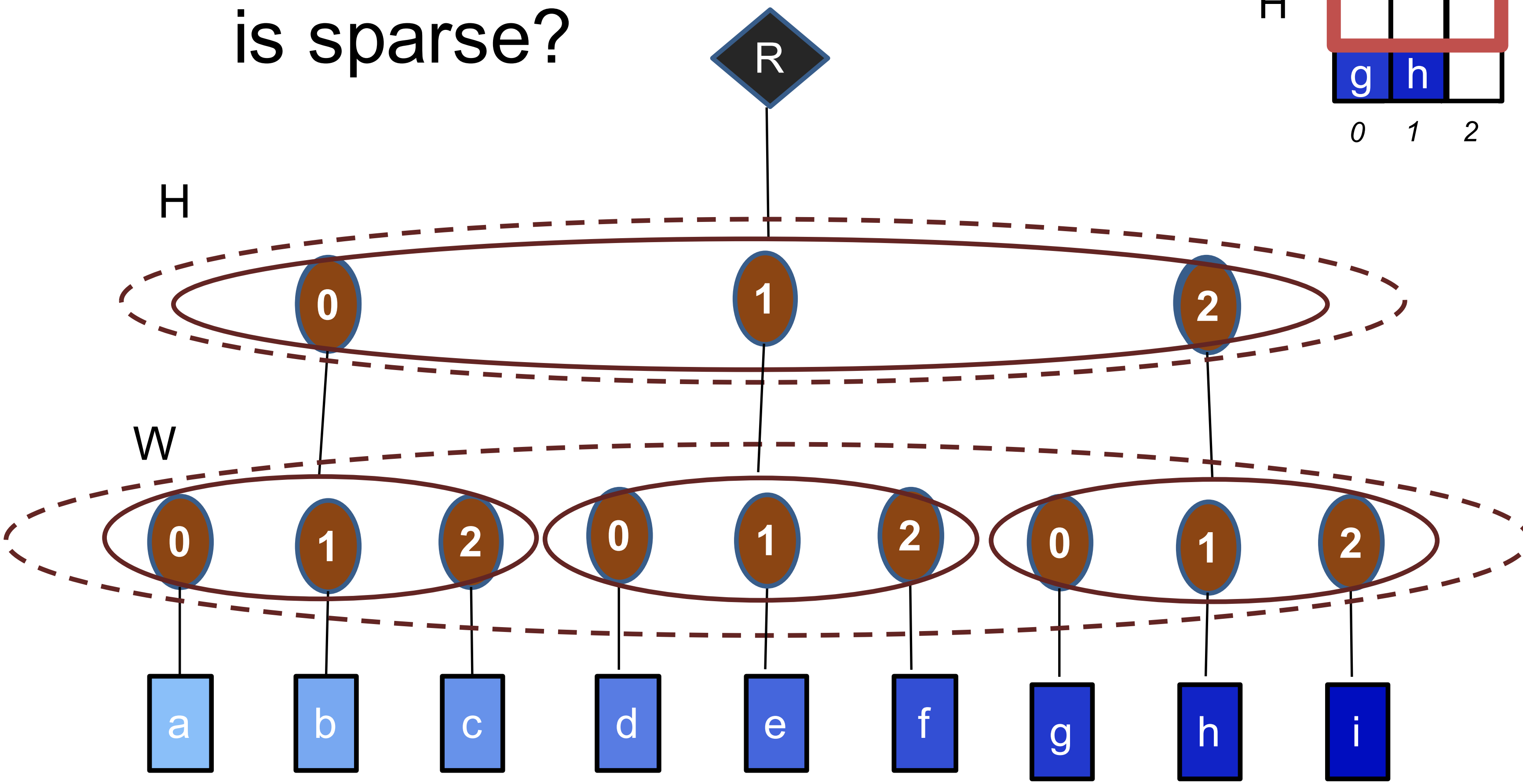

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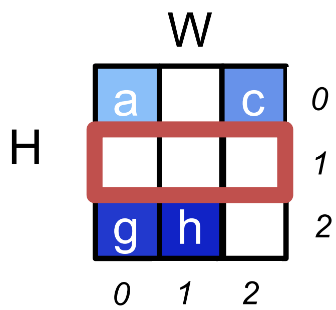


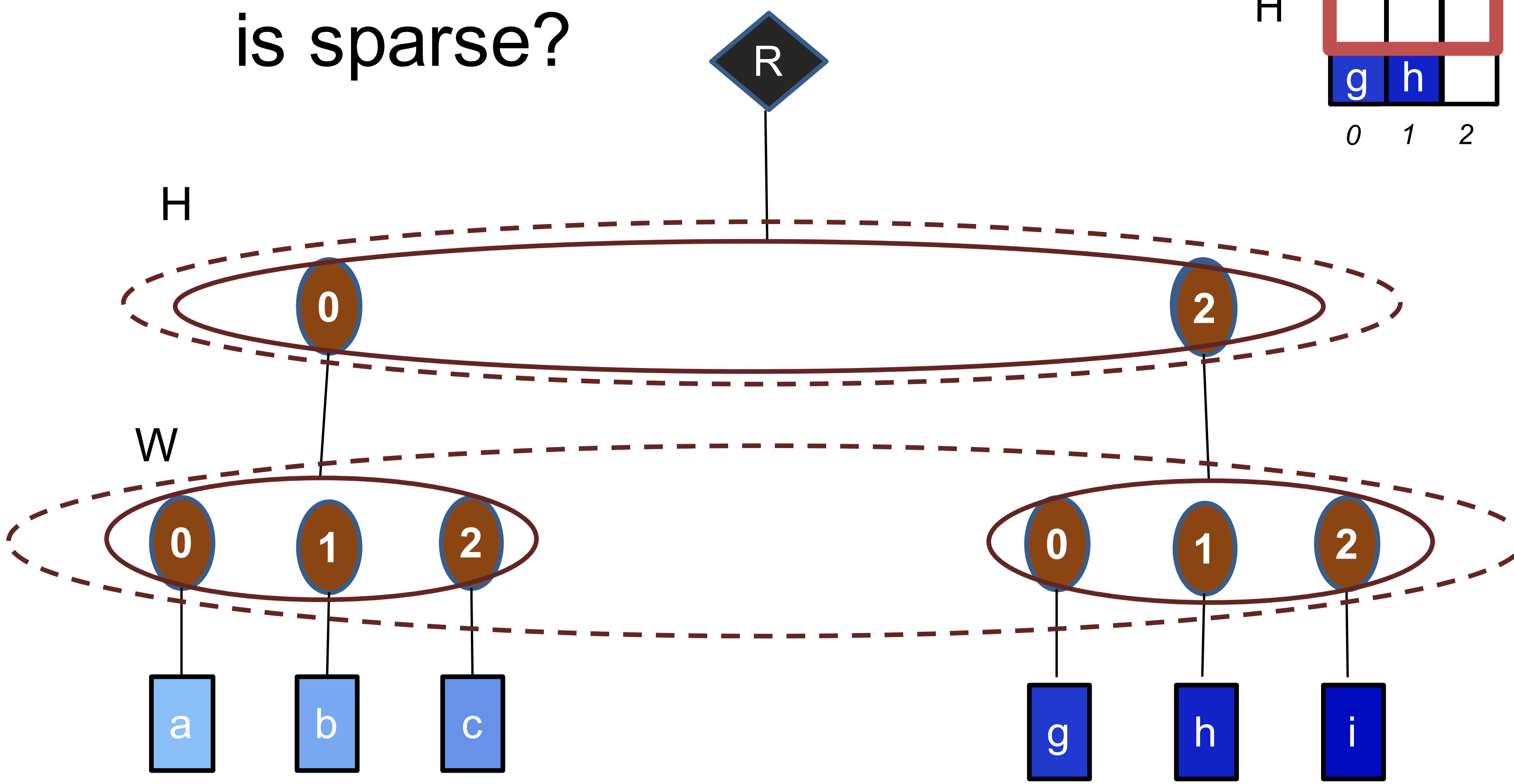

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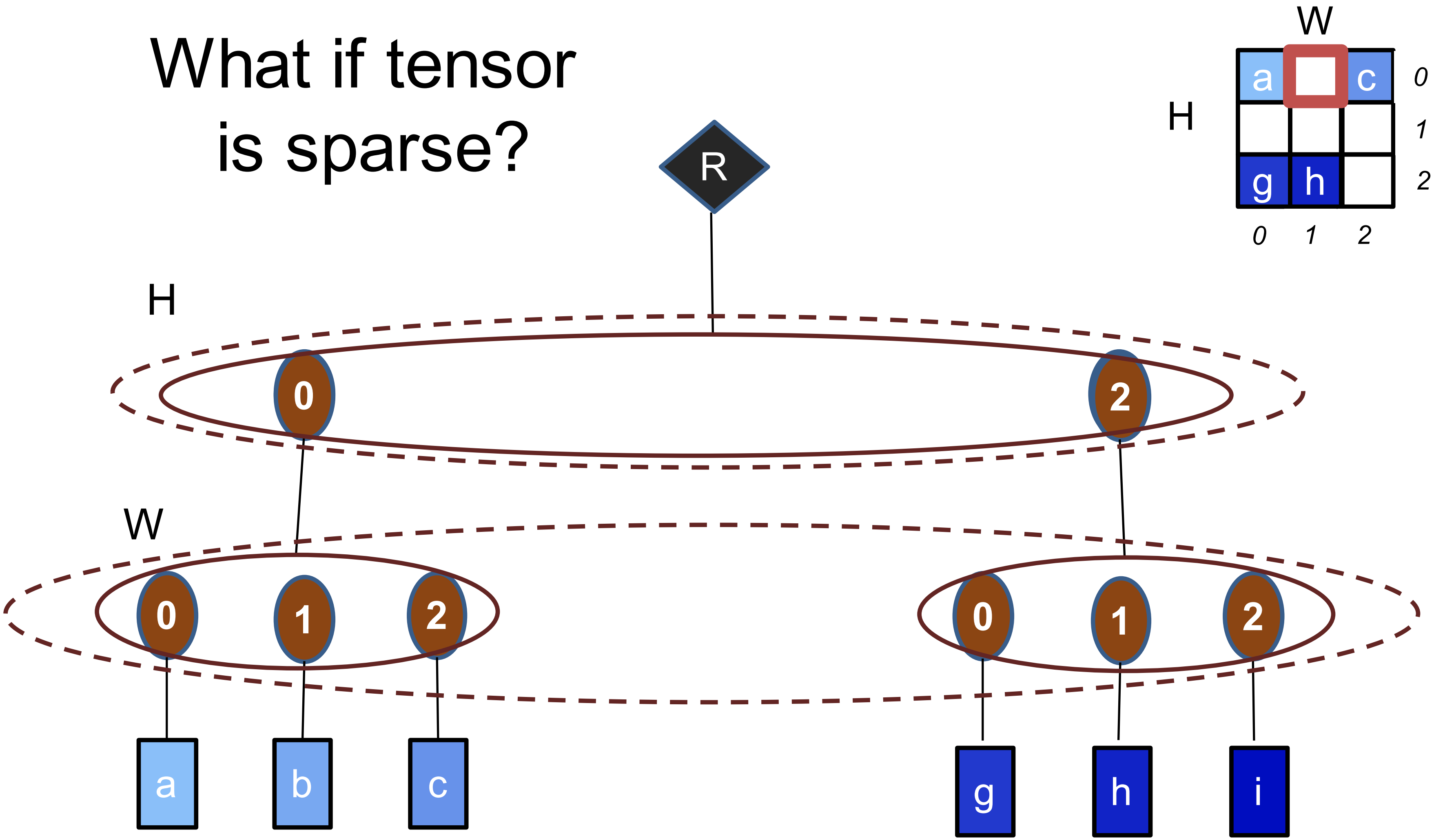

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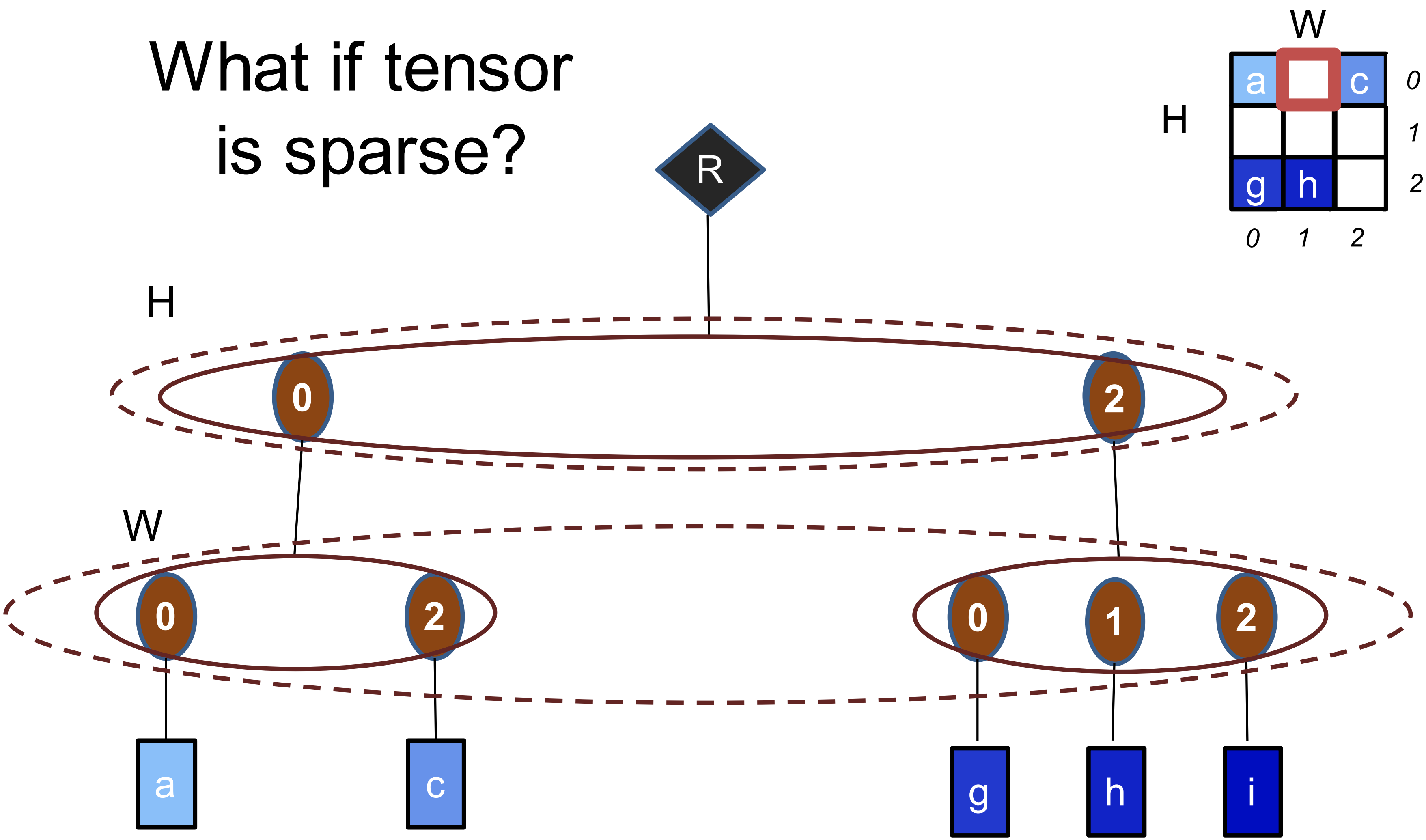

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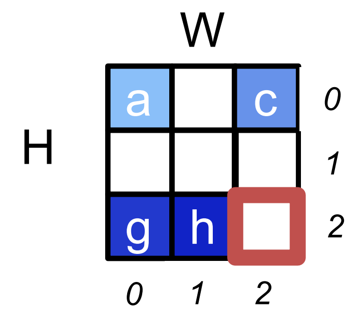


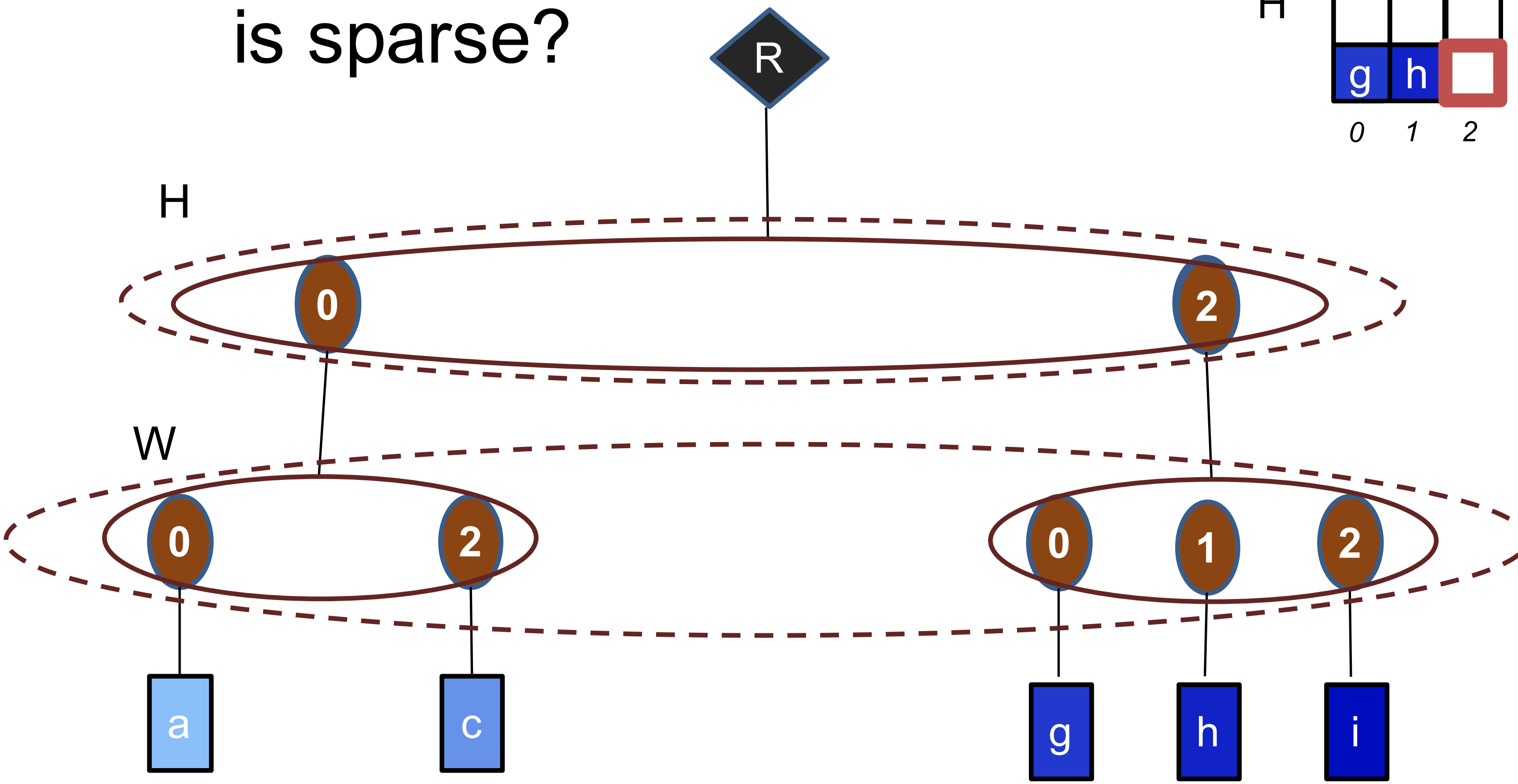

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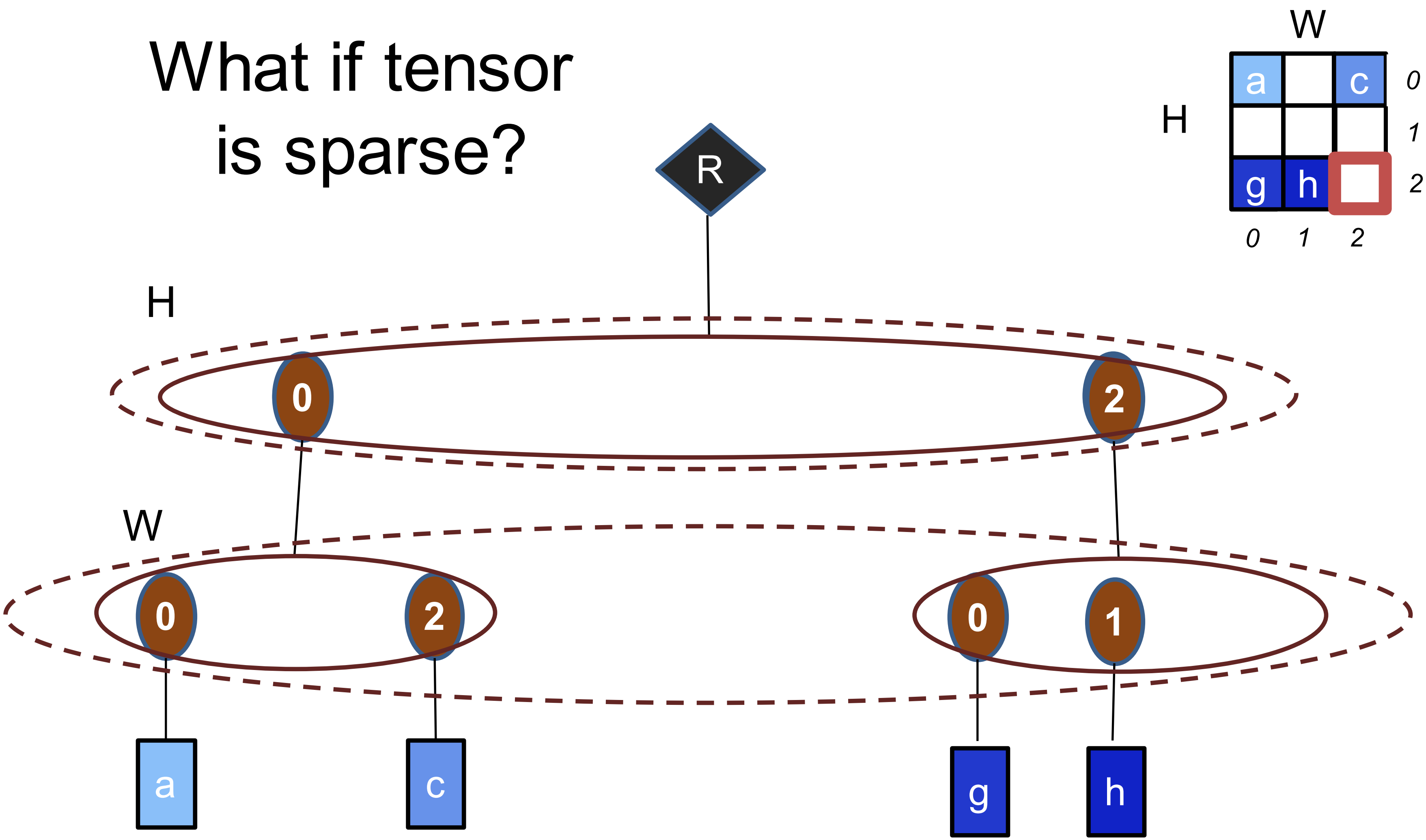

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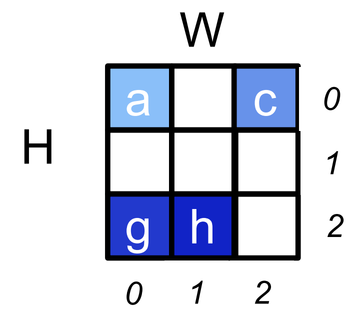


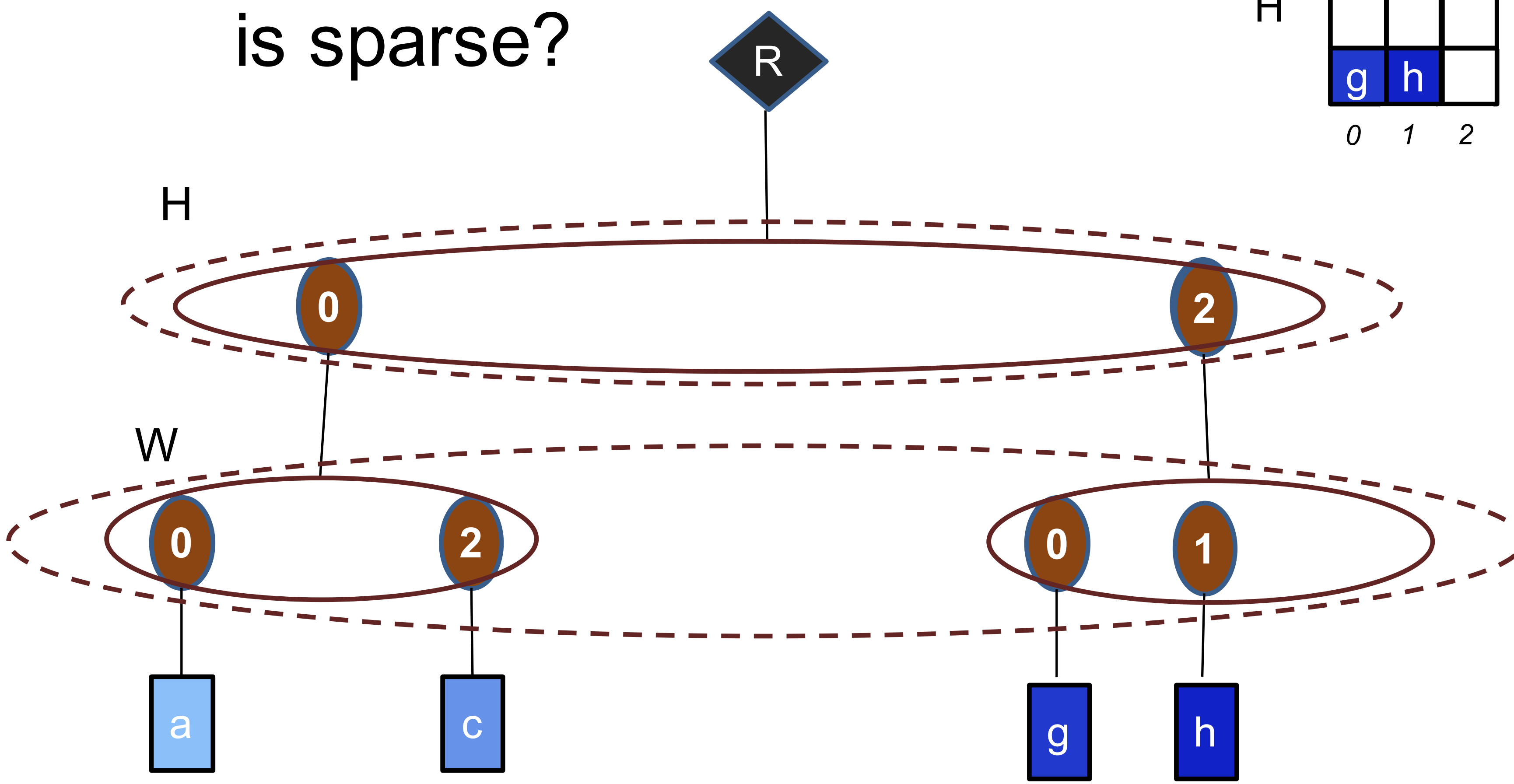

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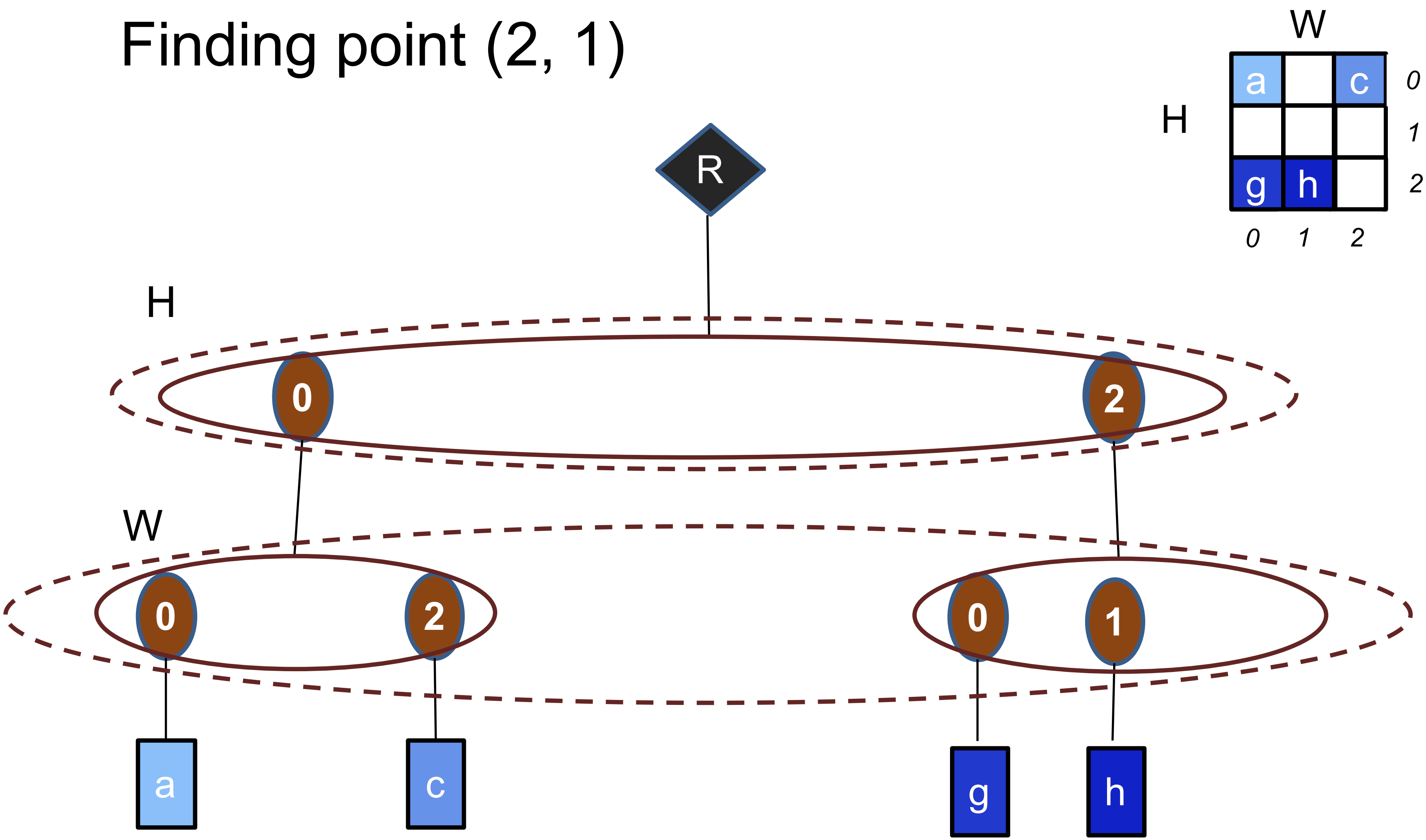

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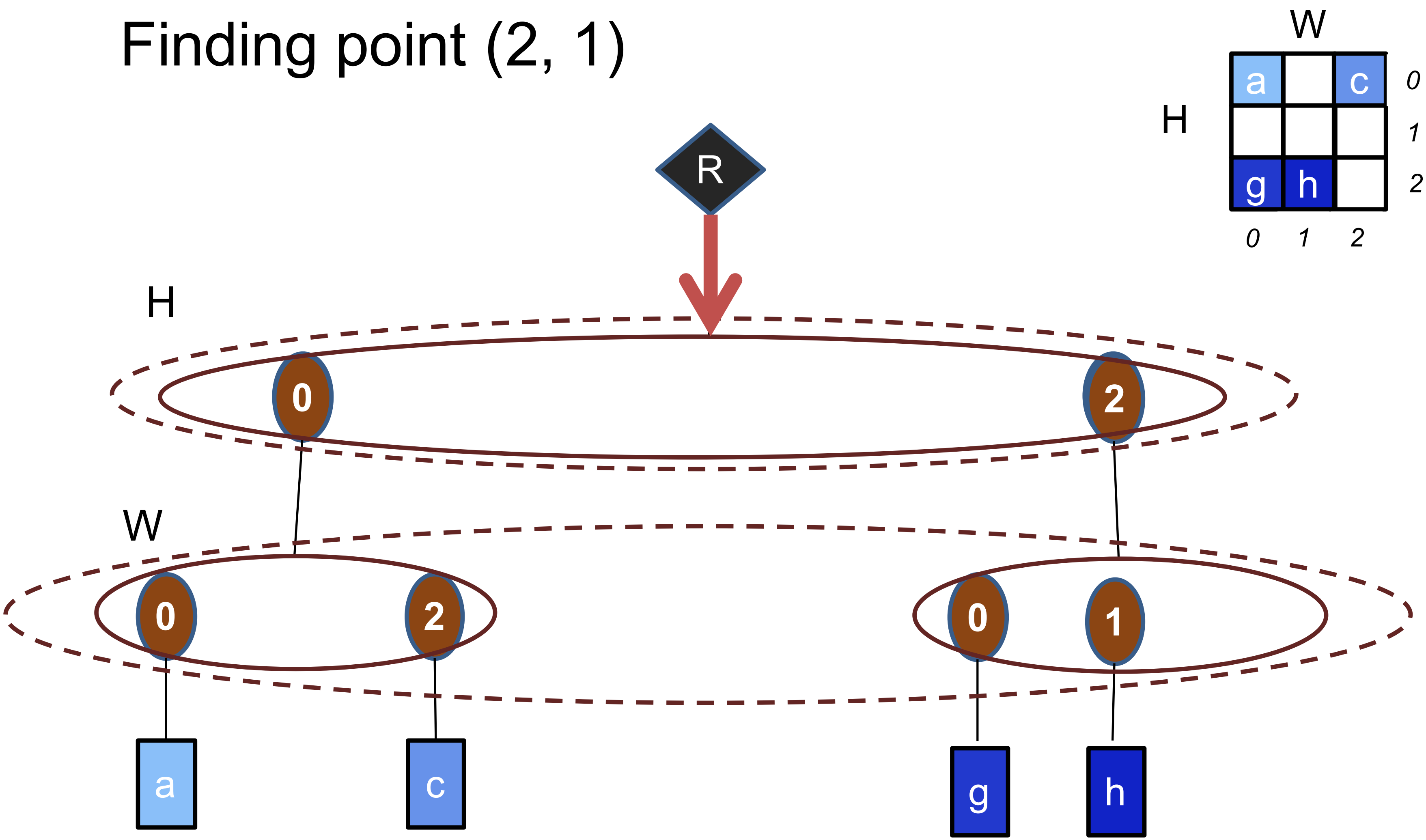

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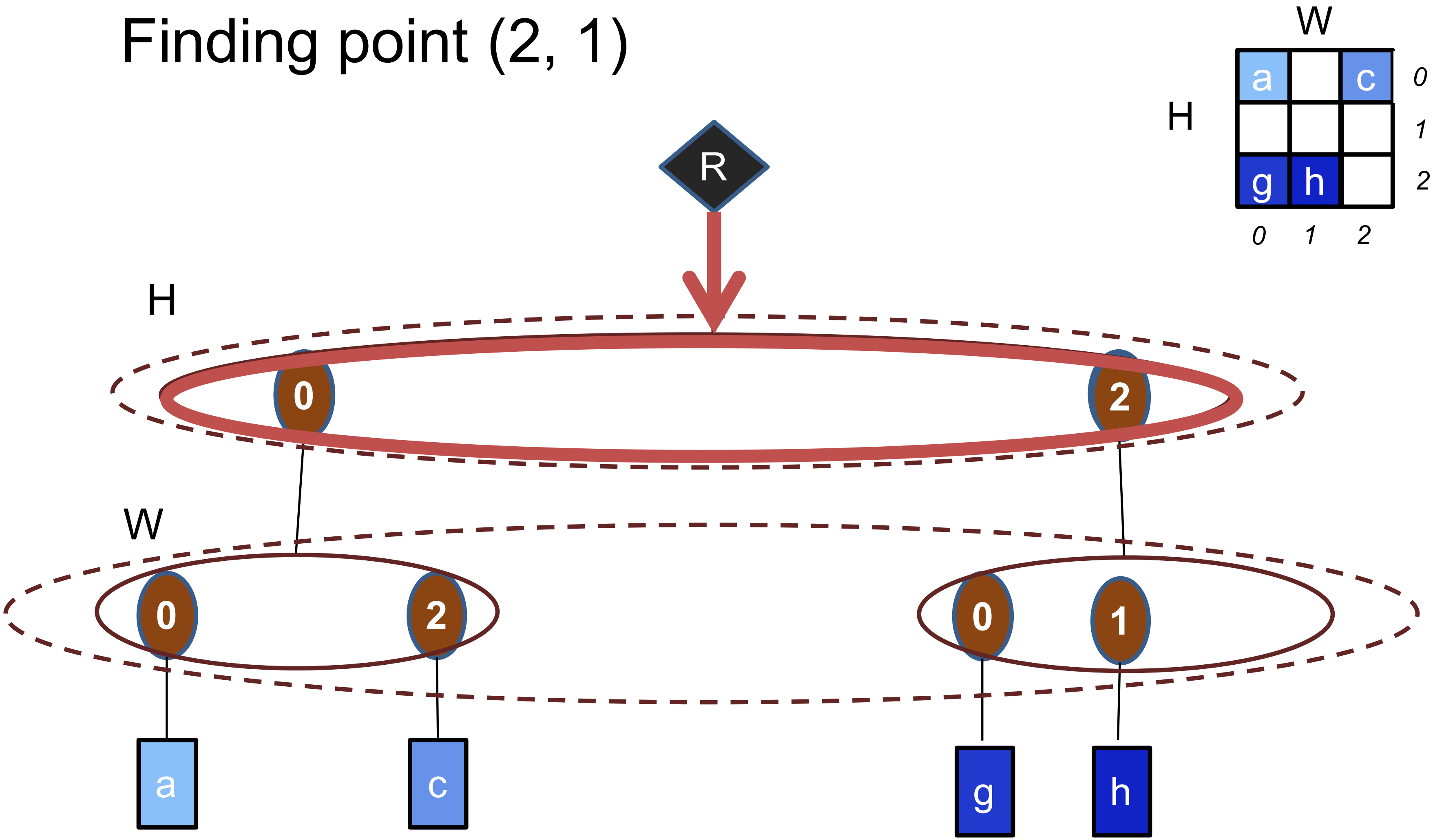

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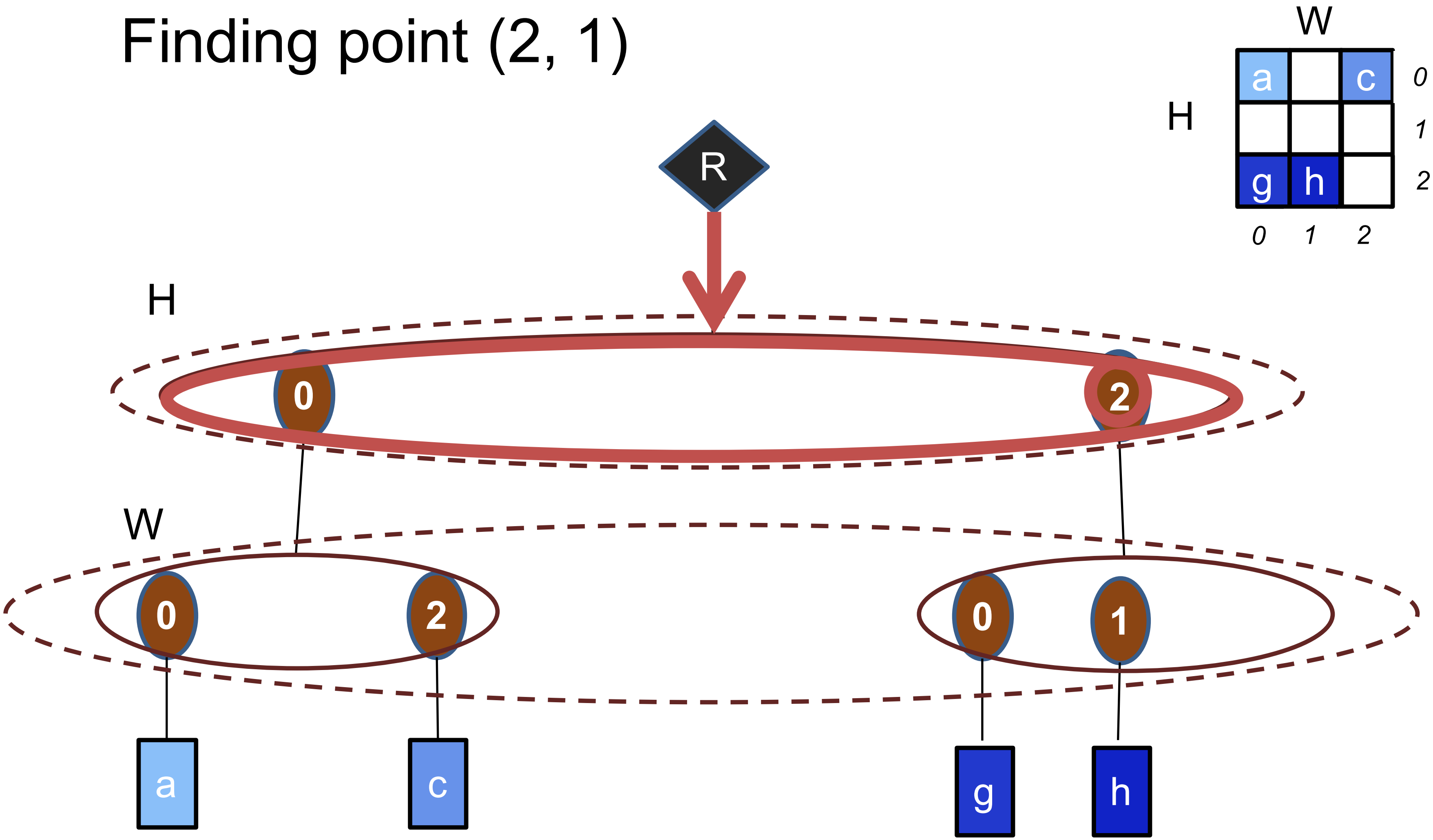

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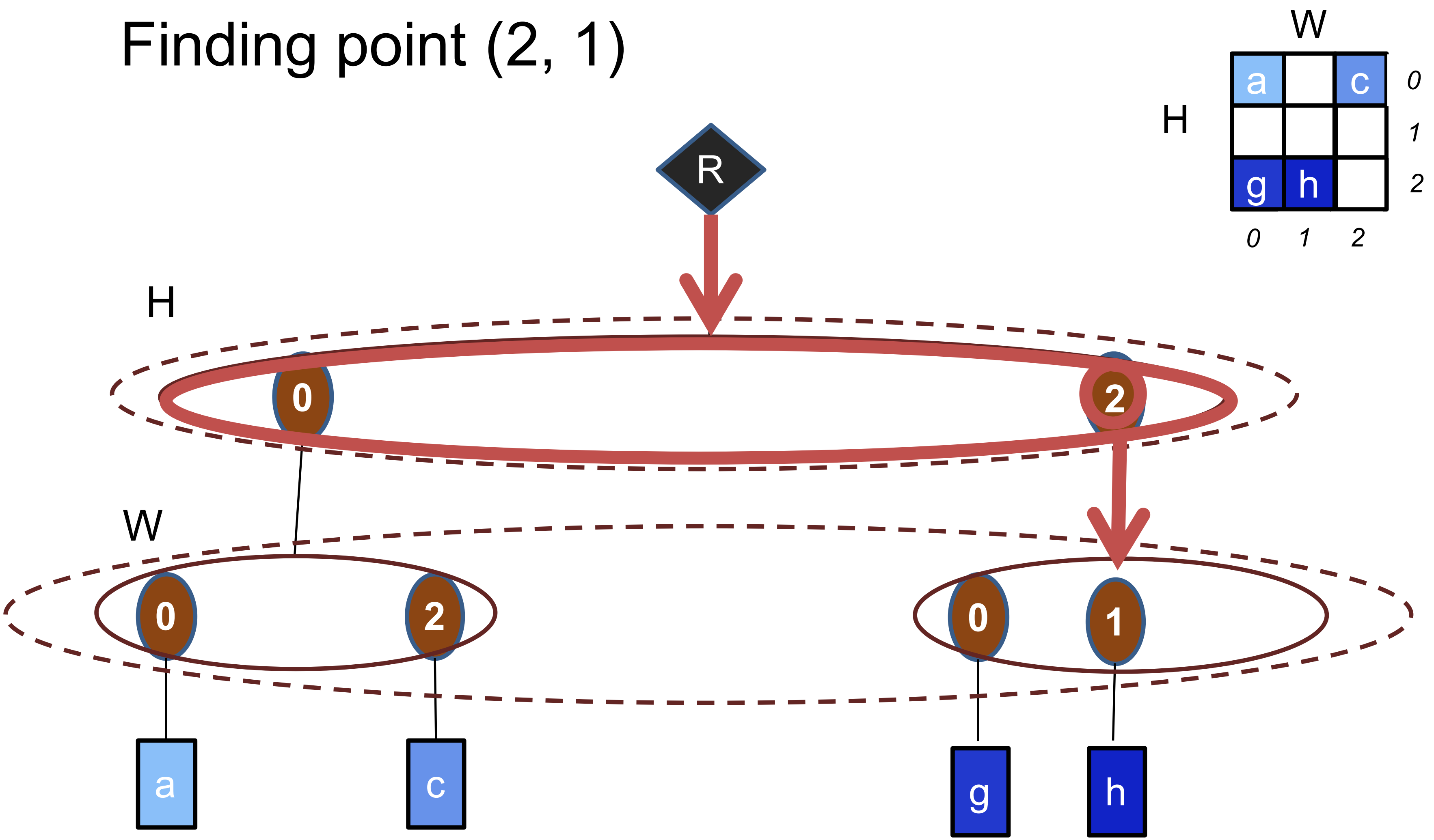

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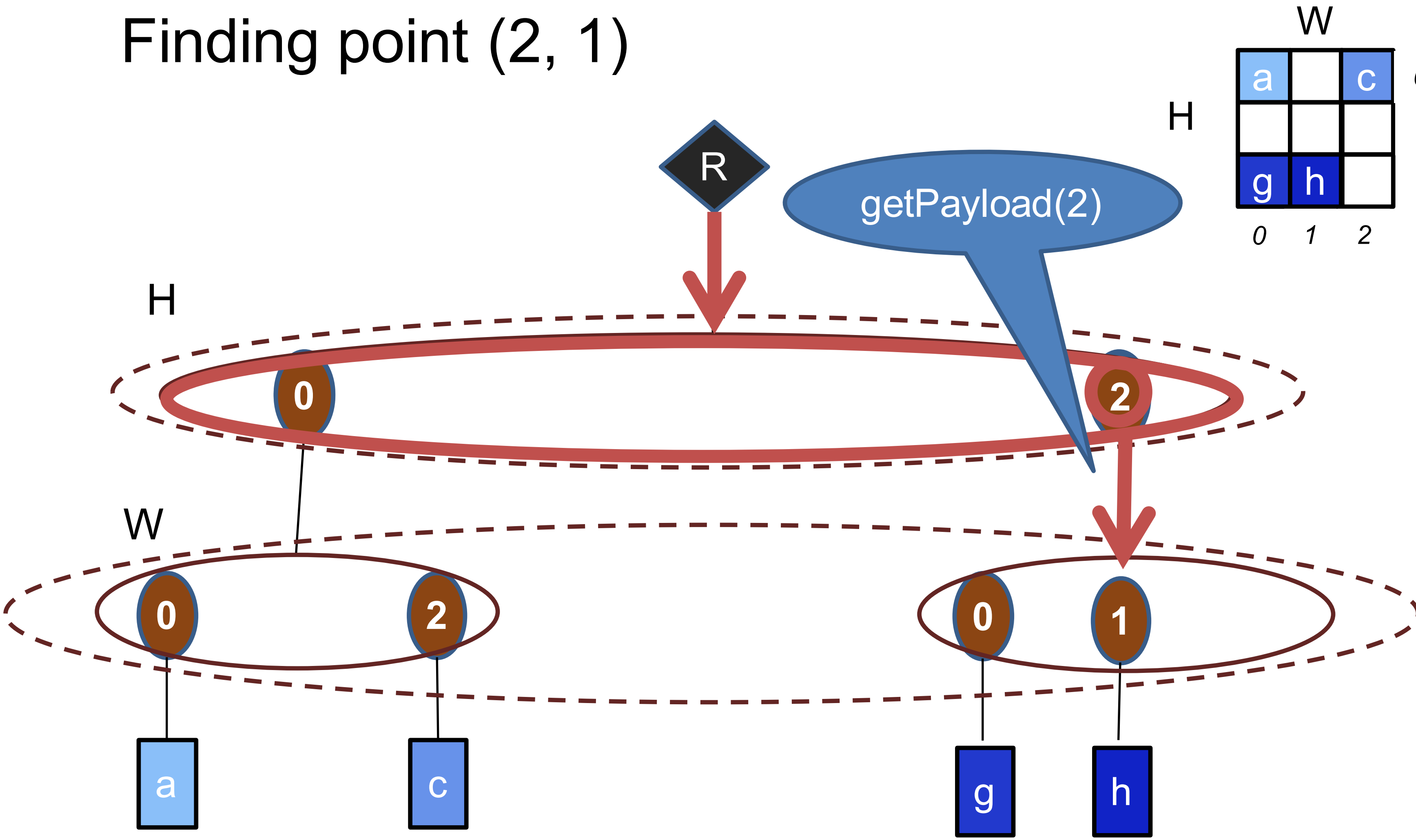

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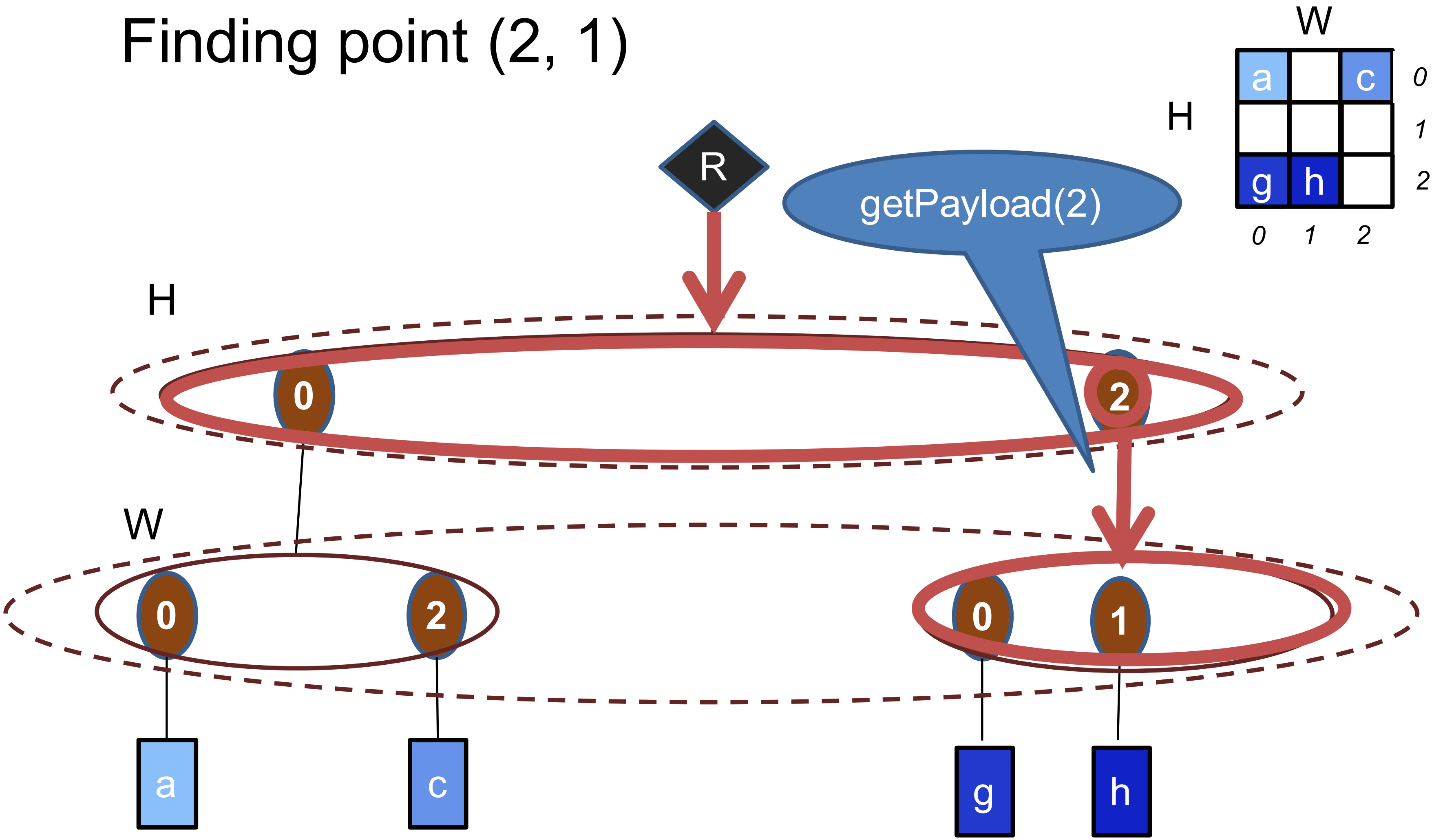

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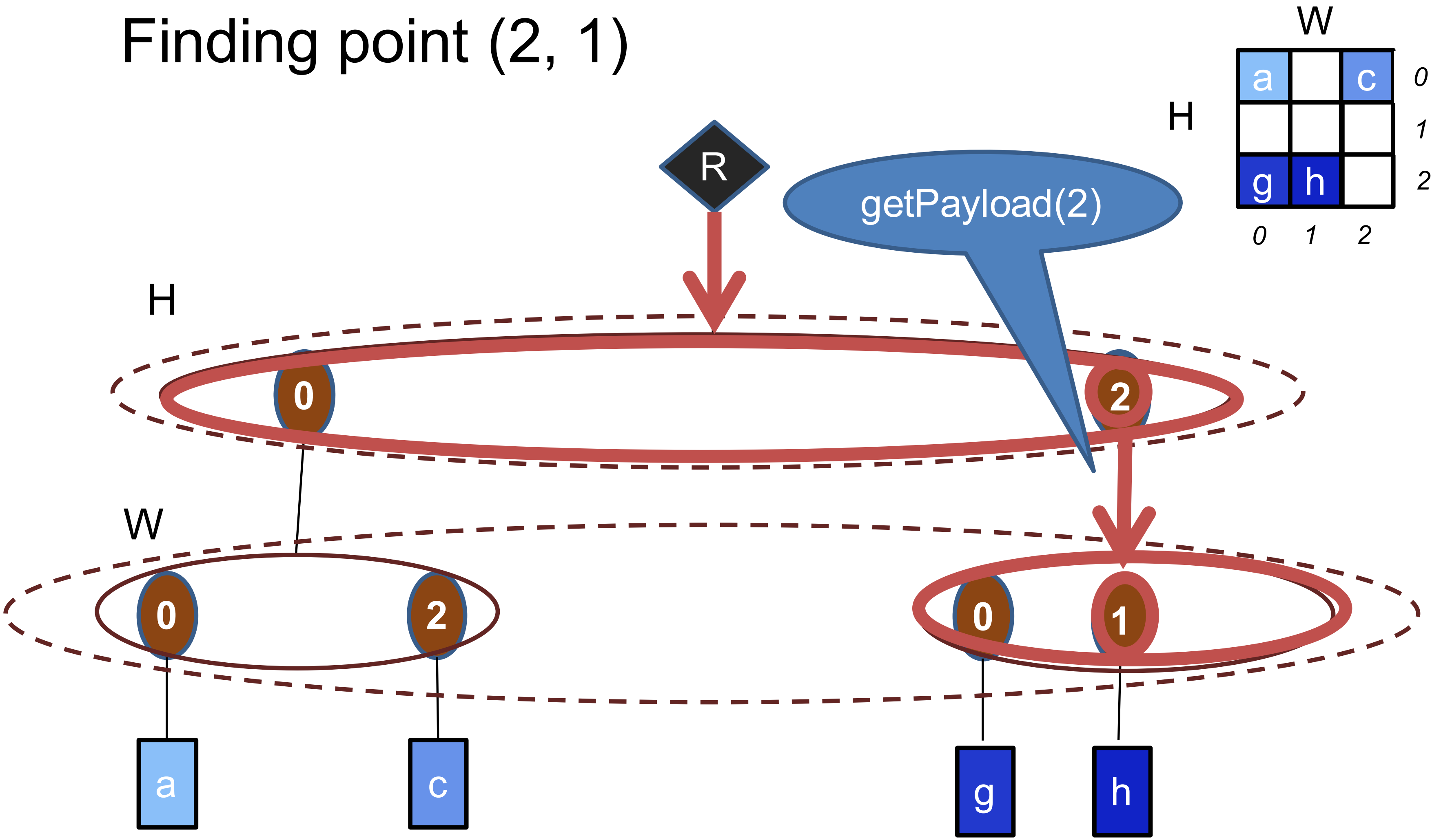

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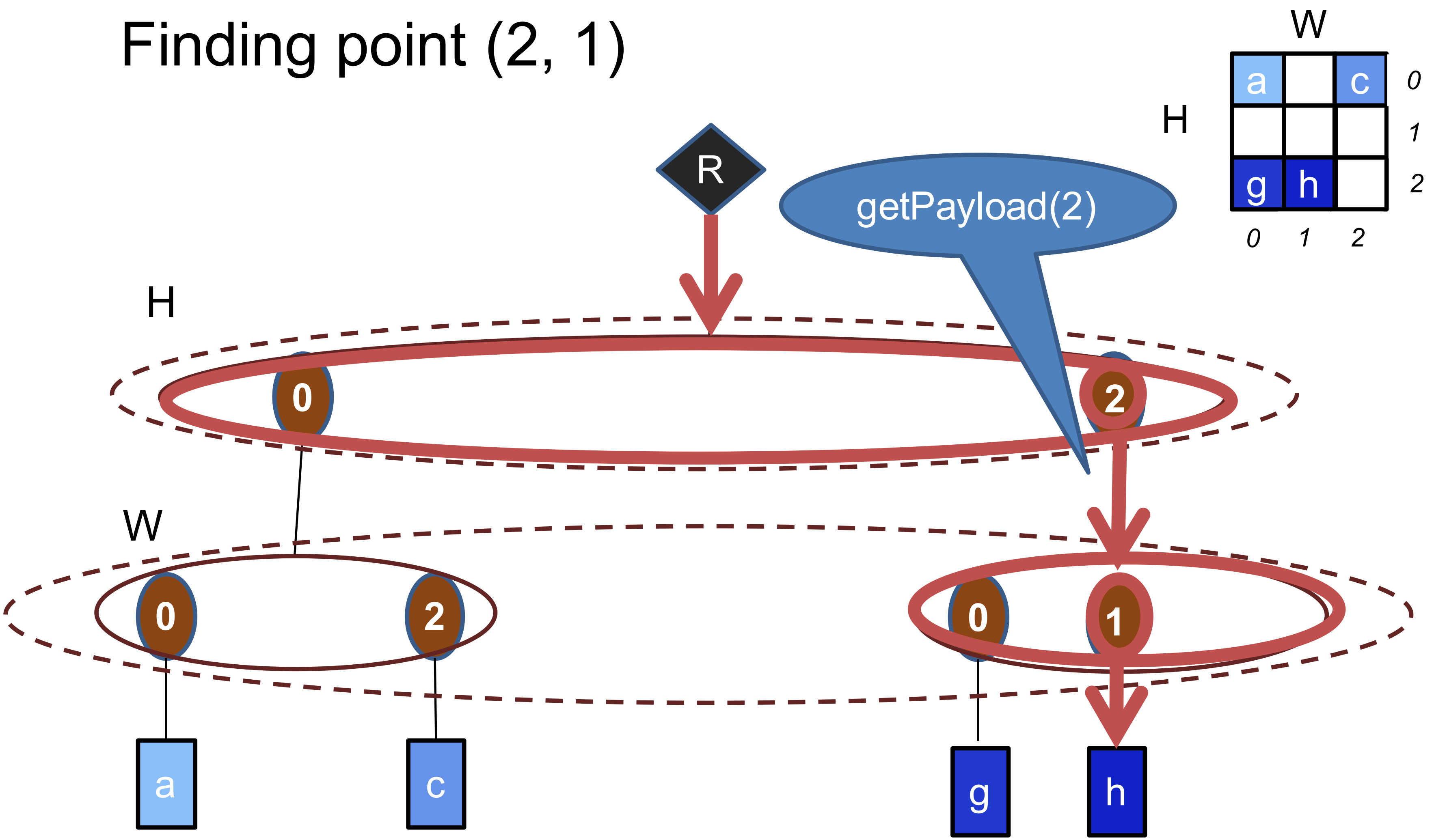

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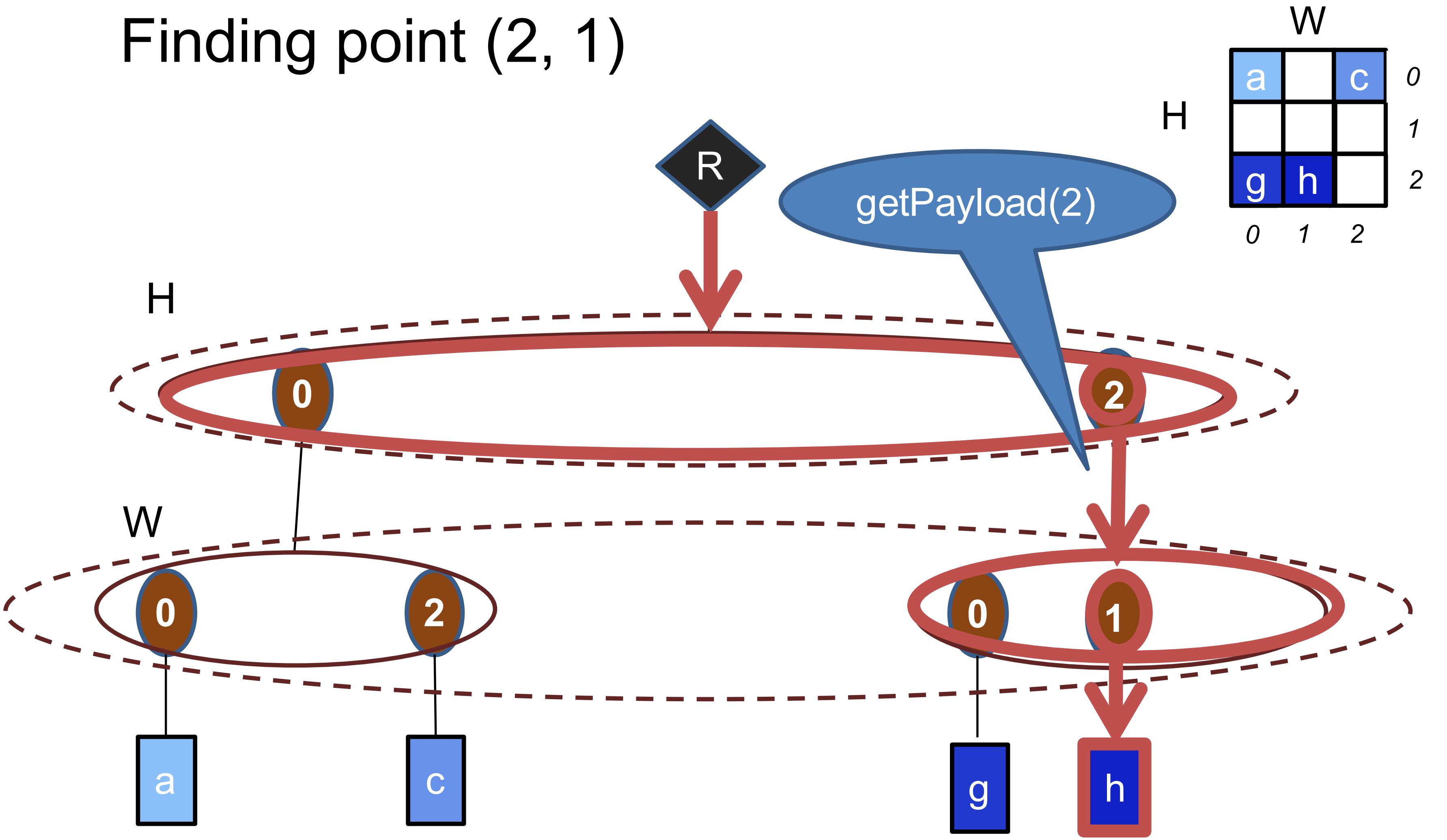

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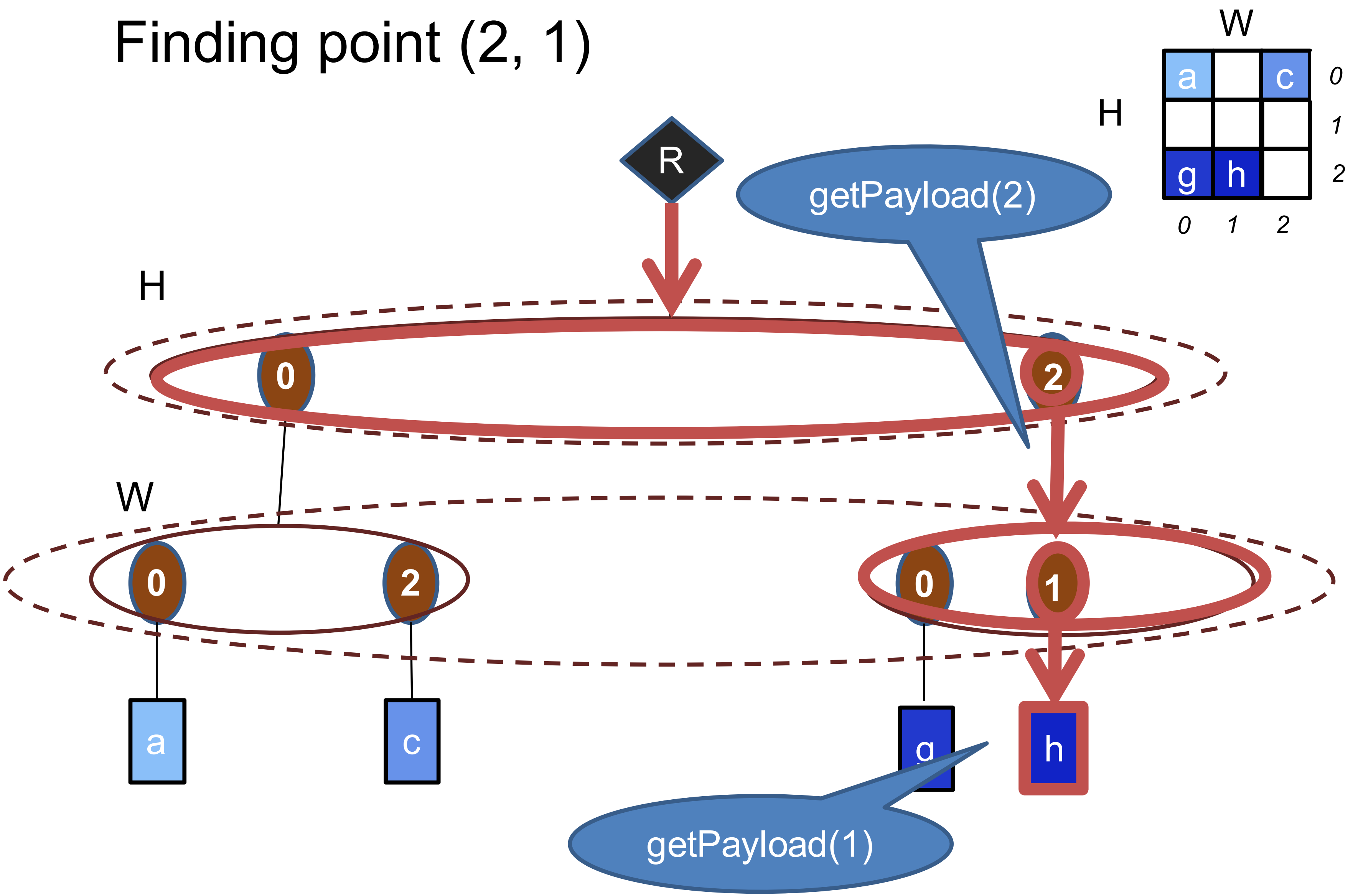
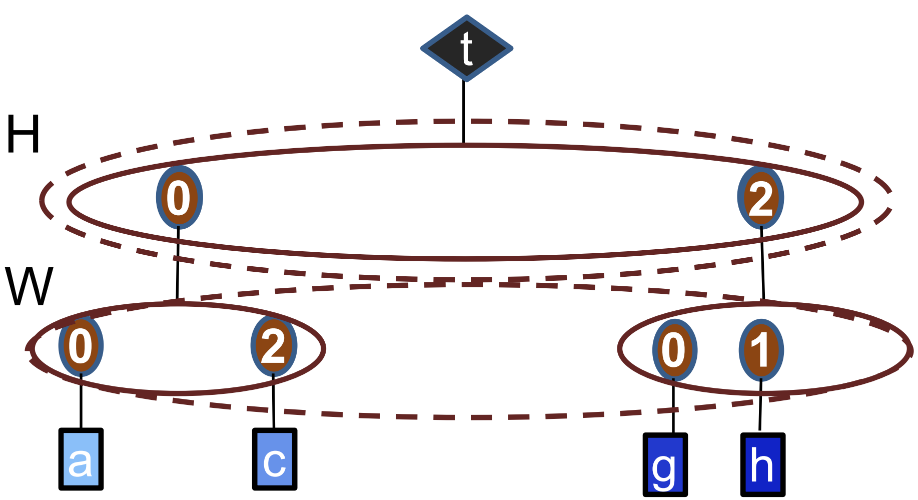


# Tensor Traversal (2-D)

```
# 2-D Tensor Traversal

t = Tensor(H,W)

sum = 0
for (h, t_h) in t:
  for (w, t_val) in t_h:
     sum += t_val
```



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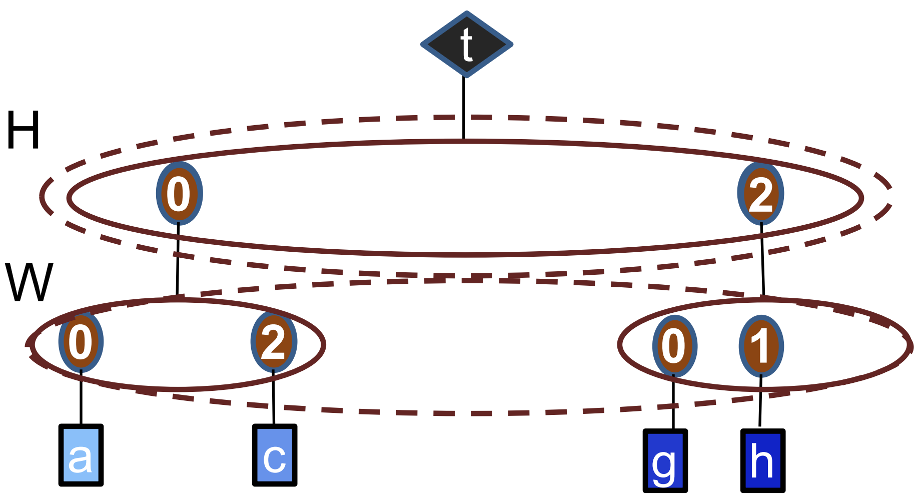
Each iteration returns a (coordinate, payload) tuple



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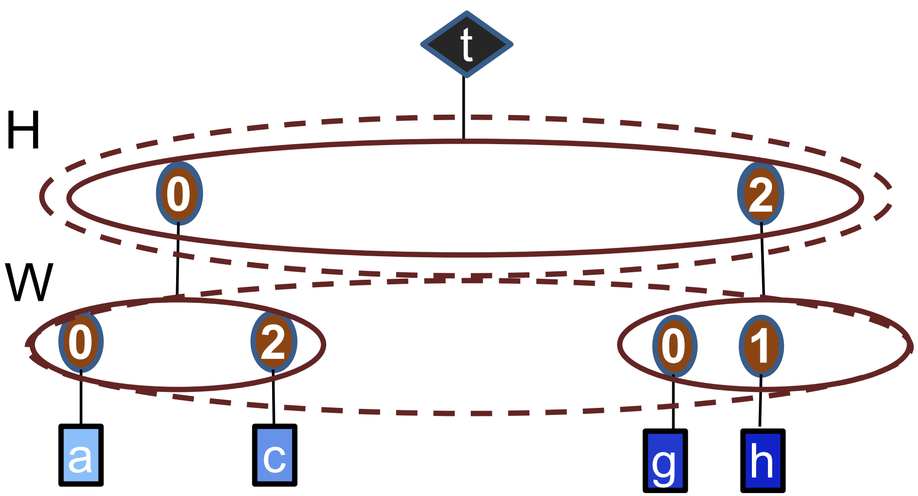


| t_pos | h | t_h_pos | w | t_val |
|---|---|---|---|---|
| 0 | 0 | ? | ? | ? |
| 0 | 0 | 0 | 0 | a |
| 0 | 0 | 1 | 2 | c |
| 1 | 2 | ? | ? | ? |
| … | … | … | … | … |

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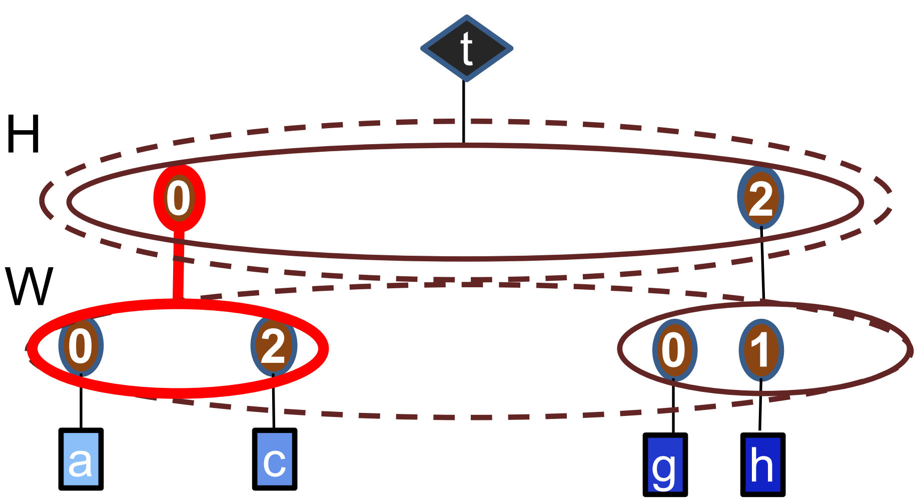


| t_pos | h | t_h_pos | w | t_val |
|---|---|---|---|---|
| 0 | 0 | ? | ? | ? |
| 0 | 0 | 0 | 0 | a |
| 0 | 0 | 1 | 2 | c |
| 1 | 2 | ? | ? | ? |
| … | … | … | … | … |

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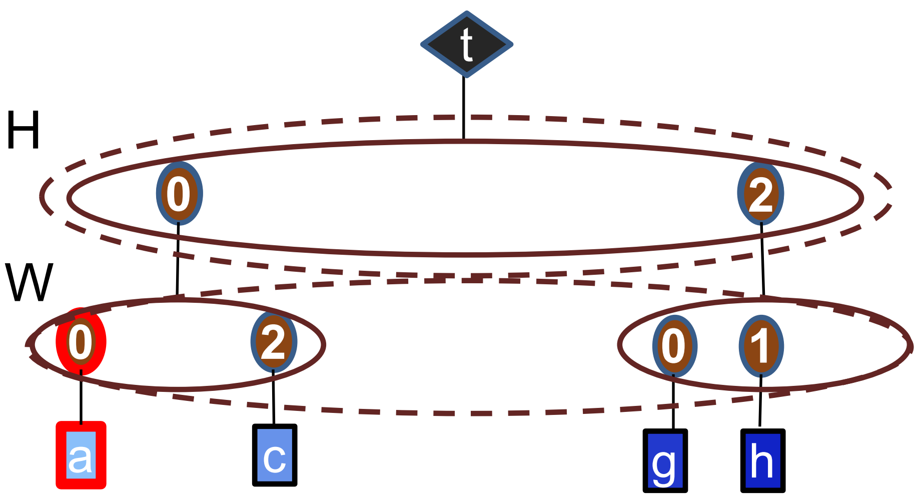


| t_pos | h | t_h_pos | w | t_val |
|---|---|---|---|---|
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| 0 | 0 | 0 | 0 | a |
| 0 | 0 | 1 | 2 | c |
| 1 | 2 | ? | ? | ? |
| … | … | … | … | … |

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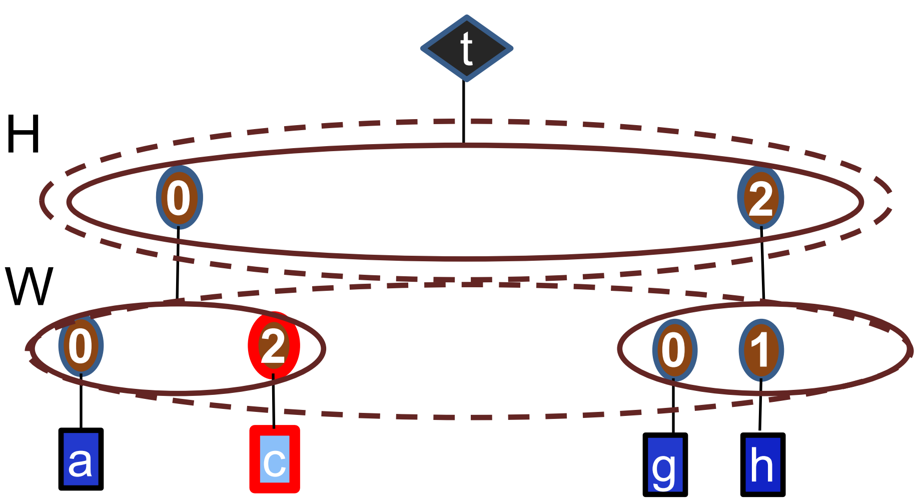
```



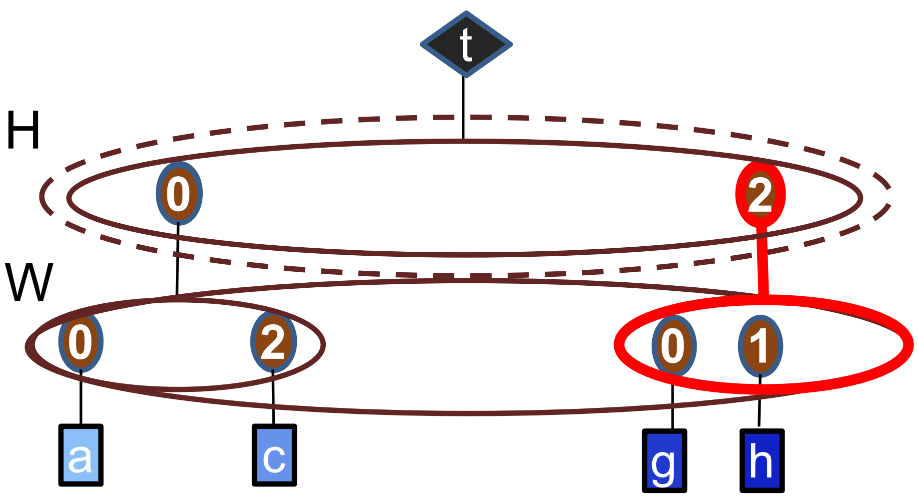

| t_pos | h | t_h_pos | w | t_val |
|---|---|---|---|---|
| 0 | 0 | ? | ? | ? |
| 0 | 0 | 0 | 0 | a |
| 0 | 0 | 1 | 2 | c |
| 1 | 2 | ? | ? | ? |
| … | … | … | … | … |

# Tensor Traversal (2-D)

```
# 2-D Tensor Traversal

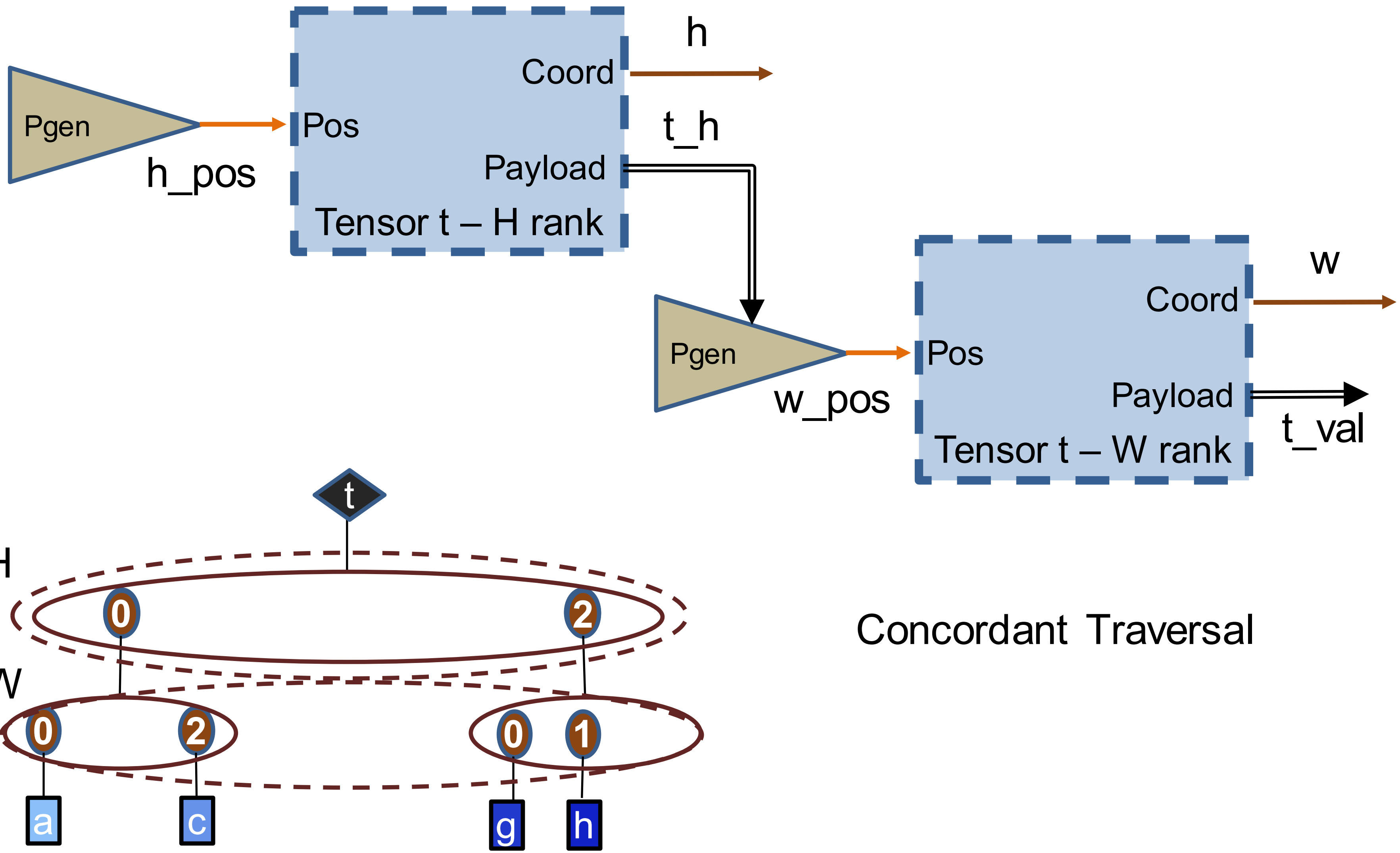
t = Tensor(H,W)

sum = 0
for (h, t_h) in t:
   for (w, t_val) in t_h:
      sum += t_val
```




| t_pos | h | t_h_pos | w | t_val |
|---|---|---|---|---|
| 0 | 0 | ? | ? | ? |
| 0 | 0 | 0 | 0 | a |
| 0 | 0 | 1 | 2 | c |
| 1 | 2 | ? | ? | ? |
| … | … | … | … | … |

# Tensor Traversal (2-D)

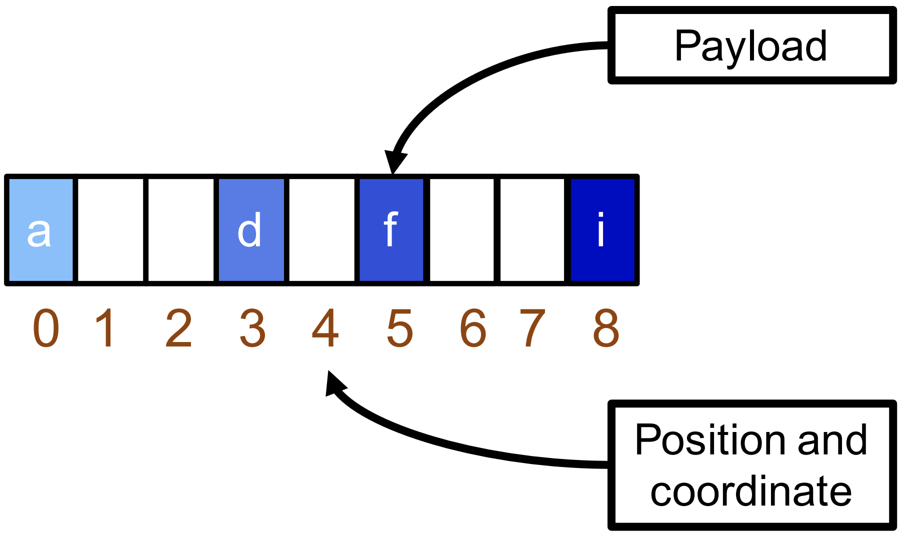



Concordant Traversal

# Example Fiber Representations

Each fiber has a set of (coordinate, “payload”) tuples

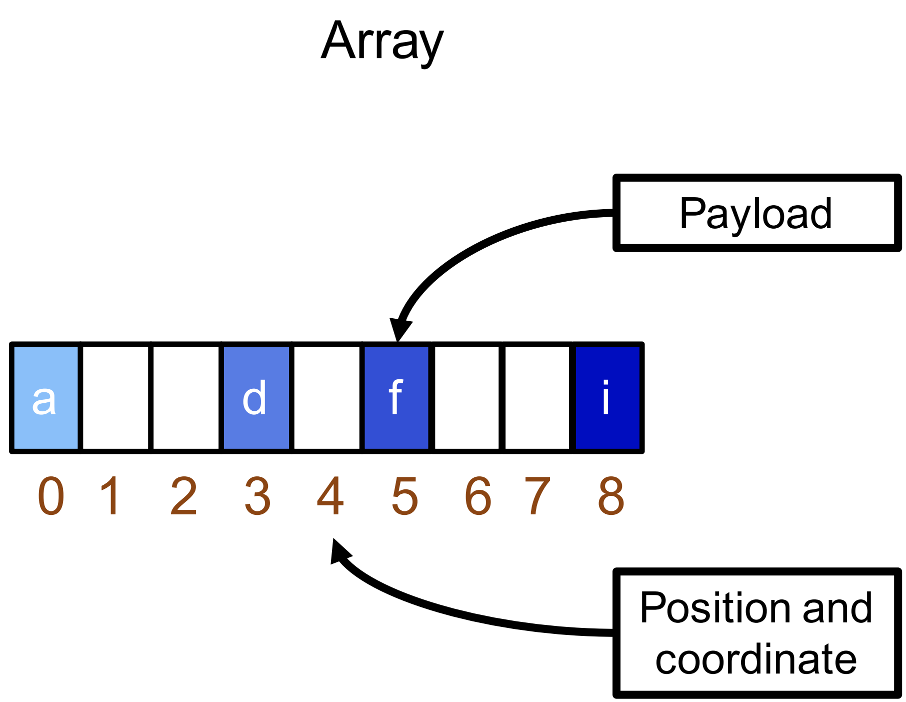
Array

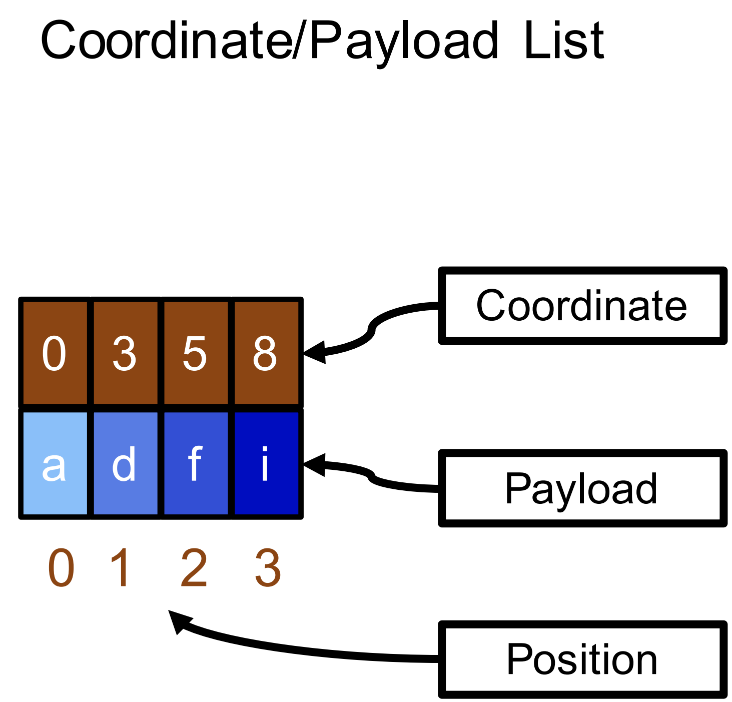



Data in a fiber is accessed by its position or offset in memory

# Example Fiber Representations

Each fiber has a set of (coordinate, “payload”) tuples




Data in a fiber is accessed by its position or offset in memory

# Fiber Representation Choices

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- Implicit Coordinates
  - Uncompressed (no metadata required)
  - Compressed – e.g., run length encoded

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  - E.g., coordinate/payload list

- Compressed vs Uncompressed
  - Compressed/uncompressed is an attribute of the representation*.
  - Uncompressed means size is proportional to maximum coordinate value
  - Compressed formats will have metadata overhead relative to uncompressed formats. For dense data, this may cost more than just using an uncompressed format.
  - Space efficiency of a representation depends on sparsity

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  - E.g., coordinate/payload list
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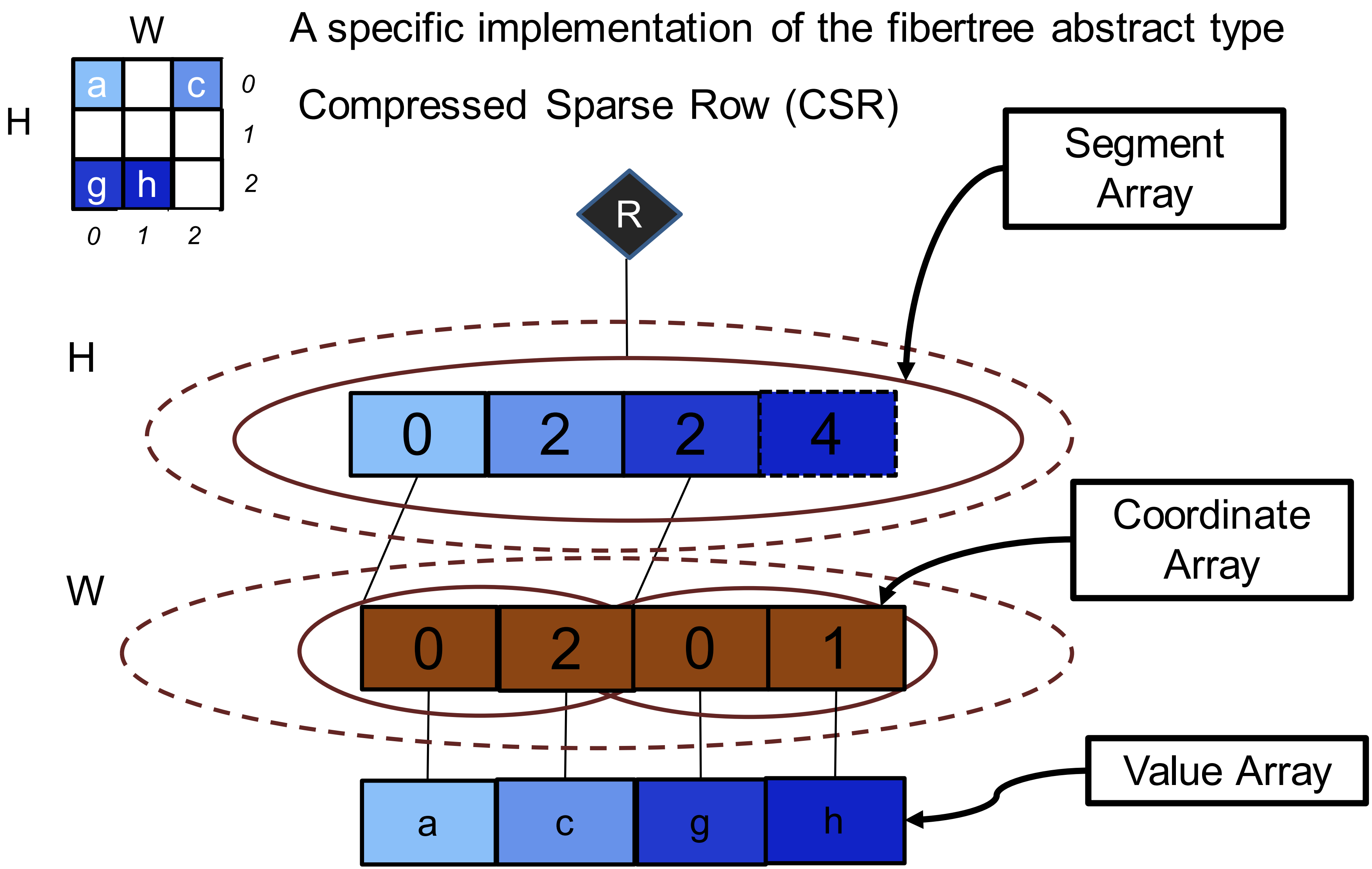
  - Uncompressed means size is proportional to maximum coordinate value
  - Compressed formats will have metadata overhead relative to uncompressed formats. For dense data, this may cost more than just using an uncompressed format.
  - Space efficiency of a representation depends on sparsity

*Note: sparsity/density is an attribute of the data.

# Uncompressed/Compressed Representation

A specific implementation of the fibertree abstract type

Compressed Sparse Row (CSR)



# Tensor Traversal (CSR Style)

```
# 2-D Tensor Traversal (CSR)

t_segs = Array(H)
t_coords = Array(W)
t_vals = Array(W)

sum = 0
for t_h_pos in [0,H):
  h = t_h_pos
  t_w_start = t_segs[t_h_pos]
  t_w_len = t_segs[t_h_pos+1]-t_w_start
  for t_w_pos in [t_w_start, t_w_len):
     h = t_coords[t_w_pos]
     t_val = t_vals[t_w_pos]
     sum += t_val
```

For uncompressed rank coordinate equals position

Coordinates not actually used in this example

# CONV: Exploiting Sparse Weights

# Hardware Sparse Acceleration Features

**Format:**
Choose tensor representations to save storage space and energy associated with zero accesses

**Gating:**
Explicitly eliminate ineffectual storage accesses and computes by letting the hardware unit staying idle for the cycle to save energy

**Skipping:**
Explicitly eliminate ineffectual storage accesses and computes by skipping the cycle to save energy and time

```
i = Array(W)          # Input activations
f = Tensor(S)         # Filter weights
o = Array(Q)          # Output activations

for (s, f_val) in f:
  for q in [0, Q):
    w = q + s
    o[q] += i[w] * f_val
```

# Weight Stationary - Sparse Weights

```
i = Array(W)          # Input activations
f = Tensor(S)         # Filter weights
o = Array(Q)          # Output activations

for (s, f_val) in f:
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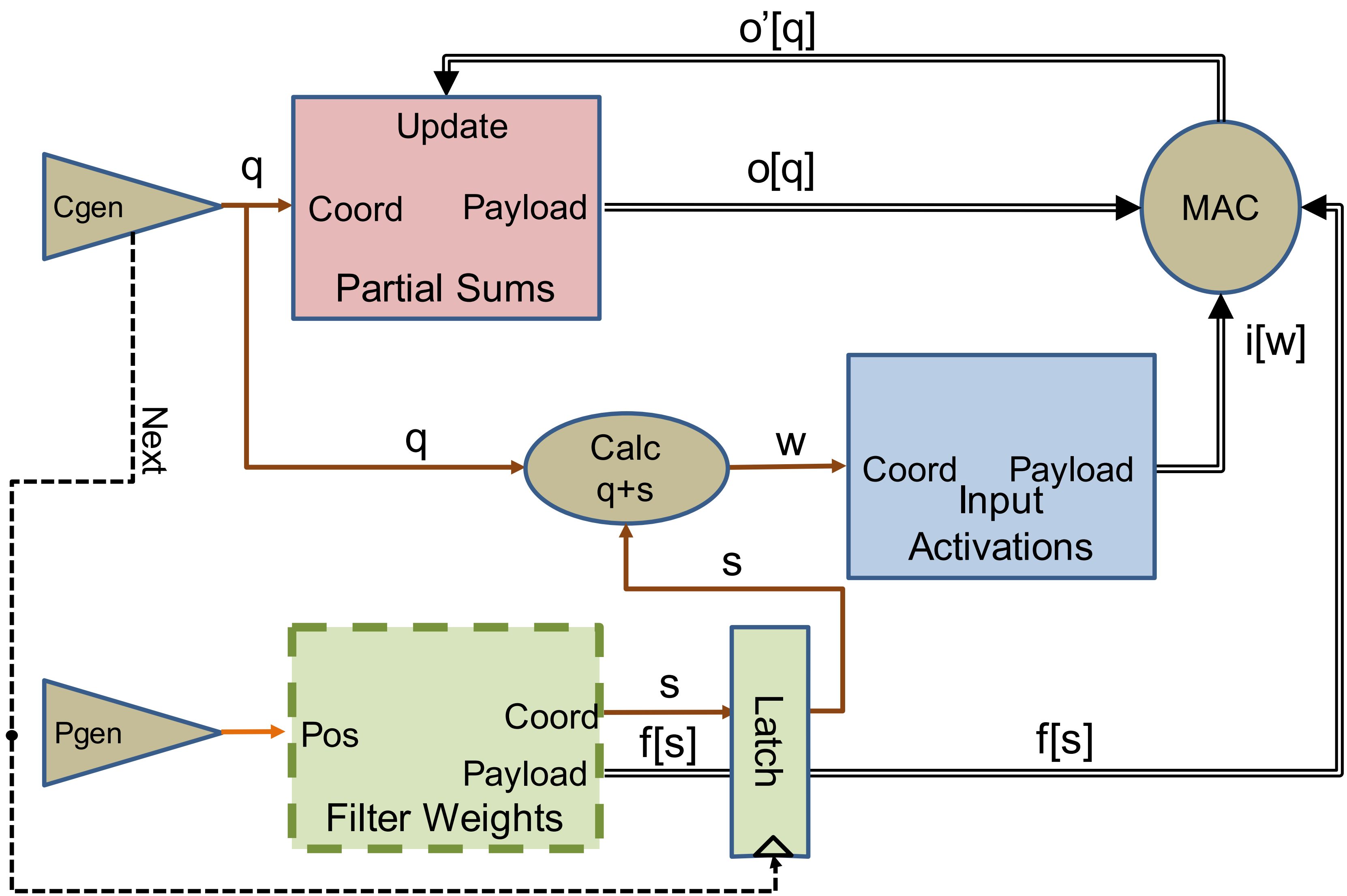
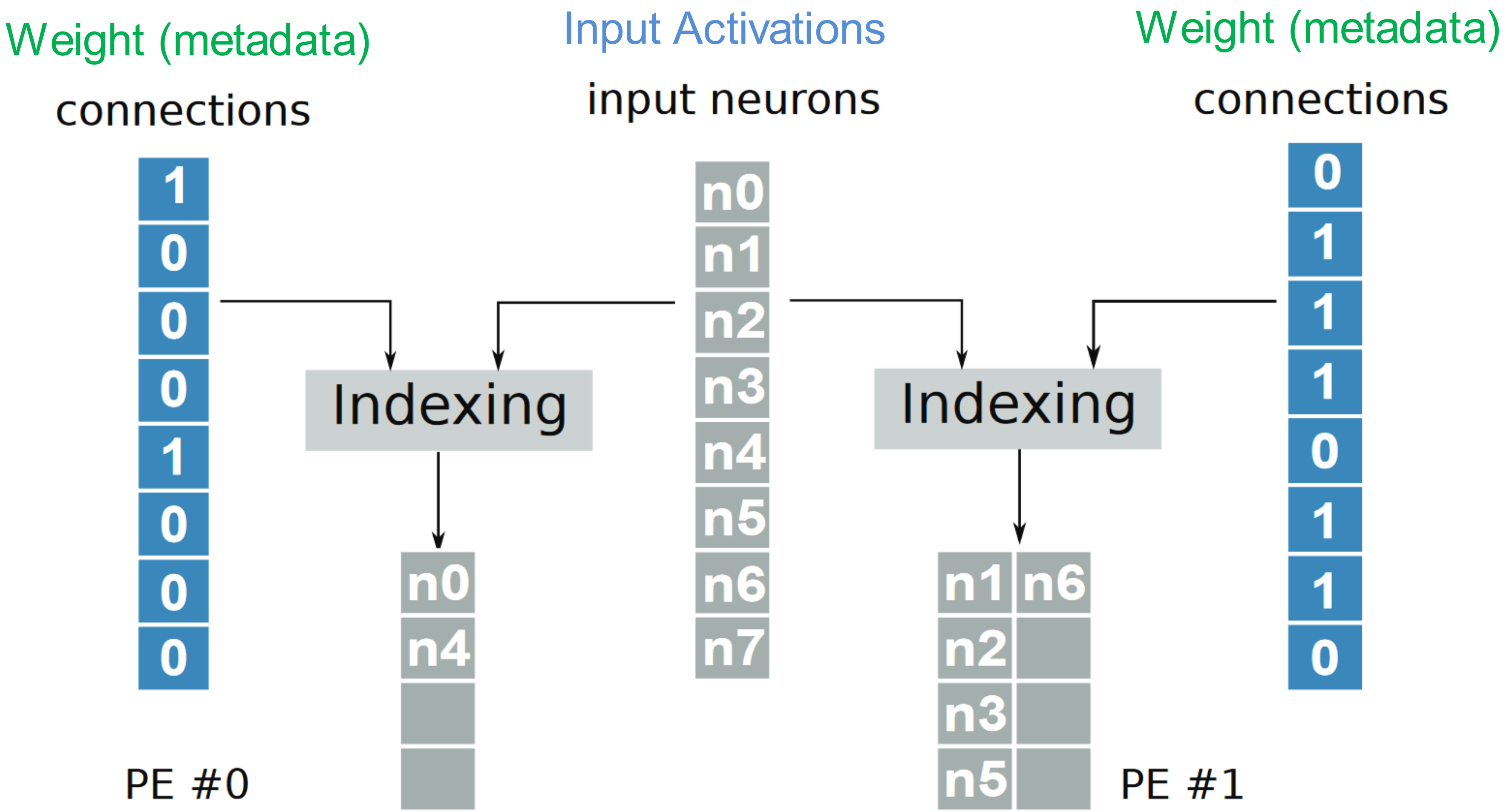
```

Concordant traversal

# Weight Stationary - Sparse Weights



# Cambricon-X – Activation Access




Cambricon-X – Zhang et.al., Micro 2016

# To Extend to Other Dimensions of DNN

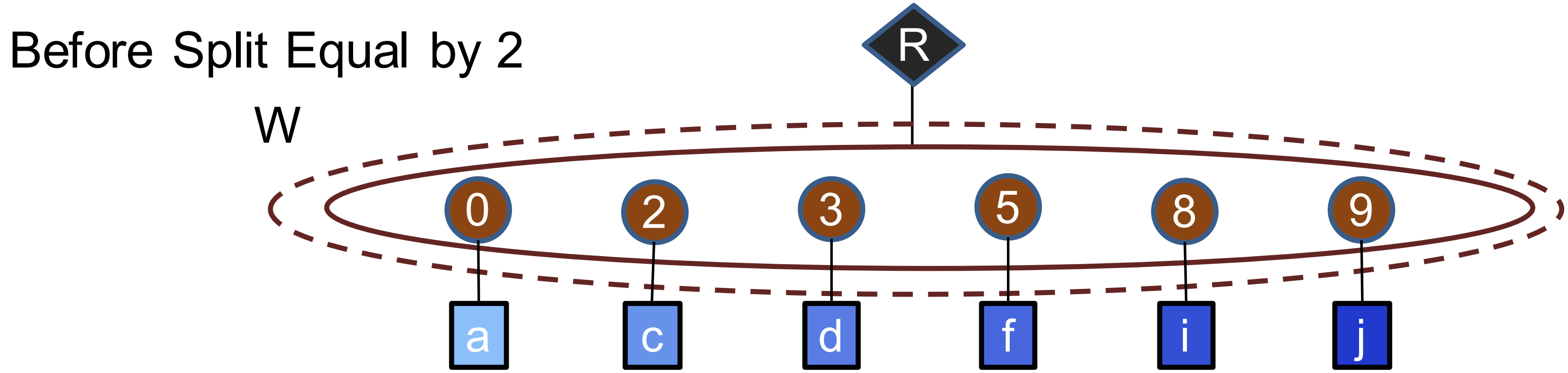
- Need to add loop nests for:
  - 2-D input activations and filters
  - Multiple input channels
  - Multiple output channels

# To Extend to Other Dimensions of DNN

- **Need to add loop nests for:**
  - 2-D input activations and filters
  - Multiple input channels
  - Multiple output channels

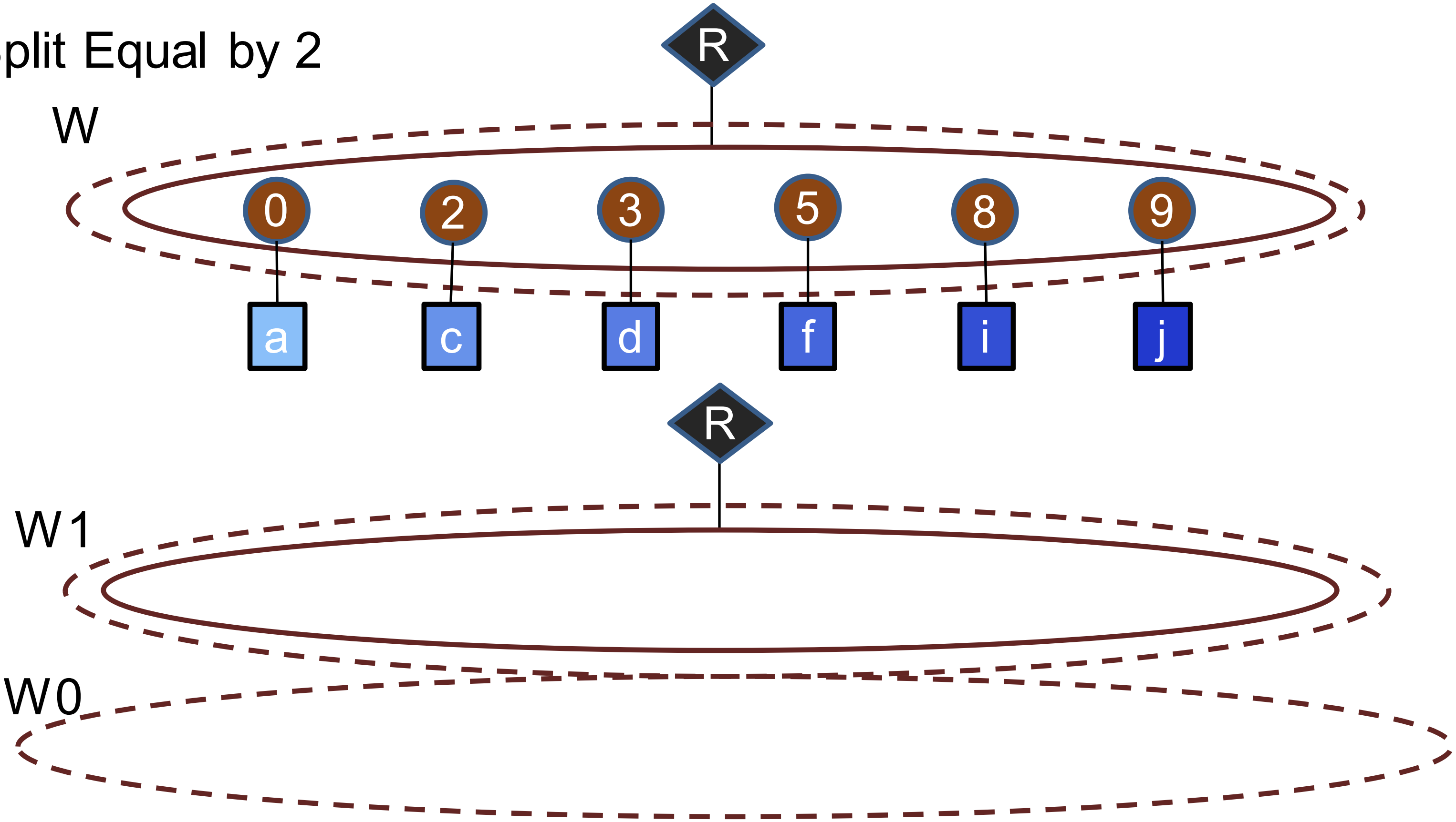
- **Add parallelism…**

# Fiber Splitting Equally in Position Space



# Fiber Splitting Equally in Position Space

Before Split Equal by 2



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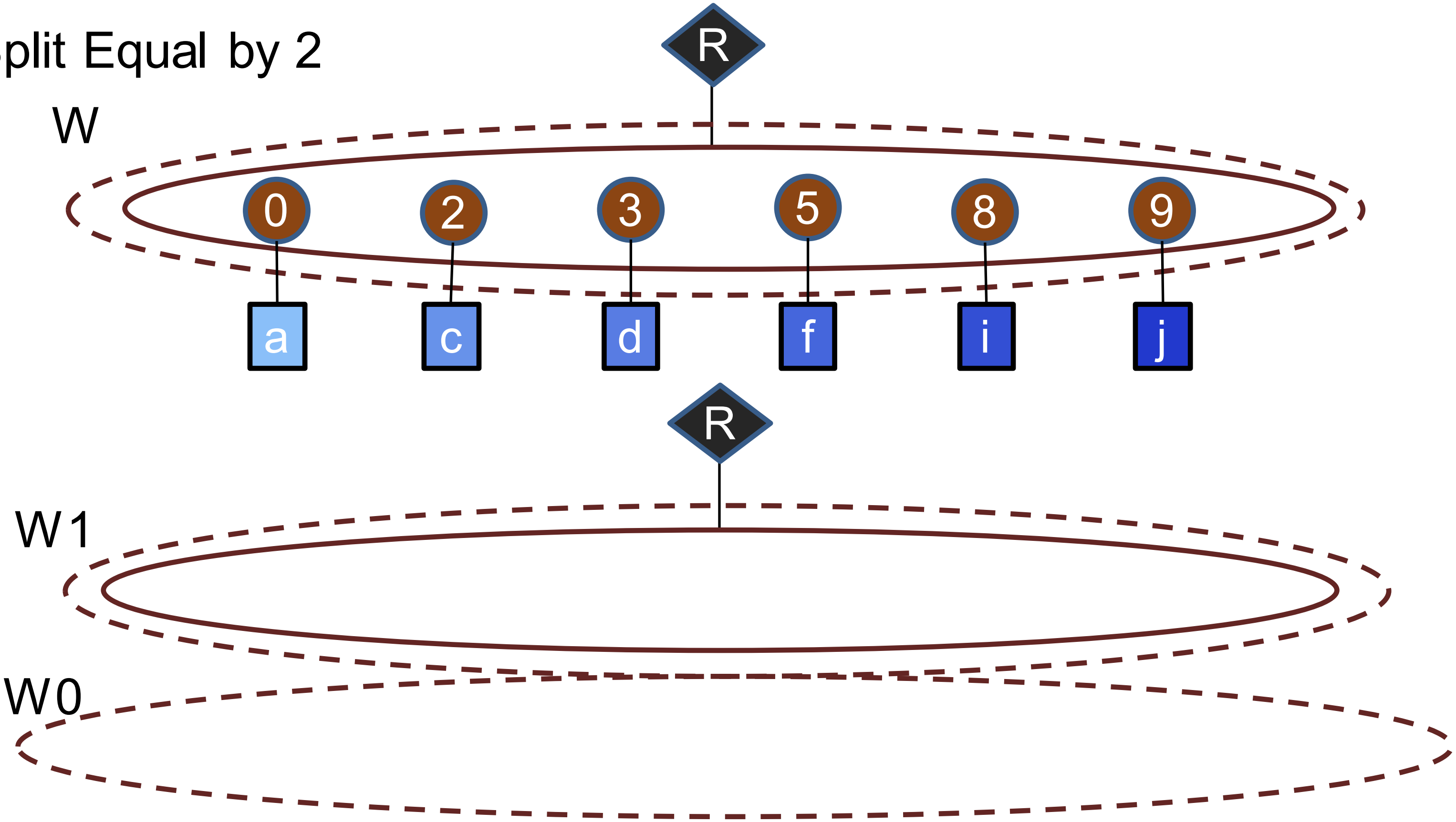
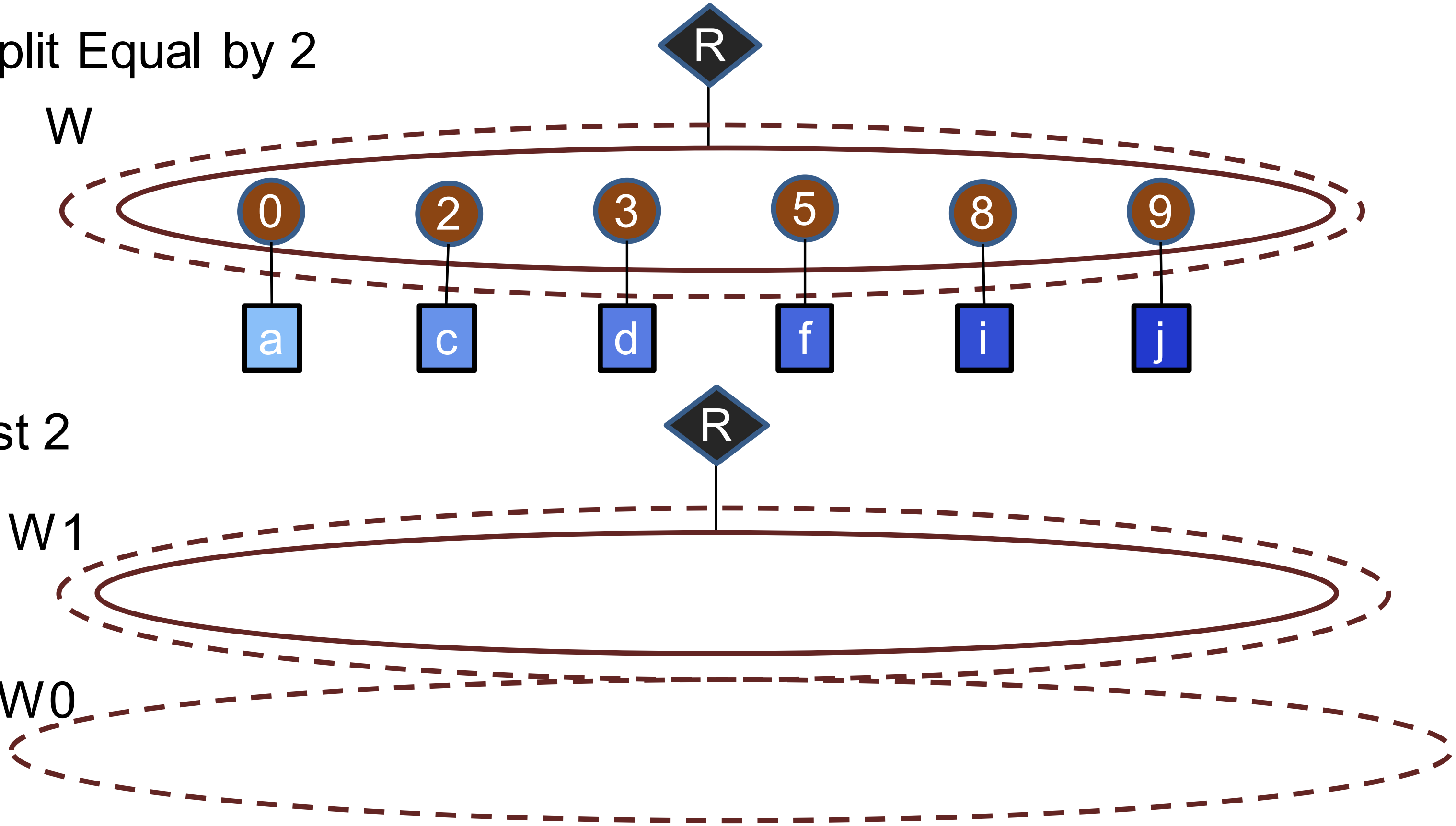


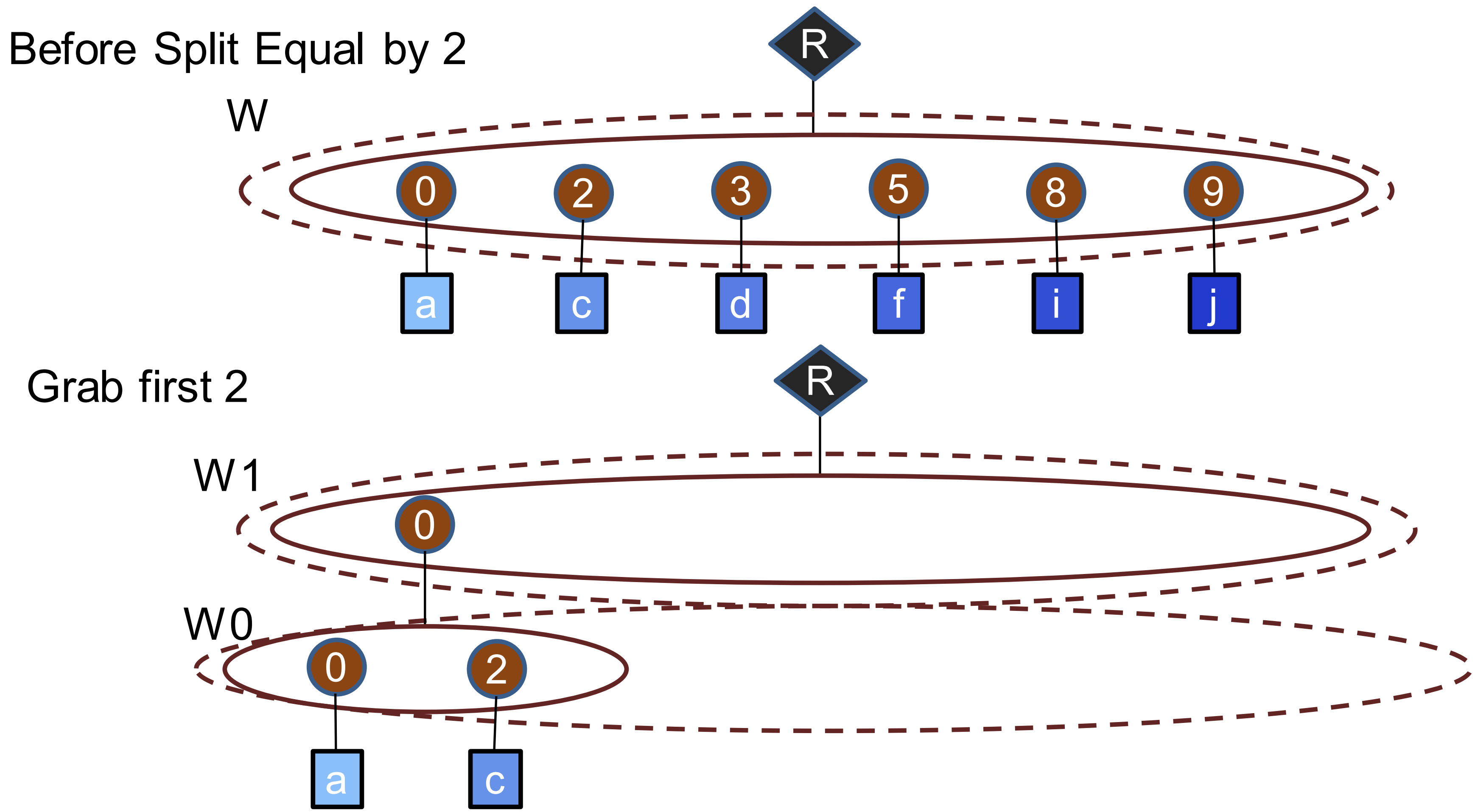
# Fiber Splitting Equally in Position Space

Before Split Equal by 2

Grab first 2



# Fiber Splitting Equally in Position Space



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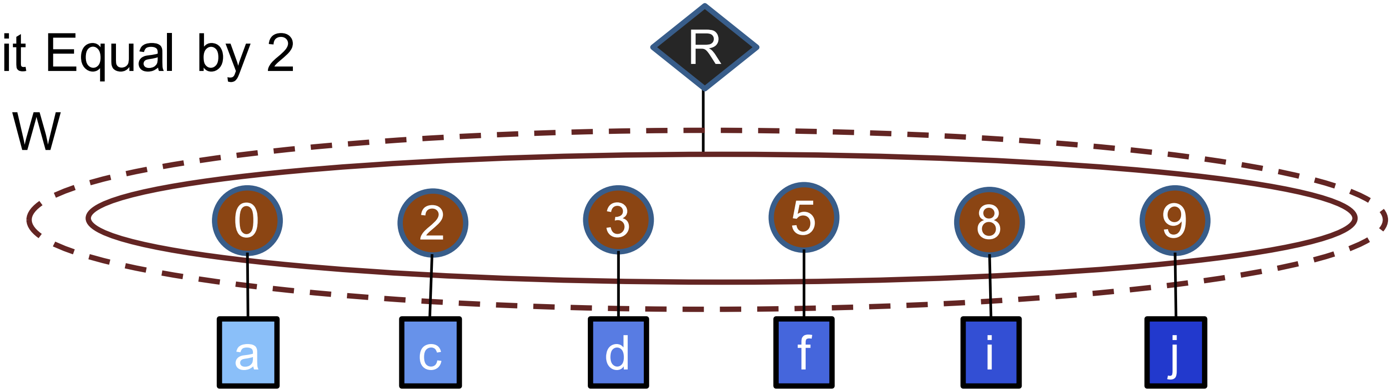
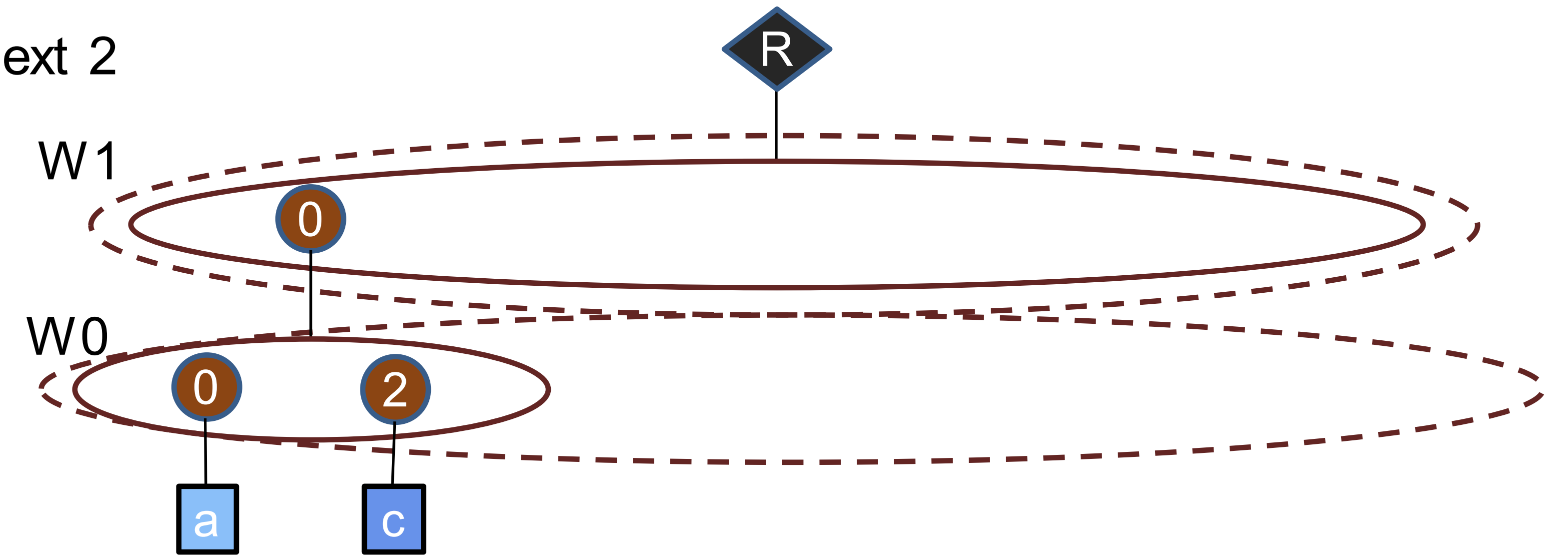



Grab next 2



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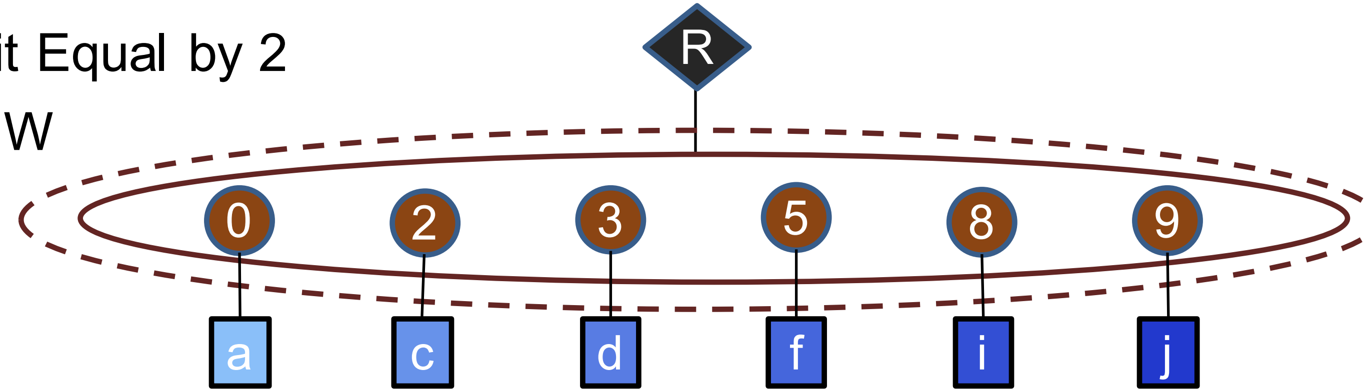
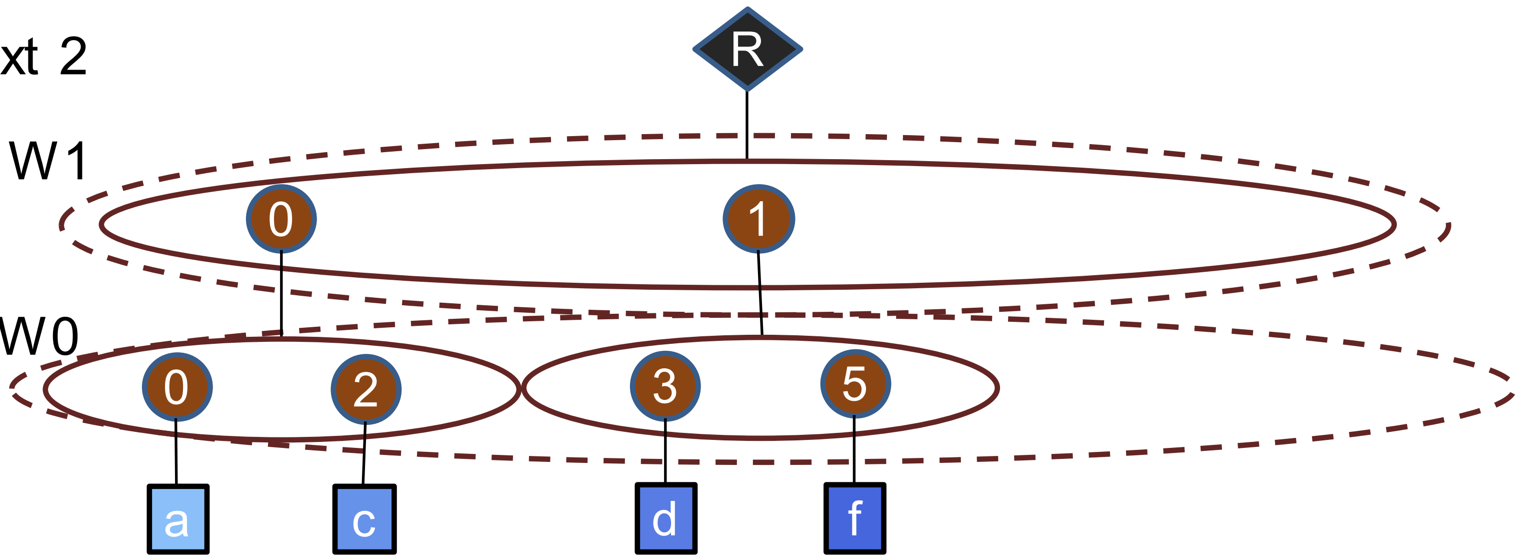



Grab next 2



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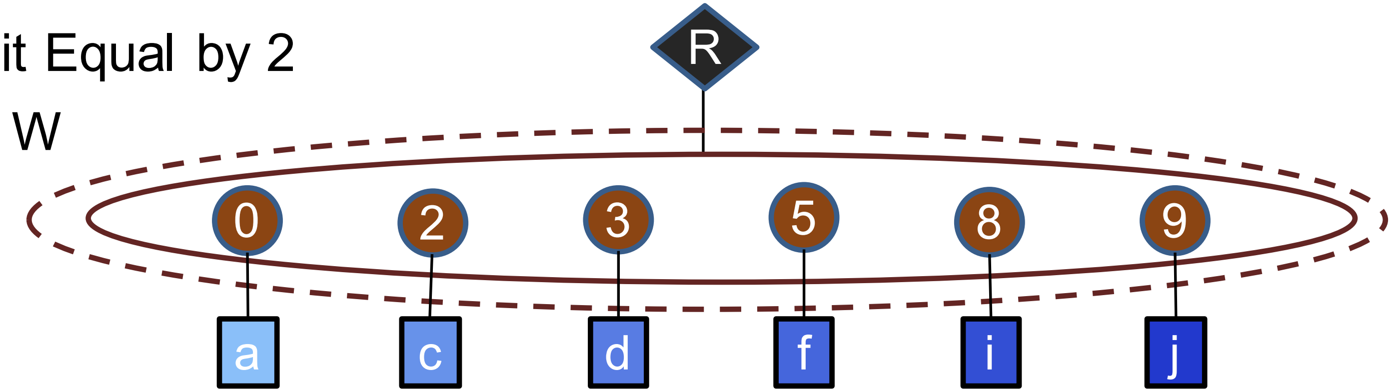
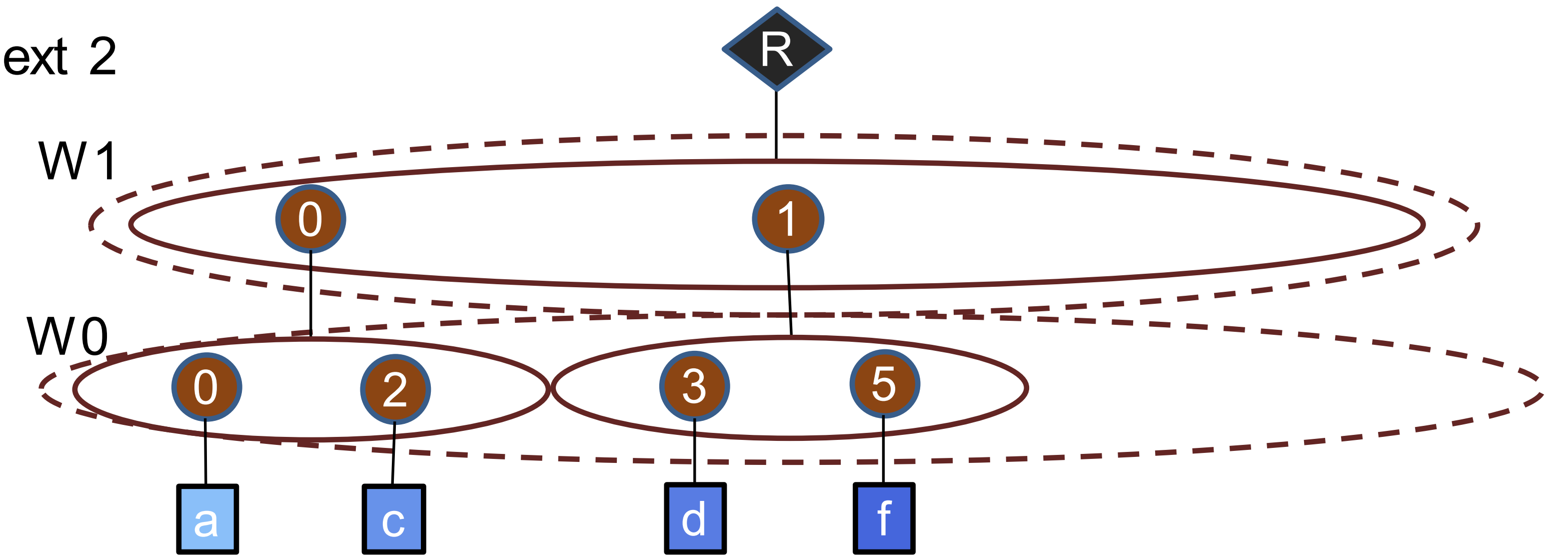



Grab next 2



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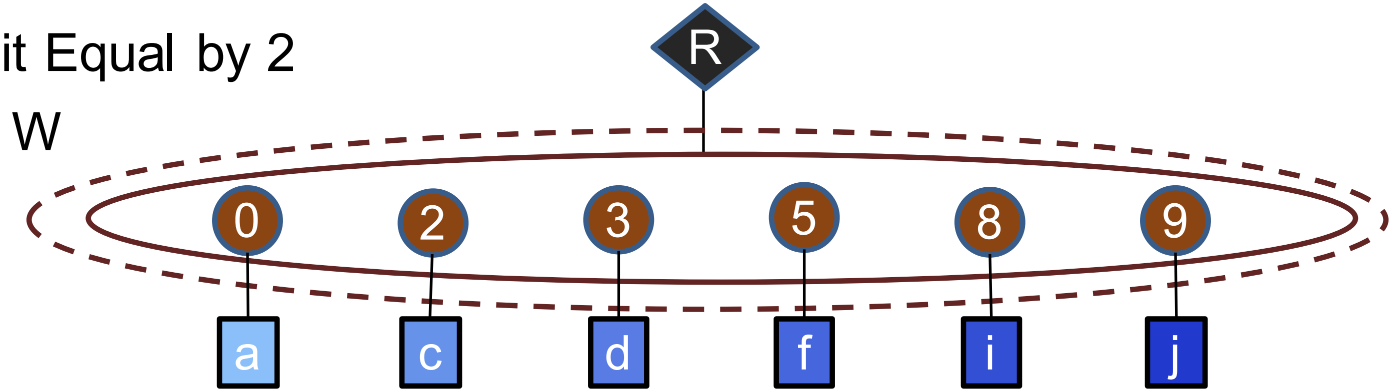


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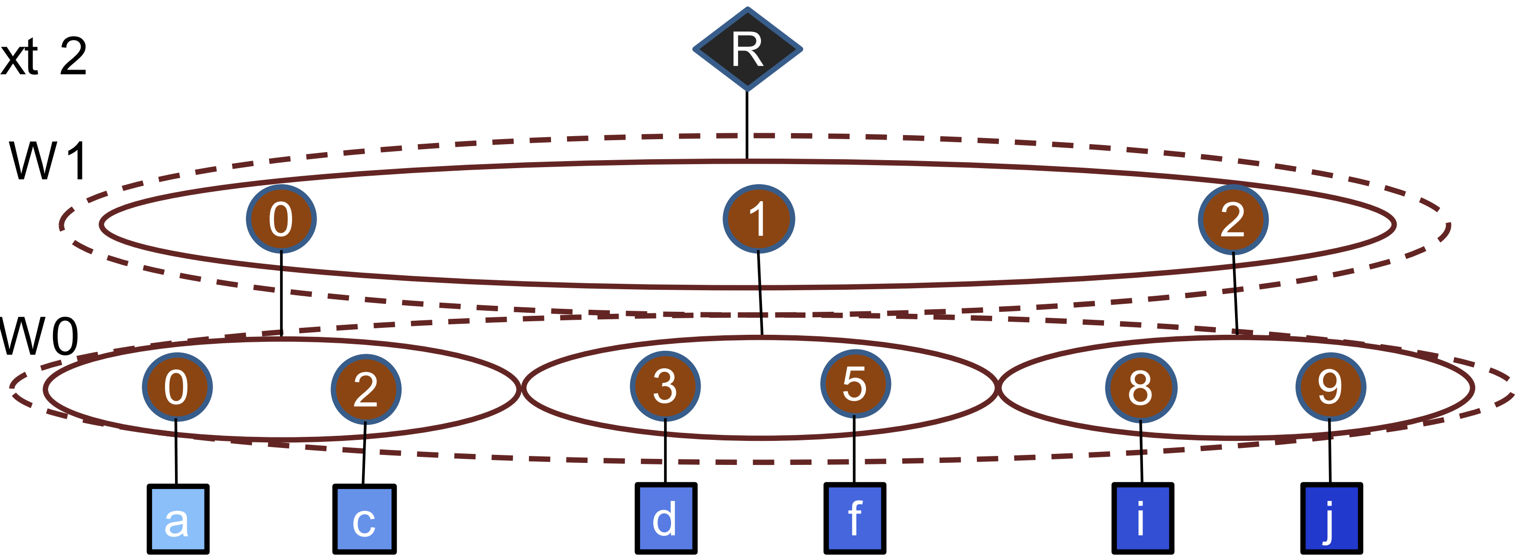

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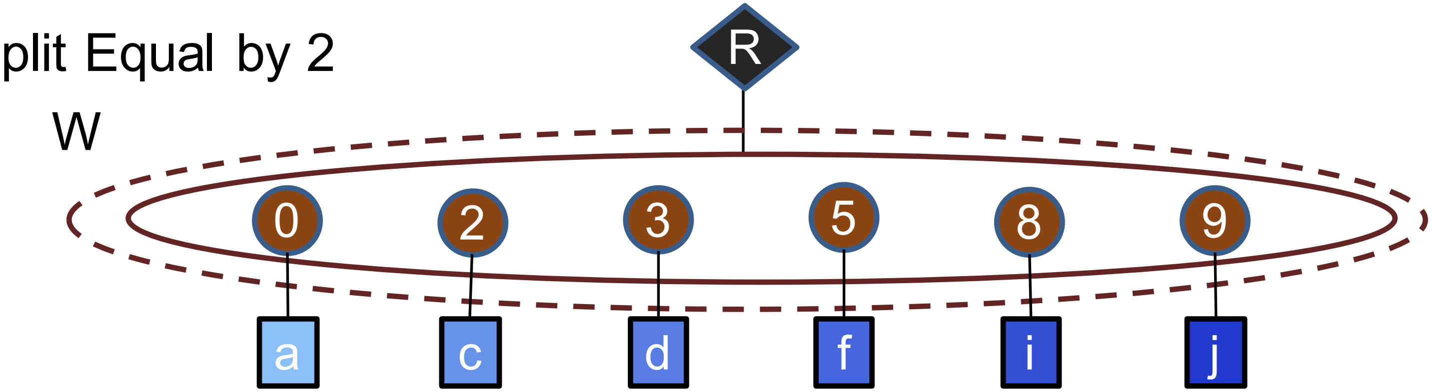


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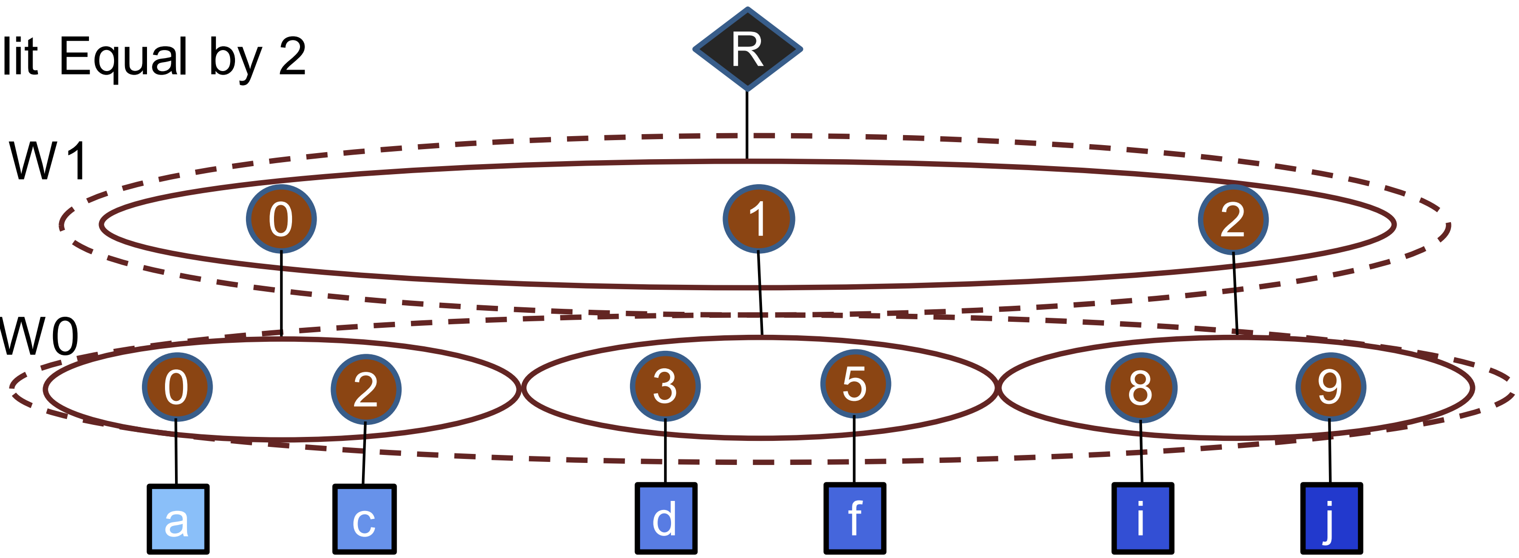


# Parallel Weight Stationary - Sparse Weights

```
i = Array(W)          # Input activations
f = Tensor(S)         # Filter weights
o = Array(Q)          # Output activations

for (s1, f_split) in f.splitEqual(2):
  for q1 in [0, Q/4):
     parallel-for (s0, f_val) in f_split:
       parallel-for q0 in [0, 4):
         q = q1*4 + q0
         w = q + s
         o[q] += i[w] * f_val
```

Get groups of two weights

Work on two weights in parallel

Work on four outputs at once

Calculate coordinates

Accumulate multiple outputs each spatially

Look up input activation

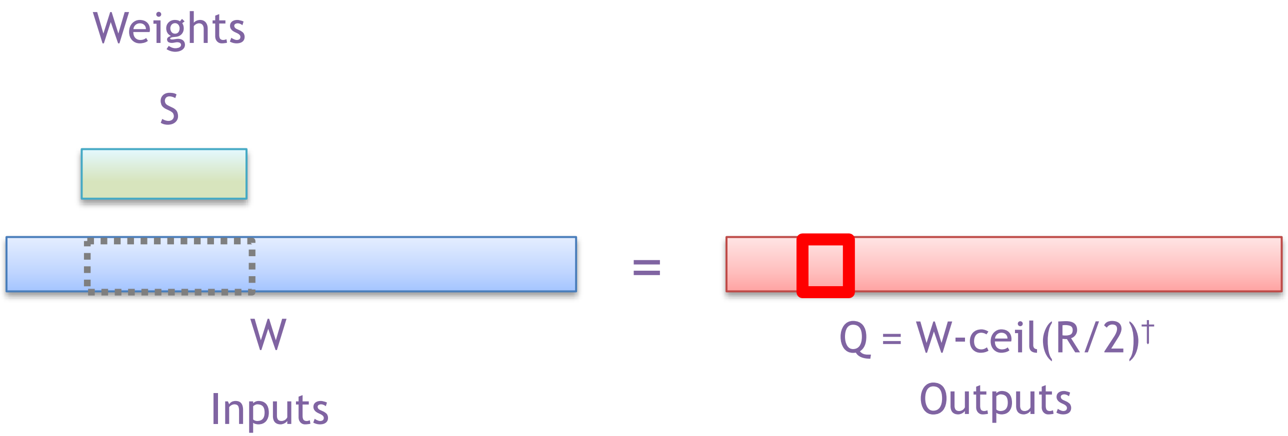
# CONV: Exploiting Sparse Inputs & Sparse Weights

# Output Stationary - Sparse Weights & Inputs

```
i = Tensor(W)          # Input activations
f = Tensor(S)          # Filter weights
o = Array(Q)           # Output activations

for q in [0,Q):
  for (s, (f_val, i_val)) in f.project(+q) & i:
    o[q] += i_val * f_val
```

# Fiber Coordinate Projection




fiber-projection

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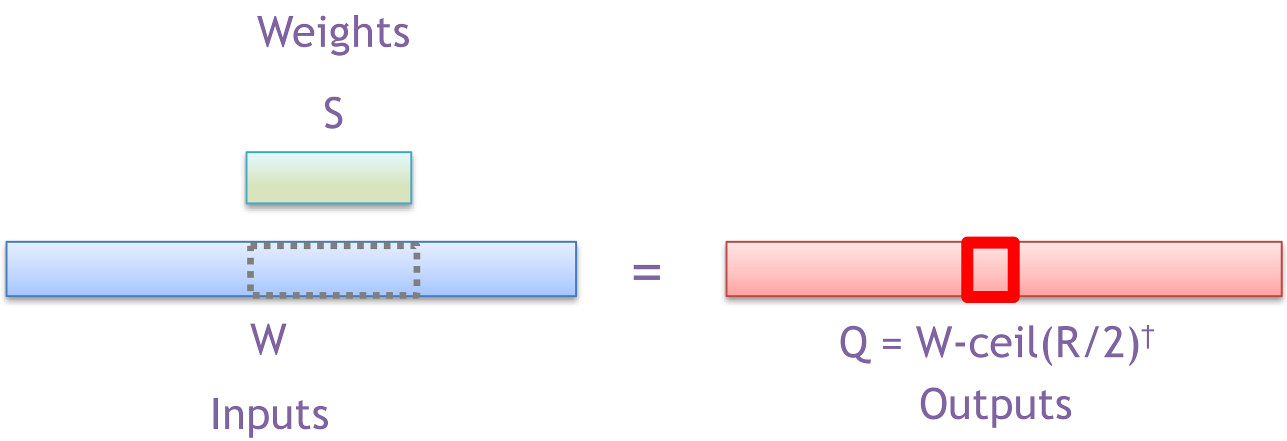


fiber-projection

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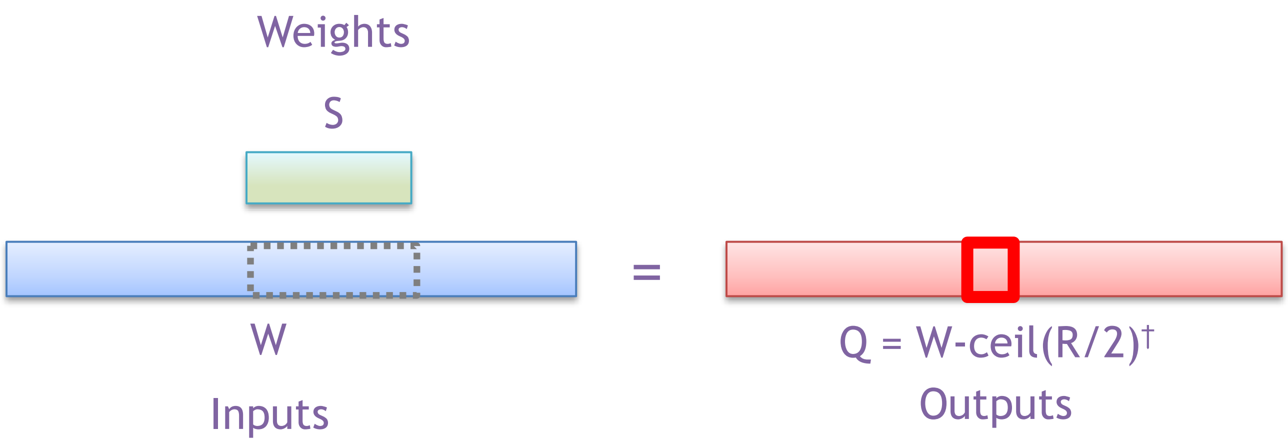


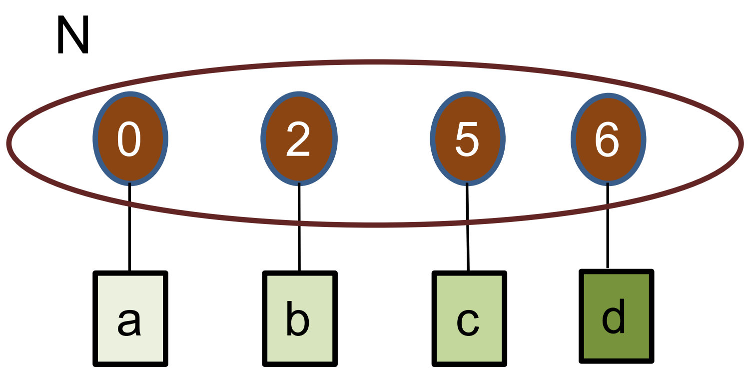


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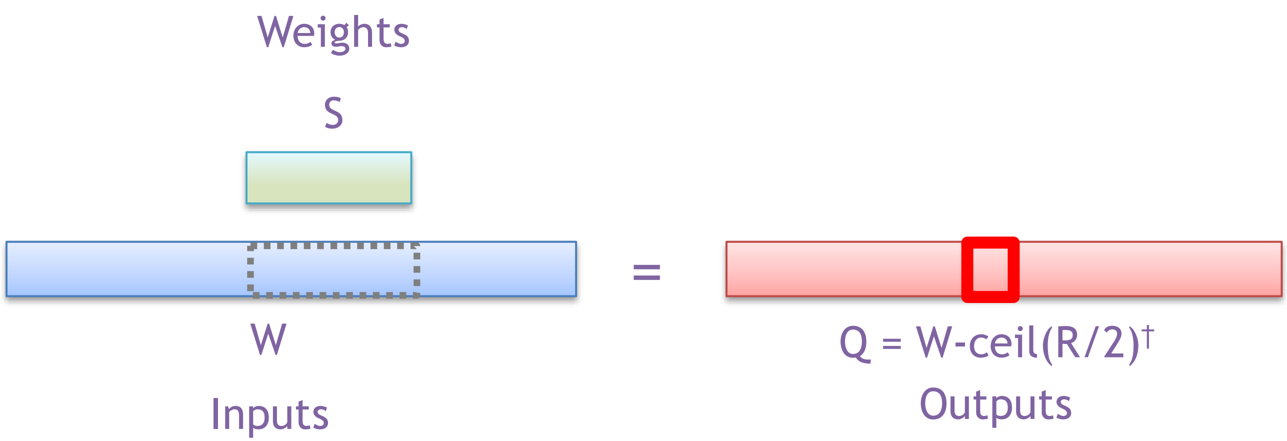
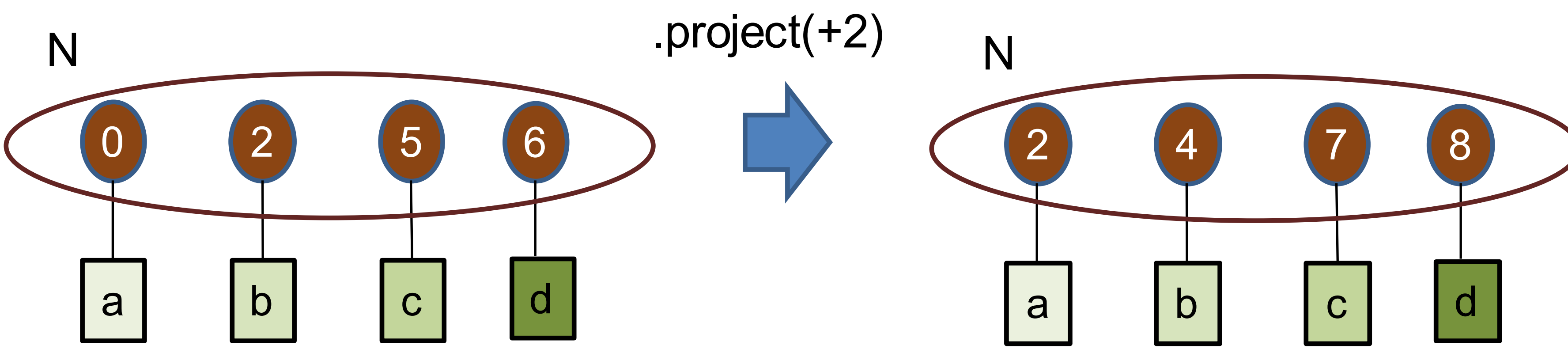
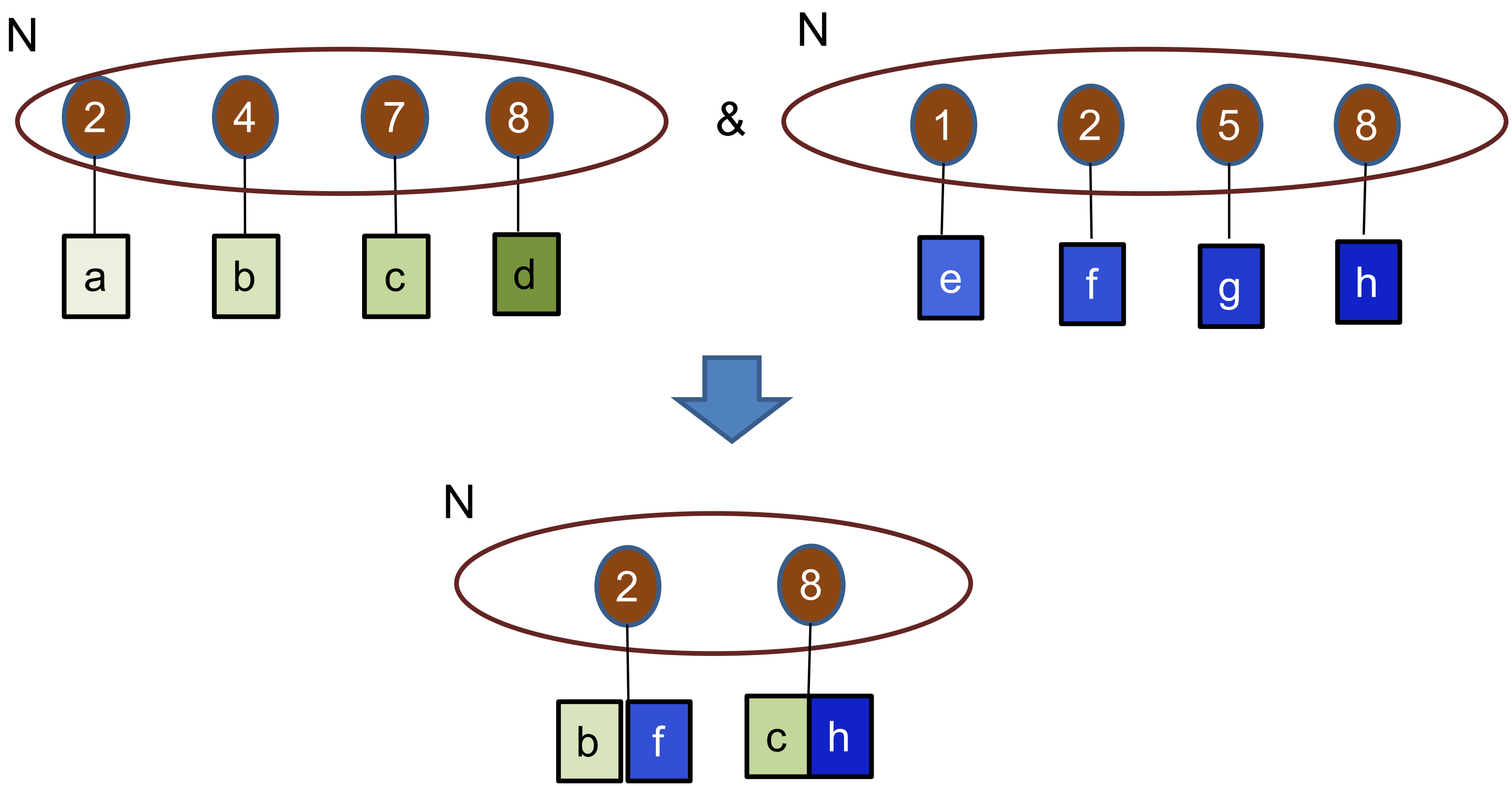


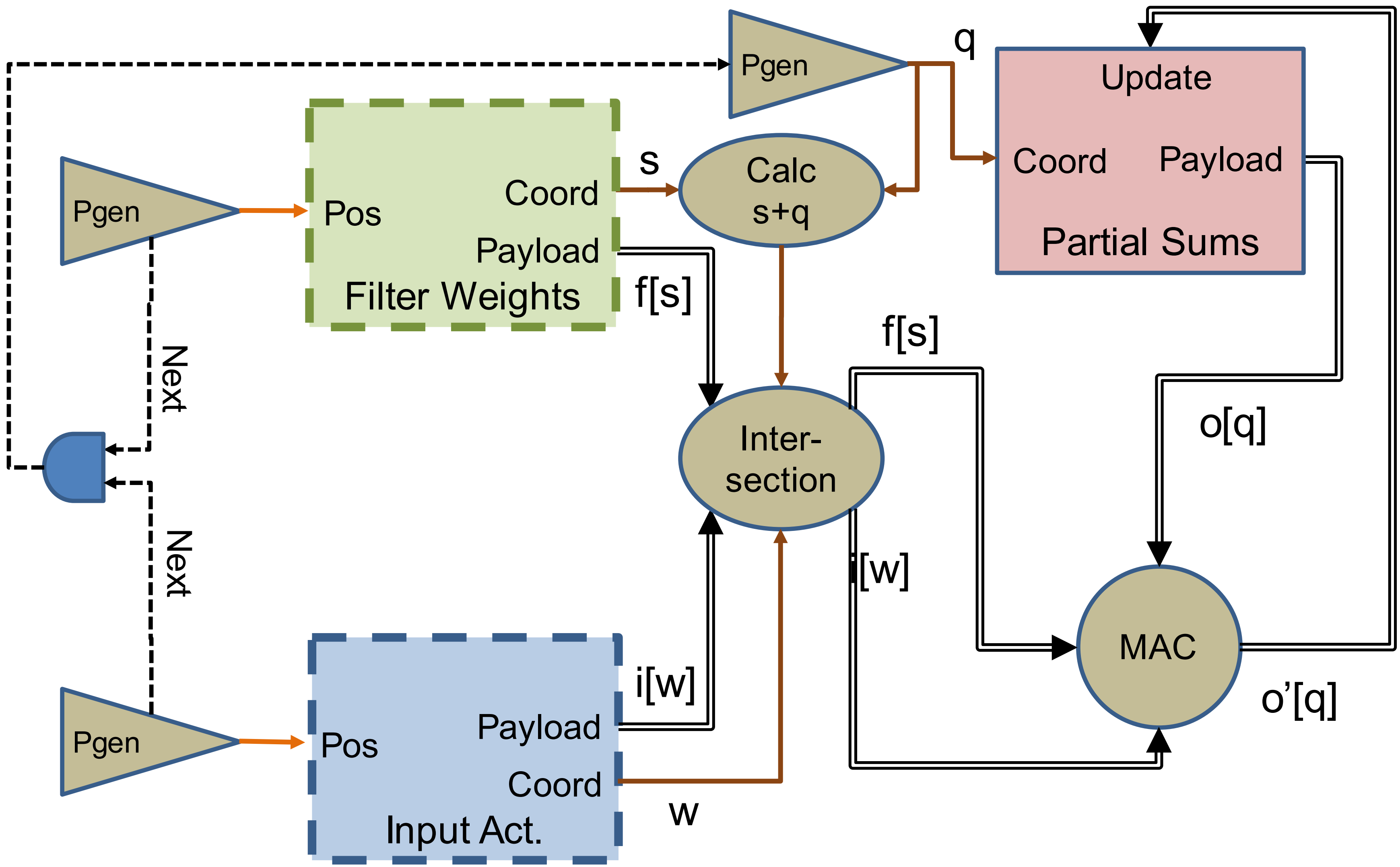




fiber-projection

# Fiber Intersection

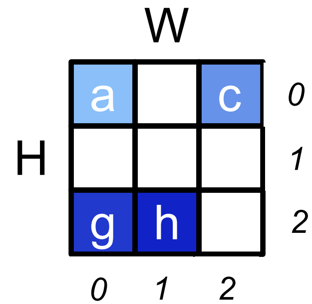


# Output Stationary - Sparse Weights & Inputs

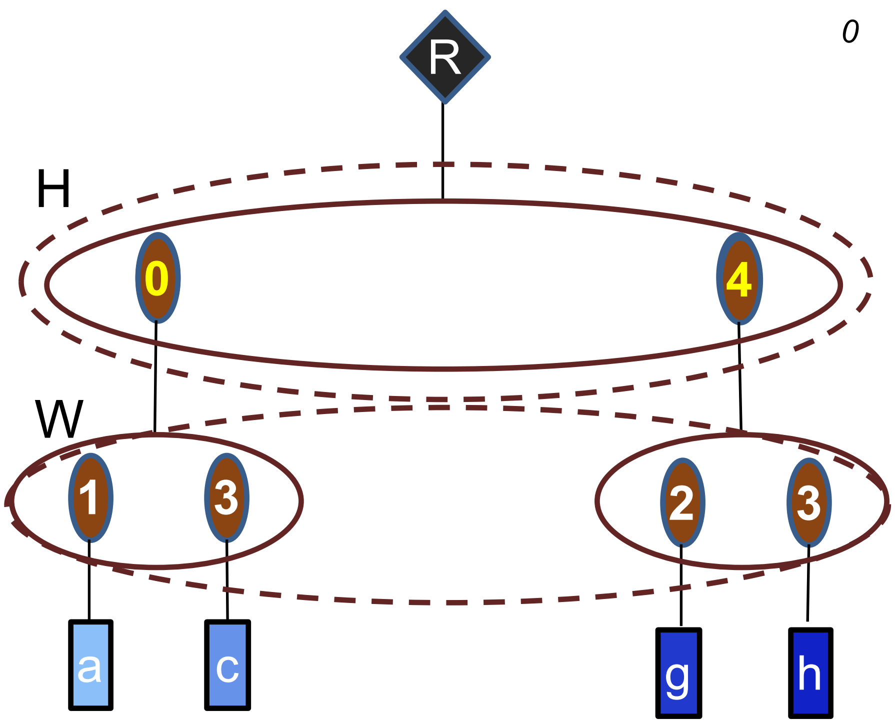


# Flattening Ranks

For efficiency one can form new representations where the data structure for two or more ranks are combined.




Tensor-<CC>(HW)



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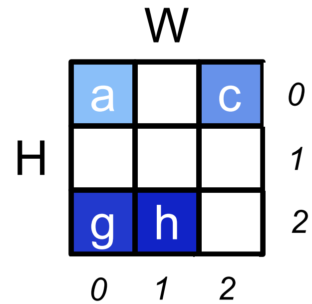


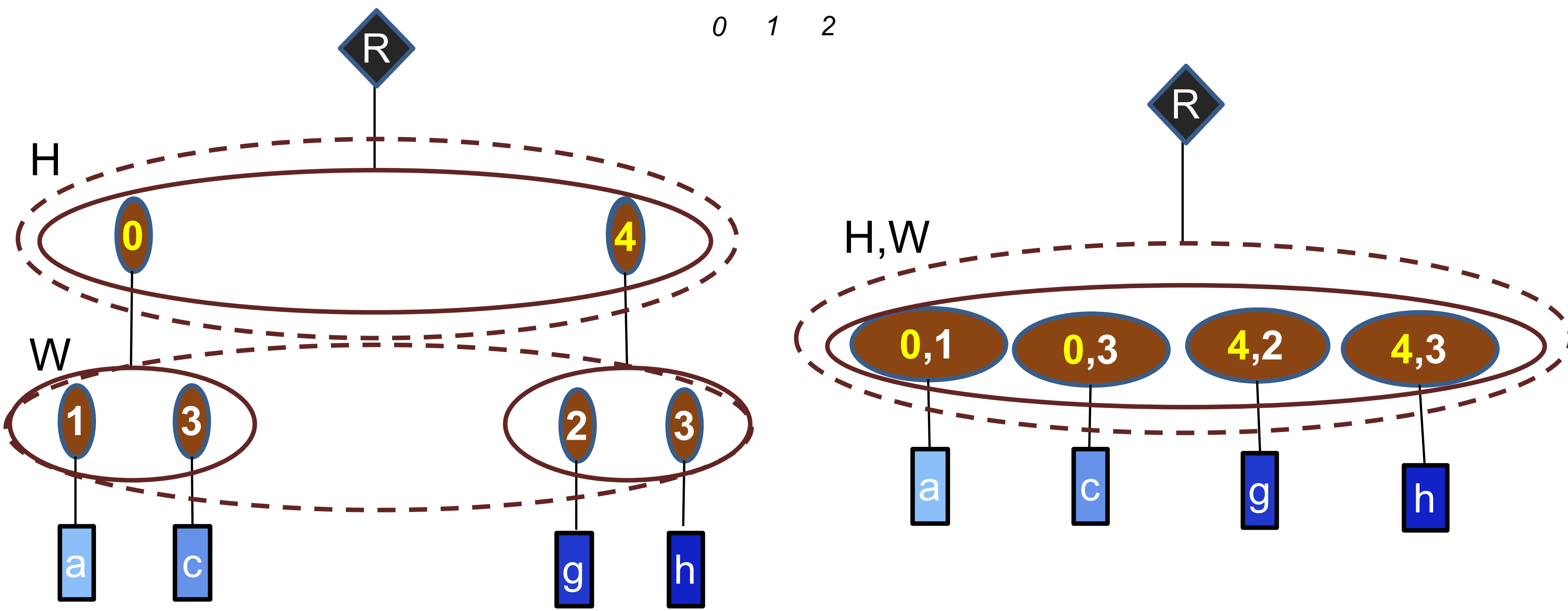

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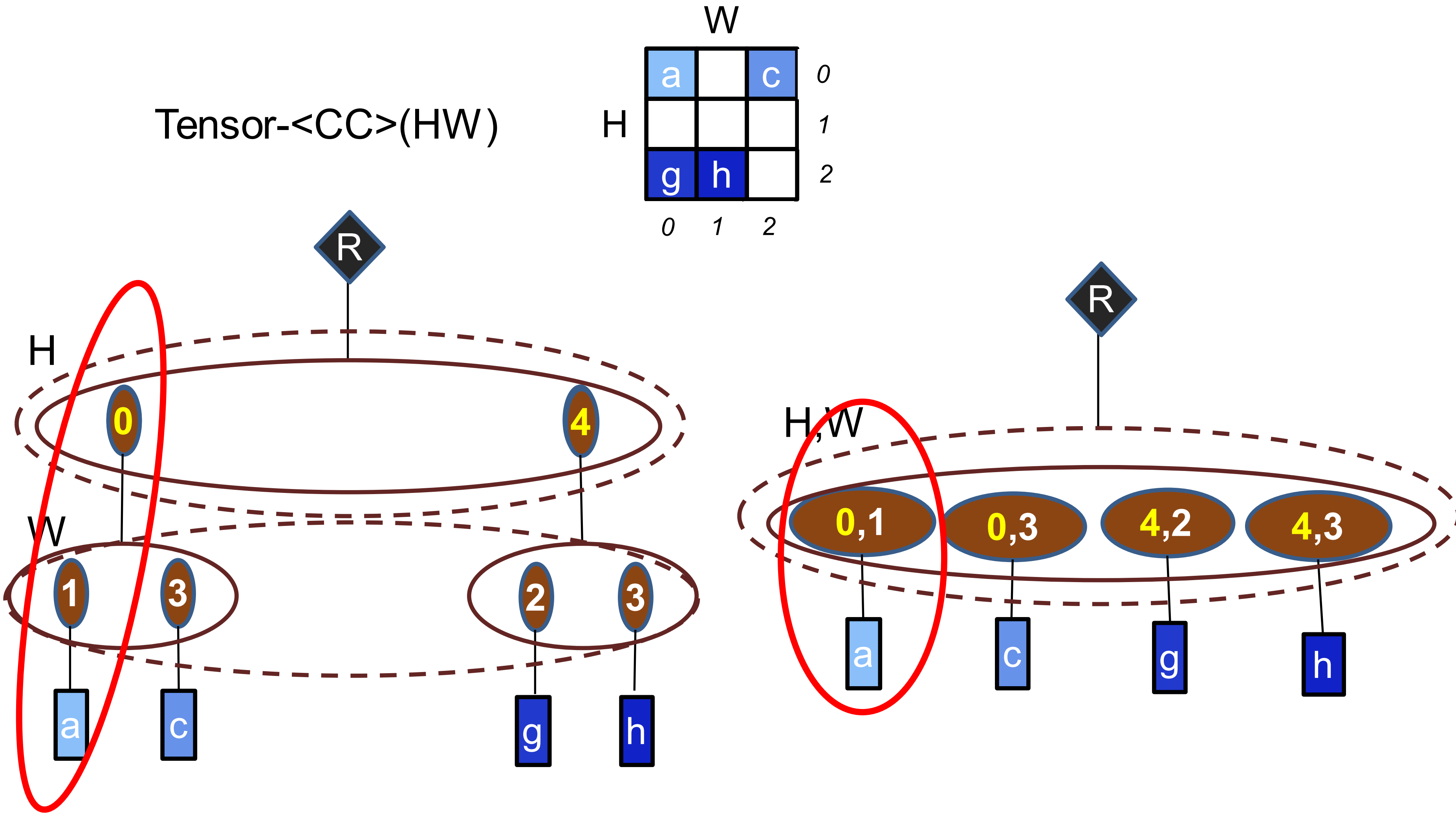

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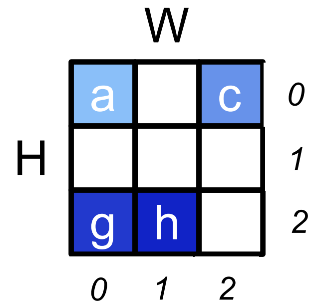


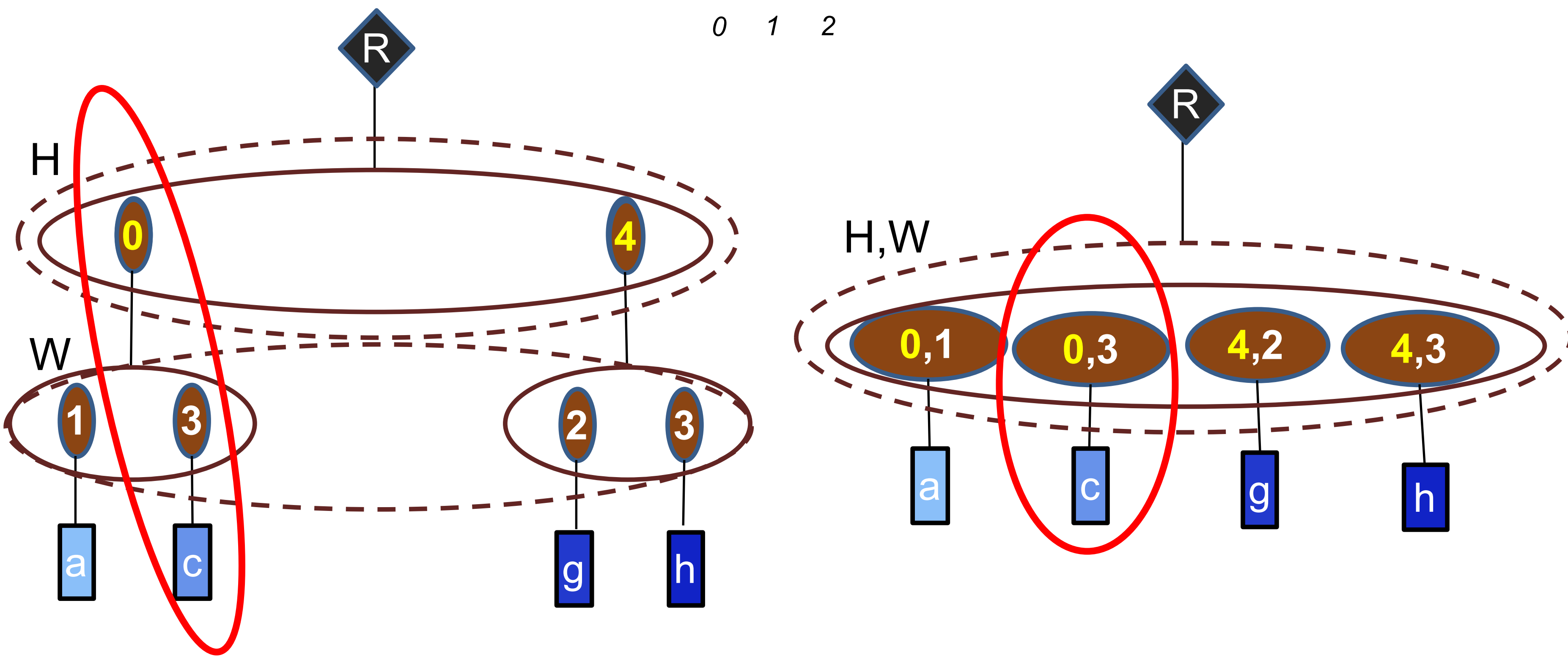

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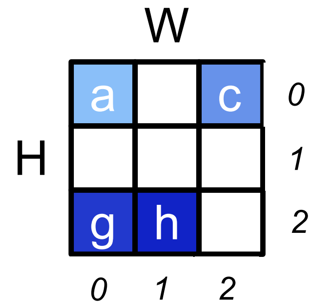


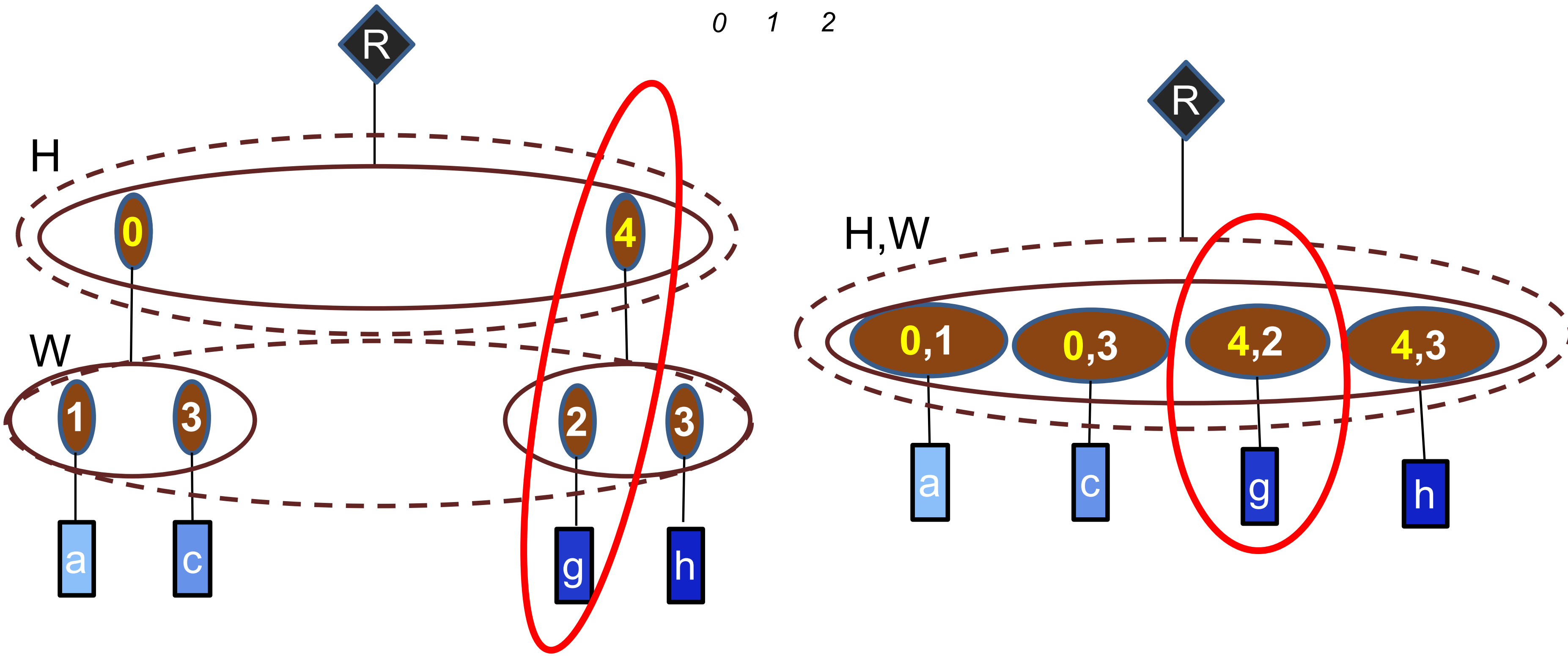

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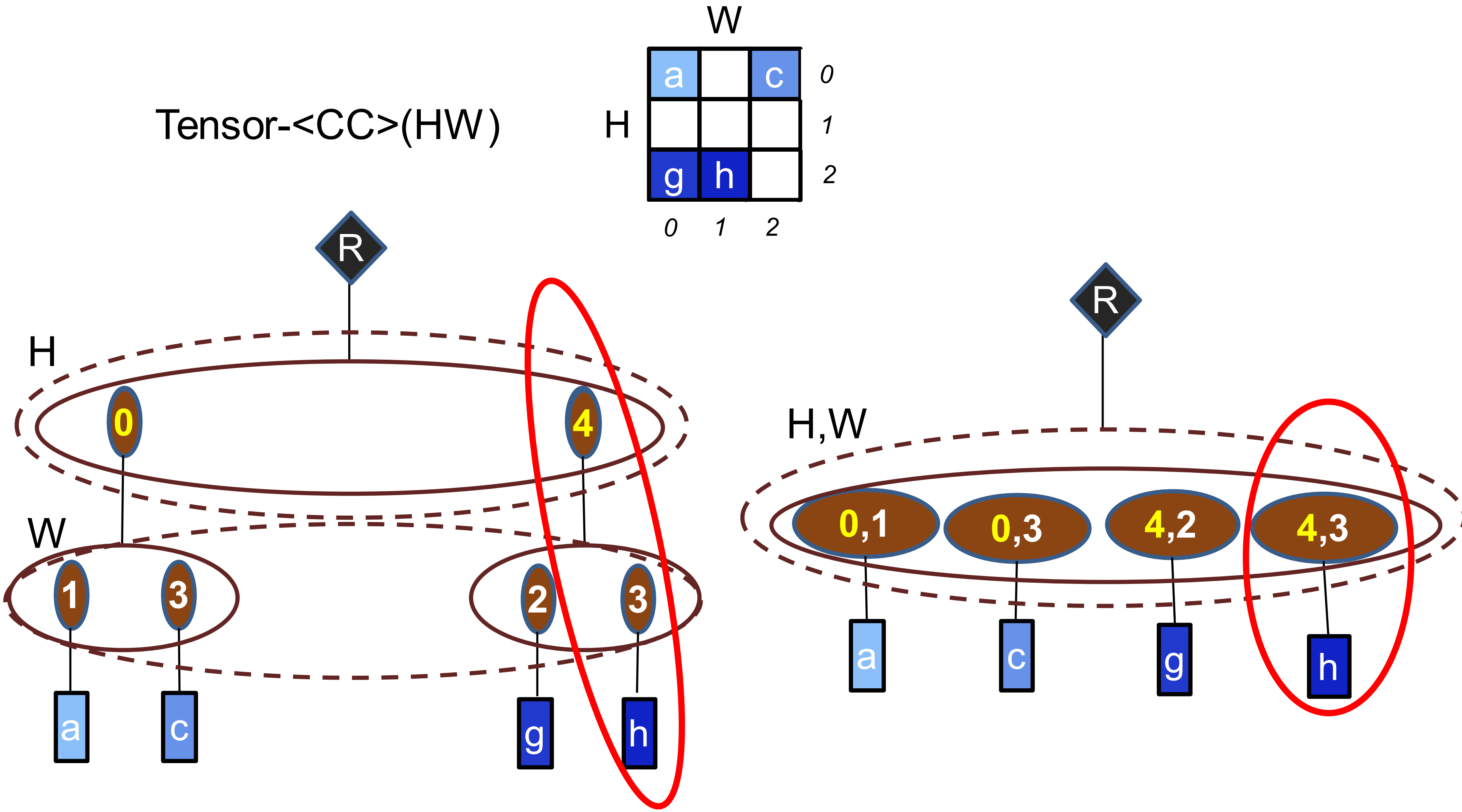

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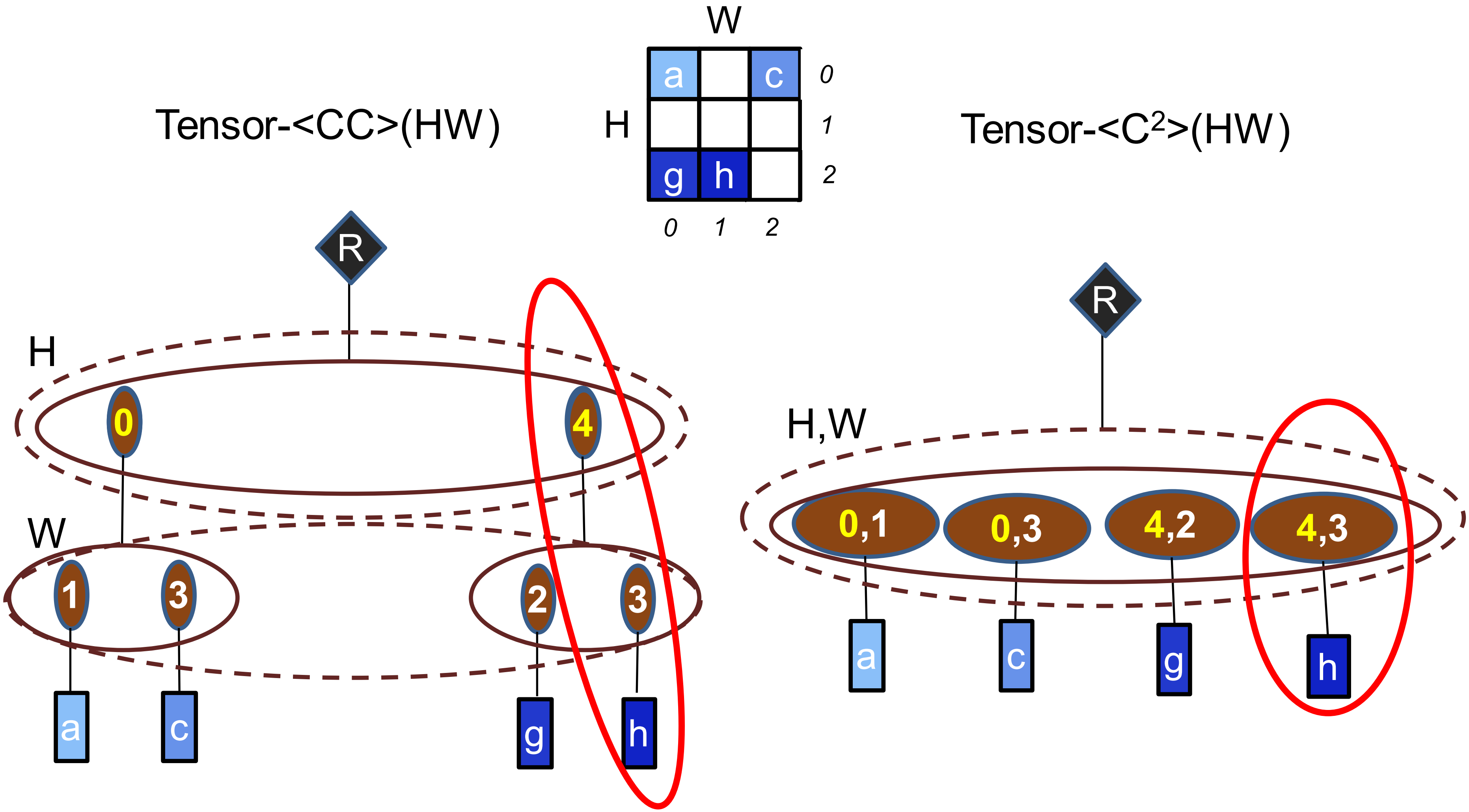


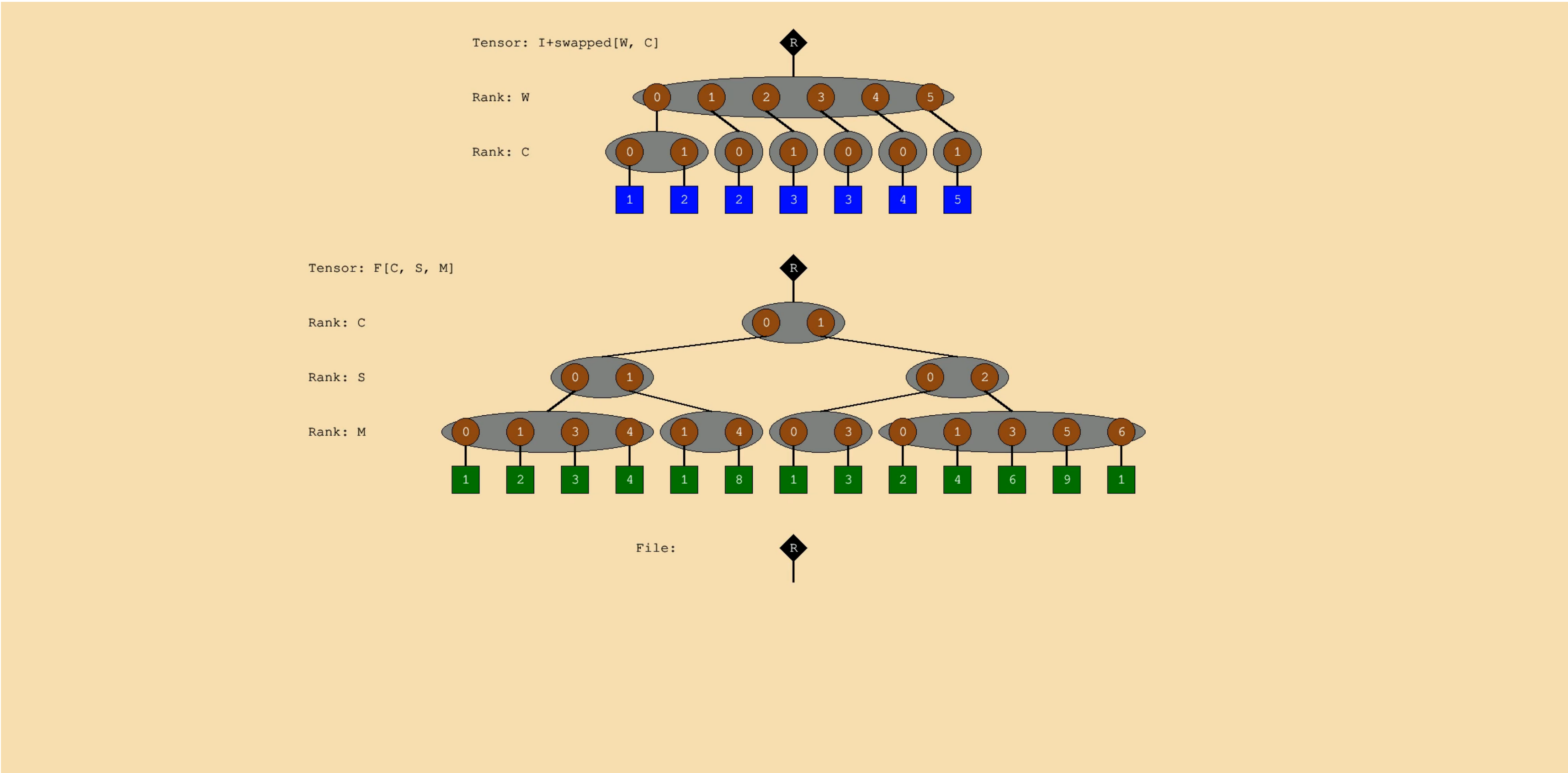
# Row Stationary – Sparse Inputs & Activations

```
i = Tensor(CW)          # Input activations (CW flattened)
f = Tensor(C,SM)        # Filter weights (SM flattened)
o = Array(M, Q)         # Output activations

for ((c, w), i_val) in i:
  f_c = f.getPayload(c)
  f_c_split = f_c.splitEven(2)
  parallel-for (_, f_sm) in f_c_split:
    for ((s, m), f_val) in f_sm if w-Q <= s < w:
      q = w - s
      o[m, q] += i_val * f_val
```

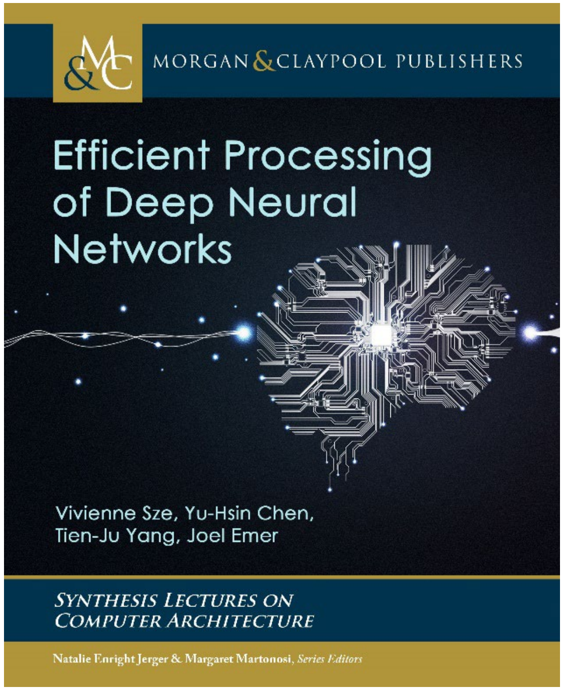
Eyeriss V2 – Chen et.al., JETCAS 2018

# Row Stationary – Sparse Inputs & Activations




Eyeriss V2 – Chen et.al., JETCAS 2018

# Hardware Architecture for Deep Learning




***Part I Understanding Deep Neural Networks***
*Introduction*
*Overview of Deep Neural Networks*

***Part II Design of Hardware for Processing DNNs***
*Key Metrics and Design Objectives*
*Kernel Computation*
*Designing DNN Accelerators*
*Operation Mapping on Specialized Hardware*

***Part III Co-Design of DNN Hardware and Algorithms***
*Reducing Precision*
*Exploiting Sparsity*
*Designing Efficient DNN Models*
*Advanced Technologies*

6.593[01] – Coming Spring 2023

*Thank you!*

*Next Lecture:*
*Virtualization and Security*